Sparse Unmixing using an approximate L_0 Regularization

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Abstract. Recently, sparse unmixing focuses on finding an optimal subset of spectral signatures in a large spectral spetral library. In most previous work concerned with the sparse unmixing, the linear mixture model has been widely used to determine and quantify the abundance of materials in mixed piexels[1]. In this paper, we propose a new sparse unmxing method based on an approximate sparsity regularization model[2]. The approximate sparsity regularizer is much easier to solve than the L0 regularizer and has stronger sparsity than the L1 regularizer. What's more, a variable splitting and augmented Lagrangian methods introduced in to solve the proposed problem. Our numerical results on sparse unmixing illustration the efficiency of approximate sparsity a under the SUnSAL algorithm framework, compared to the L1 norm.

Introduction

Spectral unmixing is an effectively way to hyperspectral date analysis. To deal this problem, the spectral unmixing technique was proposed, which estimates the fractional signatures of pure spectral signatures in each mixed pixel. Based on the relationship of photons interact with material, mixed pixel model can be divided into two basic models: the linear mixture model and the nonlinear mixture model[3]. As the linear mixture model is ease of implementation and flexibility, It's has been widely used into many different applications.

In this paper, we proposed a novel compound regularization based hyperspectral unmixing method, which exploits the approximate sparsity. The approximate sparsity L0 regularized, which provides much easier to solve than L0 regularized, and better sparsity than L1 regularized. Experimental results also show that the proposed method can effectively improve the SRE of hyperspectral unmixing.

Sparse unmixing

Linear mixture model

The linear mixture model assumes minimal secondary reflections and multiple scattering effects in the data collection procedure, and hence the measured spectra can be expressed as a linear combination of the spectral signatures of materials present in the mixed pixel. The LMM can be formulated as follows:

$$y = M\alpha + n, \tag{1}$$

where y is a $\mathrm{L}\times 1$ column vector of observed hyperspectral pixel, L is the number of the spectral bands, M is an $\mathrm{L}\times \mathrm{q}$ matrix standing for the q endmembers, α is a $\mathrm{q}\times 1$ abundance vector, n represents a $\mathrm{L}\times 1$ vector of error and noise. There are two constraints are widely used in the linear mixture model: the abundance non-negativity and abundance sum-to-one, as follows:

$$\alpha \ge 0, \tag{2}$$

$$\sum_{i=1}^{q} \alpha_i = 1, \tag{3}$$

Sparse Unmixing

The idea of Sparse unmixing is to find a linear combination of endmembers for each observed pixel from a large spectral library. Given a known large spectral library A, the sparse unmixing can be written as[4] :

$$y = Ax + n \tag{4}$$

Where A acts as the available spectral library, which is a large matrix, $A \in R^{L \times p}$ containing p endmembers. L denotes the number of bands, and p is the number of endmembers in A.

Bearing the LMM and sparse unmixing theory in mind, we consider a general minimization problem as:

$$\min_x \|x\|_0 \ s.t. \|y - Ax\| \le \delta, x \ge 0, 1^T X = 1 \tag{5}$$

Where $\|x\|_0$ denotes the L_0 norm of the vector x, $\delta \ge 0$ is the tolerated error and modeling error. However, in terms of computational complexity, the L_0 norm optimization problem is a typical NP-hard problem, and it was difficult to solve until Candes and Tao[5,6] proved that the L_0 norm can be replaced by the L_1 norm under a certain condition of the restricted isometric property(RIP). Therefore, the optimization problem is relaxed to alternative convex optimization problem can be written as:

$$\min_x \|x\|_1 \ s.t. \|y - Ax\| \le \delta, x \ge 0, 1^T X = 1 \tag{6}$$

Where $\|x\|_1$ denotes the L_1 norm. The constrained optimization problem can be converted into an unconstrained Lagrangian version,as follows:

$$\min_x \frac{1}{2} \|Ax - y\|_2^2 + \lambda \|x\|_1 + \ell_{R_+^q}(\mathrm{x}) + \ell\{1\}(1^T x) \tag{7}$$

Where λ is a non-negative regularization parameter which controls the relative weights of two objective function. The $\ell_{R_+^q}(\mathrm{x})$ and $\ell\{1\}(1^T x)$ represent the ANC and ASC, respectively.

