

Revision of thickness design of frozen walls considering frost heave

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Abstract. The paper outlines development of the thickness design of cylindrical frozen walls in artificial ground freezing (AGF). A plain strain mechanical model coupled with infinite surrounding soil and rock takes into account the frost heave ratio to investigate the influence of frost heave on the thickness design of frozen wall, and superposition method is used to solve the complicated problem of frozen wall swelling. A revised formula referred to as “Baoshen” formula has been proposed. This formula provides a convenient analytic solution for any AGF problem involving not only frost heave but also the action of surrounding soil.

Introduction

Artificial ground freezing (AGF) technology has been widely used in shaft sinking. There has been more than 1,000 shafts successfully sunk by AGF [1]. The thickness design formula has developed significantly over the time. In the early age (1910 to 1950) the shaft sink is very shallow (the depth is less than 100m), the Lamé formula, based on plain strain elastic model of cylindrical frozen wall, was used for design. Later, as the depth becomes deeper and deeper, the Lamé formula was found to be too conservative. Various formulae have been developed based on partially elastic and partially plastic [2], fully plastic, or fully viscous behavior of the frozen wall. Examples include Domke formula in 1915 and Klein formula in 1968, etc. In 2009, Zhou [3, 4] proposed a new formula based on plain strain elastic model considering the effect of the infinite surrounding rock or soils, which can be used for the design of both shaft lining and frozen wall, especially for shaft sinks built in rocky aquifer

Despite these improvements, there are still many occurrences of freeze tube breakage observed in field applications. Many attribute these failures to over-deformation of the ice wall, while others think the strength of the tube material is too low or the joints of freeze tube are too weak.

Frost heave has not been widely considered in the past. Since good performance has been achieved by prolonged deep freezing, we think frost heave can be a very important factor that needs to be considered in artificial ground freezing. In this paper, we present a plain strain model of cylindrical frozen wall enclosed in infinite rock or soil considering frost heave ratio.

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1. Mechanical principal of AGF of deep shaft

Before the shaft is excavated and lined, a circular configuration of freeze tubes with a designed spacing is installed in the ground, then chilled brine circulation system are formed, and ground freezing can be achieved if the cooling system is allowed sufficient time to work. After the development of frozen wall meets the requirements for the ground stress support and protection, shaft sinking can be carried out step by step. Such a procedure of shaft sinking with AGF is illustrated with Fig.1. Fig.1-① represents the original state of ground stress, Fig.1-② describes the

second state of ground stress distribution with the forming of frozen wall. Fig.1-③ presents the third state of ground stress distribution when the excavation is carried out without lining.

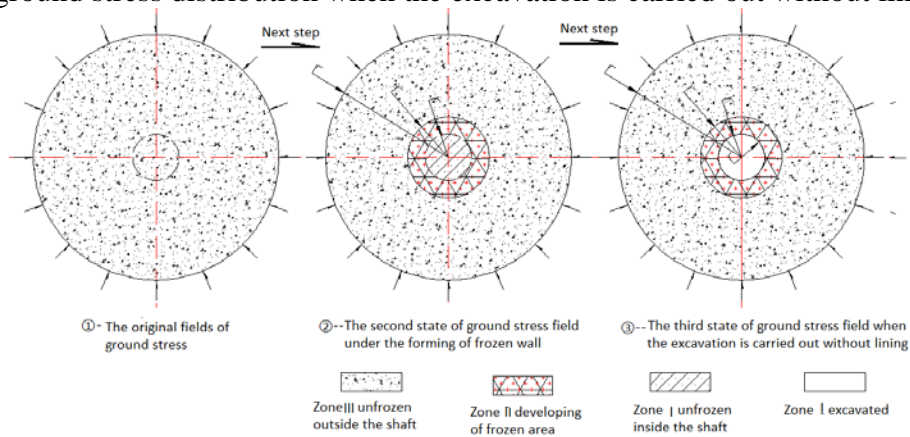


Fig.1 Mechanical Chart of ground freezing and shaft sinking procedure

We aim to analyze the stress distribution at every zone in the third state and to study the safety of the frozen wall. In the second state as described by Fig.1- ②, frost heave occurs, and induces additional stress to the original stress field. Fig.1-③ shows the excavation, so the stresses inside the shaft area are released completely, and this causes the stress re-distribution.

Assume the entire procedure is completely elastic, we can apply superposition method to solve the problem as shown in Fig.2.

