3-D linear shot-source modeling

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Abstract. Numerical methods using the linear shot sources are described for the simulation of 3-D wave phenomena with application to the modeling of seismic data. In the method, time stepping is performed with a second-order differencing operator. The use of linear source in modeling can efficiently provide data for testing some schemes that deal with the areal shot-records. The implementation of the methods is demonstrated on numerical examples.

Introduction

3-D forward modeling represents a challenge for computer technology. Current 3-D modeling algorithms [1-3] can not efficiently provide data for testing purposes. For example, it is hard to provide all point shot-records for testing areal shot-record migration for intolerable computation time. The use of linear shot-source is illuminated by the areal shot-record migration plus that in the residual migration velocity analysis [4] either of the pair of ray parameters is fixed. Then many computations are reduced and nearly a magnitude of computations is saved. Meanwhile, the controlled illumination technique can be used directly.

In some cases, the synthesized areal shot-records are required to evaluate some seismic data processing schemes. Instead of computing many shot-records and synthesizing them into the areal shot-records, we can compute the records with liner shot sources and then synthesize the records into the areal shot-records. . The utilization of linear source is based on the linearity property of wave equation. An enormous data reduction can be obtained with the idea. In this paper, the rightness of the linear shot source modeling is given first and then the results are shown.

Linear shot source modeling

When the density is constant, the acoustic wave equation reads

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}+s=\frac{1}{v^2}\frac{\partial^2 p}{\partial t^2}, \tag{1}$$

where $p=p(x,y,z,t)$ represents pressure, $v(x,y,z)$ represents the velocity and $s(x,y,z,t)$ represents the source item which equals the divergence of the body forces[5]. The operator L is defined as $L=\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 p}{\partial t^2}$. In frequency domain, L is a Helmholtze operator:

$$L=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\omega^2}{v^2}, \tag{2}$$

where ω denotes angular frequency.

Assume there are $nshot$ records p^k and sourcs s^k satisfying $Lp^k=s^k,(k=1,2,...,nshot)$, it is easy to testify L has the following properties：

(1) Linearity

$$L\sum_{k=1}^{nshot} a_k p^k=\sum_{k=1}^{K} a_k s^k\ , \quad \alpha_k \text{ could be any real numbers.}$$

(2) Shifting

$$Lp^k(t-t_1) = s^k(t-t_1).$$

A linear synthesizing operator $\Gamma_k(\omega), k = 1,2,...,nshot$ has the form:

$\Gamma_k^l(\omega) = \exp(i\omega t_k) = \exp\{i\omega[(y_j - y_c)p_y]\}$, $l = 1,2,...,NX$, where NY is the sampling numbers along y direction and NX along x direction. If $p_y = 0$, $\Gamma_k^l(\omega) = 1$. The plane waves and the corresponding records on the surface z_0 are obtained by

$$S_{syn}^l(x,y,z_0,\omega) = \sum_{k=1}^{nshot} S_k(x,y,z_0,\omega)\Gamma_k^l(\omega) \tag{3}$$

and

$$P_{syn}^l(x,y,z_0,\omega) = \sum_{k=1}^{nshot} P_k(x,y,z_0,\omega)\Gamma_k^l(\omega). \tag{4}$$

With the properties of L, $LP_{syn}^l = S_{syn}^l$ is obtained, which suggests that the linear shot source and corresponding records satisfy the wave equation. The records can be obtained by utilizing the linear shot sources in the modeling scheme. To synthesize the linear shot records into arel shot records is similar to that in 2-D cases. A linear shot record can be seen as a shot record in the 2-D case. The synthesizing operator is

$$\Gamma_l(\omega) = \exp(i\omega t_k) = \exp\{i\omega[(x_i - x_c)p_x]\}, l = 1,2,...,NX. \tag{5}$$

The source vector of a single point source (dipole) at the surface reads:

$$S_{i,j}(z_0) = S'(\omega)\begin{bmatrix} 0,...,0,0,0,...,0 \\ .... \\ 0,...,0,1,0,...,0 \\ .... \\ 0,...,0,0,0,...,0 \end{bmatrix}, \tag{6}$$

where the only nonzero value of the vector in the ith row and jth column indicates the position of the dipole, and $S'(\omega)$ is the source signature. However, source vector $S_{i,j}(z_0)$ can be used for any kind of source wave field. For areal shot- source, we may write

$$S(z_0) = \begin{bmatrix} S'_{1,1}(\omega), S'_{1,2}(\omega),..., S'_{1,NY}(\omega) \\ S'_{2,1}(\omega), S'_{2,2}(\omega),..., S'_{2,NY}(\omega) \\ .... \\ S'_{NX-1,1}(\omega), S'_{NX-1,2}(\omega),..., S'_{NX-1,NY}(\omega) \\ S'_{NX,1}(\omega), S'_{NX,2}(\omega),..., S'_{NX,NY}(\omega) \end{bmatrix}. \tag{7}$$

Assume the areal source wavefield has a single wavefront with a space-invariant signature

$$S'_{i,j}(\omega) = S'(\omega)\cos(\omega t_{i,j}) + S'(-\omega)\sin(\omega t_{i,j}) \tag{8}$$

with $t_{i,j} = p_x(x_k - x_c) + p_y(y_j - y_c)$ and $p_x = \sin\alpha / v_0, p_y = \sin\beta / v_0$, where α and β are incidence angles of the source wavefield at (x_i, y_j) along x axis and y axis, respectively, x_c and y_c represent the lateral positions with zero time shift.