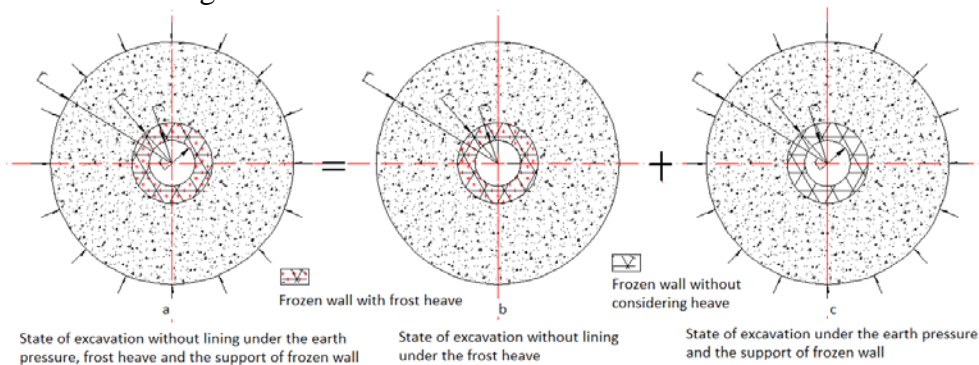


Fig.2 Superposition of mechanics solutions of shaft ground freezing and sinking

Based on the superposition method, the stress field described in Fig.2-a equals to the stress field described in Fig.2-b plus the stress field described in Fig.2-c. we already have the stress field and displacement solutions of Fig.2- c[4] which is the case without consideration of frost heave: Eqn. (1) and (2) are the stress field functions; Eqn.(3) is the displacement field in Area II of frozen wall (refer to Fig. 1 for zone identification).

$$\sigma_{2r}^{II-c} = - \frac{\left(1 - \frac{r_1^2}{r^2}\right)}{\left[\frac{G_1}{G_2}(1-2\mu_2)+1\right] + \left(\frac{G_1}{G_2}-1\right)\frac{r_1^2}{r_2^2}} p_\infty \quad (1)$$

$$\sigma_{2\theta}^{II-c} = - \frac{\left(1 + \frac{r_1^2}{r^2}\right)}{\left[\frac{G_1}{G_2}(1-2\mu_2)+1\right] + \left(\frac{G_1}{G_2}-1\right)\frac{r_1^2}{r_2^2}} p_\infty \quad (2)$$

$$u_f^{II-c}(r) = - \frac{r p_\infty}{2G_2} \frac{\frac{r_1^2}{r^2} + (1-2\mu_2)}{\frac{G_1}{G_2}(1-2\mu_2)+1 + \left(\frac{G_1}{G_2}-1\right)\frac{r_1^2}{r_2^2}} \quad (3)$$

In the next step we aim to find the solution of Fig.2- b.

2. Solution

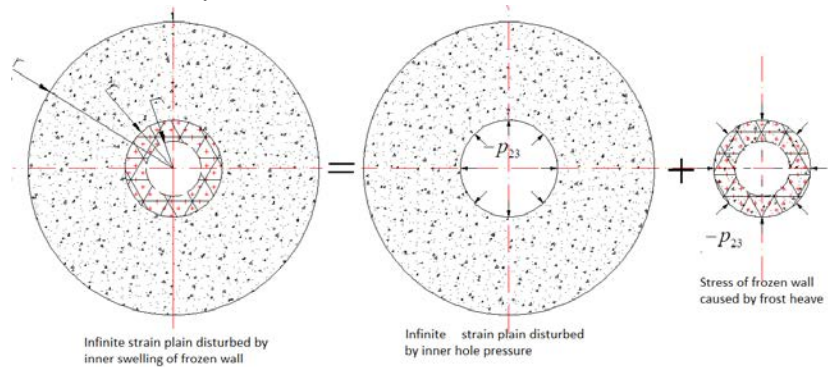
To solve the problem of Fig.2- b, we divide the model into two parts, and apply the rotational symmetry example of polar coordinate based on elastic theory [5].

Frost heave is produced by the phase change of pore water into pore ice; the pore water commonly exists in various types of soils or rocks, and is affected by the movement of moisture and heat transfer in frozen and unfrozen soil; the frost heave ratio δ is a function of pressure, temperature gradient and ice penetration rate, time, and so on [6]. Takashi, et al proposed an empirical formula for frost heave ratio:

$$\delta = \frac{\Delta h}{h} = \delta_0 + \frac{\sigma_0}{\sigma} \left(1 + \sqrt{\frac{U_0}{U}}\right) \quad (4)$$

Where δ is frost heave ratio [no dimensional], Δh is the total heave height, h initial soil specimen height, σ effective stress which acts parallel to heat flow [MPa], U freezing speed of soil [mm/day], δ_0, σ_0, U_0 are the characteristic constants of the soil. In elastic plain analysis of polar coordinates, we can take the frost heave as a strain for constitutive relationship^[7]:

$$\begin{cases} \sigma_r = \frac{E}{1-\mu^2} (\varepsilon_r + \mu\varepsilon_\theta) - \frac{E\delta}{1-\mu} \\ \sigma_\theta = \frac{E}{1-\mu^2} (\varepsilon_\theta + \mu\varepsilon_r) - \frac{E\delta}{1-\mu} \end{cases} \quad (5)$$



Thus the stress and displacement fields of the frozen wall are as follows:

$$\sigma_r'' = \frac{2G_u \left(1 - \frac{r_1^2}{r^2}\right)}{\left[\frac{G_u}{G_f}(1-2\mu_f)+1\right] + \left(\frac{G_u}{G_f}-1\right)\frac{r_1^2}{r^2}} (1+\mu_f)\delta \quad (6)$$

$$\sigma_\theta'' = \frac{2G_u(1-2\mu_f)\left(1 + \frac{r_1^2}{r^2}\right)}{\left[\frac{G_u}{G_f}(1-2\mu_2)+1\right] + \left(\frac{G_u}{G_f}-1\right)\frac{r_1^2}{r^2}} (1+\mu_f)\delta \quad (7)$$

$$u_2''(r) = -\frac{\frac{G_1}{G_2}\left(\frac{r_1^2}{r_2^2} + \frac{r_1^2}{r^2}\right) + 1 - \frac{r_1^2}{r_2^2}}{\left[\frac{G_1}{G_2}(1-2\mu_2)+1\right] + \left(\frac{G_1}{G_2}-1\right)\frac{r_1^2}{r^2}} (1+\mu_2)\delta r \quad (8)$$

3 Final solution

With the solutions of Fig.2 b and Fig.2 c, we combine them based on superposition method to obtain the following stress and displacement field solutions.

$$\sigma_r'' = \frac{\left(2G_2\delta - \frac{G_2}{G_1}p_\infty\right)\left(1 - \frac{r_1^2}{r^2}\right)}{\left(\frac{G_2}{G_1}+1-2\mu_2\right) - \left(\frac{G_2}{G_1}-1\right)\frac{r_1^2}{r^2}} \quad (9)$$

$$\sigma_{\theta}^{\prime\prime} = \frac{\left(2G_2\delta - \frac{G_2}{G_1} p_{\infty}\right) \left(1 + \frac{r_1^2}{r^2}\right)}{\left(\frac{G_2}{G_1} + 1 - 2\mu_2\right) - \left(\frac{G_2}{G_1} - 1\right) \frac{r_1^2}{r_2^2}} \quad (10)$$

From Eqn. 10, when $r = r_1$, the hoop stress reaches the maximum value as the first principal stress at the inner surface of frozen wall:

$$\sigma_{\theta}^{\prime\prime} \Big|_{\max} = \frac{2\left(2G_2\delta - \frac{G_2}{G_1} p_{\infty}\right)}{\left(\frac{G_2}{G_1} + 1 - 2\mu_2\right) - \left(\frac{G_2}{G_1} - 1\right) \frac{r_1^2}{r_2^2}} \quad (11)$$

According to third strength theory of material mechanics that is the maximum shearing stress theory of failure, $|\sigma_{\theta}^{\prime\prime} - \sigma_r^{\prime\prime}| = |\sigma_{\theta}^{\prime\prime}| \leq [\sigma]$, the following formula for frozen wall design are obtained.

$$th = r_2 - r_1 = r_1 \left(\sqrt{\frac{\alpha[\sigma]}{\beta[\sigma] + 4G_u\delta - 2p_{\infty}} - 1} \right) \quad (12)$$

$$\text{Where: } \alpha = \left(1 - \frac{G_u}{G_f}\right); \beta = \left[1 + \frac{G_u}{G_f}(1 - 2\mu_f)\right]$$

Eqn.12 is very similar to “Baoshen” formula, with an additional item in the denominator, $4G_u\delta$, which reflect the action of frost heave and shearing modulus. This formula for frozen wall thickness can be called a revised “Baoshen” formula.

4. Conclusion

Frost heave is one of the most important factors in artificial ground freezing and yet it has rarely been paid attention in the past studies. This paper proposes a revised “Baoshen” formula for the design of frozen wall thickness and it for the first time takes into account the impact of frost heave and the surrounding soil enclosing the frozen wall. Further experimental is needed to validate this formula.

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