In most cases when performing residual migration velocity analysis， either p_x or p_y is set to be constant and only a few constant ray parameters are required. Here we assume p_y to be constant. The linear shot-source S_i has the following form:

$$S_i(z_0) = \begin{bmatrix} 0,...,0,0,0,.............,0 \\ .... \\ S_{i,1}(\omega), S_{i,2}(\omega),..., S_{i,NY}(\omega) \\ .... \\ 0,...,0,0,0,.............,0 \end{bmatrix}. \qquad (9)$$

To avoid boundary reflections, an absorbing boundary as a 20-point weighting function[6] is applied on the bottom and lateral edges of the spatial grid.

Computer implementation and example

Let NX, NY, NZ denote the number of grid points in the x, y and z directions of a 3-D array data. We set linear shot-source along y axis. To avoid boundary reflections and meanwhile make linear shot-sources as long as possible when modeling, ten marginal grid points are excluded on both sides of y axis. For the same sake, we need not compute all the NX linear shot-source but for the middle $NX-10*2$ linear shot-source. In other words, $NX-10*2$ records are required by time extrapolations to provide data for testing. Even in the worst cases, five ray parameters p_y would be enough and $5(NX-10*2)$ records are required. For conventional pseudospectral method, $(NX-20)*(NY-20)$ shot records are needed. In this way, a magnitude of computations is saved. And there is no need any more to retrieve so many records from hard disk to synthesize them in areal shot-record migration.

The following example shows the result and feasibility of the method. The physical model has a central symmetrical structure with three flat layers and two anticlines. To get an intuitionistic view of the central part of the model, only a quarter of the 3-D model is shown by Fig.1. The following figures have the same form. The calculation used $256\times256\times400$ mesh points with grid spacing of 25m and 5m in the x, y and z directions and was carried to 4500 time steps with time step 0.4ms.

Fig.2 a-d display the snapshots of the wave field with linear shot-source x=x128 and ray parameter 0us/m at different time steps. From the figure, we can see that since the absorbing boundary has been applied on the lateral edges and the bottom of the 3-D volume the energy has been attenuated to be very weak near boundaries and little energy has penetrated through the boundaries. The recorded data is shown in Fig.3a . And a synthesized record with ray parameters (-77.1us/m, 0us/m) is shown in Fig.3b .

We have implemented our codes on SGI Origin computer with 64 processors. The algorithms have been parallelized with respect to the position of linear shot-source. Since the model has a central symmetrical structure, we have calculated only the left half linear shot-sources and the right half linear shot-source records are obtained according to the symmetry quality. That is to say, only 118 linear shot-source records are needed. The total extrapolations for one linear source-shot response take 29 hours and 46 minutes and the whole computations take about 60 hours.

Summary

3-D linear shot-source modeling method yields some individual linear shot-source records and then these records are synthesized to produce the synthesized areal-shot record used in areal shot-record migration. In this way the extrapolations need not be done for all individual shot-source, but for all the individual linear shot-source. And the needed number of retrieved records becomes less and it saves much time. So the computations time become tolerable and efficiently provides data for testing 3-D areal shot-record migration and 3-D residual migration velocity analysis. And the resulted Hartley algorithm is efficient in computation time and memory requirements. The example shows that the method is practicable and attractive.

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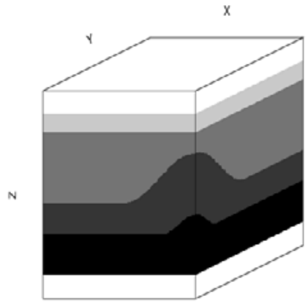

Fig. 1. Velocity model with three flat layers and two anticlines. The velocities of the layers from the top to the bottom are 2100m/s,2700m/s,3500m/s,4100m/s,4600m/s,2100m/s, respectively.

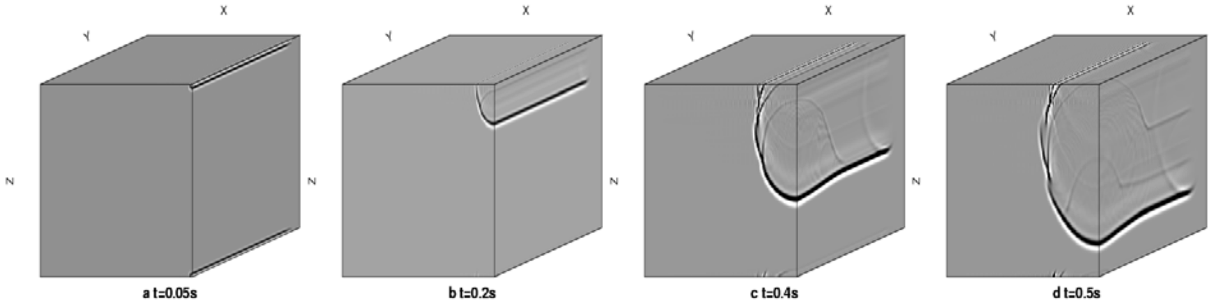

Fig. 2. Time sequence showing the evolution the wave from the linear shot-source $x = x_{128}$.

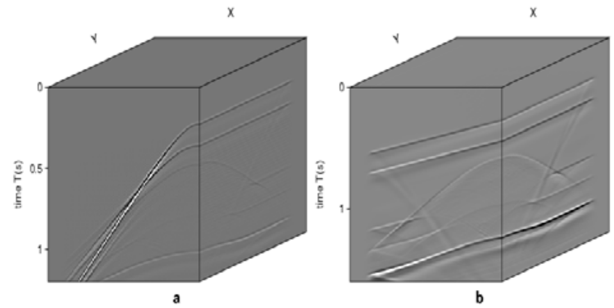

Fig. 3. Reflected waves recorded and the synthesized record at the surface $z = 0$.

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