Proposed model and algorithm

Unmixing Model Based Approximation L_0 norm

While L_1 regularization provides the best convex approximation to L_0 regularization and it is computationally efficient. However, L_0 regularizer can not obtain a satisfactory solution. In this paper, we consider using a continuous function to approximate L_0 norm sparse unmixing method, which provides smooth measure of L_0 norm and better sparsity than L_1 regularizer. The smoothed L_0 norm can be written as[7]:

$$f_\sigma(\mathrm{x}) = \frac{\exp(\frac{x^2}{2\sigma^2}) - \exp(-\frac{x^2}{2\sigma^2})}{\exp(\frac{x^2}{2\sigma^2}) + \exp(-\frac{x^2}{2\sigma^2})} \tag{8}$$

The parameter σ is a positive constant and $\sigma \ne 0$. As σ approaches to zero, we have:

$$\lim_{\sigma \to 0} f_\sigma(\mathrm{x}) = \begin{cases} 1 & x = 0 \\ 0 & x \ne 0 \end{cases} \tag{9}$$

or approximately can be denotes:

$$f_\sigma(\sigma) \approx \begin{cases} 1 & |x| \ll \sigma \\ 0 & |x| \gg \sigma \end{cases} \tag{10}$$

Define the continuous multicariate approximate sparsity function as:

$$F_\sigma(\mathrm{x}) = \sum_{i=1}^{m} f_\sigma(\mathrm{x_i}) \tag{11}$$

As we know, the larger value of σ, the smoother $F_\sigma(\mathrm{x})$ and worse approximation to L_0 norm; the smaller value of σ, the closer behavior of $F_\sigma(\mathrm{x})$ to L_0 norm.

Algorithms for approximative approach with smoothed L_0 norm

In this work, we use the approximative approach L_0 norm to replace the L_1 norm in(7), as follows:

$$\min_x \frac{1}{2}\|Ax-y\|_2^2 + \lambda g(x) + \ell_{R_+^q}(\mathrm{x}) + \ell\{1\}(1^T x) \tag{12}$$

$$g(x) = 1 - F_\sigma(\mathrm{x}) \tag{13}$$

In this work, we used the variable splitting and augmented Lagrangian algorithm to solve(12), and we introduce intermediate variable u, then transform problem(14) into an equivalent problem,

$$\min_x \frac{1}{2}\|Ax-y\|_2^2 + \lambda g(u) + \ell_{R_+^q}(u) + \ell\{1\}(1^T x) \tag{14}$$

The augmented Lagrangian of problem(14) is

$$L(x,u,d) = \frac{1}{2}\|Ax-y\|_2^2 + \lambda g(u) + \ell_{R_+^q}(u) + \ell\{1\}(1^T x) + \frac{\mu}{2}\|x-u-d\|_2^2 \tag{15}$$

The algorithm is shown in Algorithm 1.

Algorithm 1. Unmixing algorithm based on approimative approach L_0 norm

1. Initialization and parameters setting: set k=0, x_0, u_0, d_0

2.repeat: $x^{k+1} \leftarrow \arg\min_x L(x, u^k, d^k)$

$u^{k+1} \leftarrow \arg\min_u L(x^k, u, d^k)$

$d^{k+1} \leftarrow d^k - (x^{k+1} - u^{k+1})$

3. until some stopping criterion is satisfied

Experiments

Having presented our method in the previous section, we now turn our attention to demonstrate its utility for sparse unmixing. The proposed model are compared with the SUnSAL method. All the considered models take into account the abundance non-negativity constraint. Here, we employ synthetic data and real-world data in order to evaluate the performances of the algorithms. The signal-to-reconstruction(SRE) is used to evaluate the accuracy of unmixing mothods, which is defined as follows:

$$\mathrm{SRE}(\mathrm{dB}) = 10 * \log_{10}\left(E(\|x\|_2^2) \Big/ E(\|x-\hat{x}\|_2^2) \right)$$

Here, x denotes the true abundances matrix, $\hat{x}$ represents the estimated one each column of which corresponds with the abundances of a pixel, and $E(\bullet)$ denotes the expectation function. Generally speaking, the larger the SRE is, the more the estimation approximates the truth. The max iteration number $iter_{\max}$ and the iteration stopping criterion ε_{stop} are set to 200 and 0.001.

Synthetic data

In this experiment, the spectral library is the United States Geological Survey(USGS) digital spectral library, which contains 240 materials with 224 spectral bands distributed uniformly in the interval 0.4-2.5 μm.

Table 1 shows SRE(dB), obtained in the simulated dataset, for all the SNR levels considered and for different values of the parameter, such as λ *and* σ.

Real data

The hyperspectral dataset used in the real data experiments is the United States Geological Survey(USGS) digital spectral library. The size of the test area we chose was 250 x 191-piel subset. The USGS library containing 498 pure endmember signatures are measured for 224 spectral bands in the interval 0.4- 2.5 μm. with nominal spectral resolution of 10 μm. Prior to the analysis, bands1-2, 105- 115, 150- 170, and223- 224 were removed due to water absorption and low SNR in those bands, leaving a total of 188 spectral bands. In our experiments, we use spectral obtained from this library as input to the unmixing methods, and make a qualitative analysis of the performances of different sparse unmixing methods.

Table1. SRE(dB) comparison between different algorithms on the synthetic data

Data cube	SNR(db)	SUnSAL	ASL_0SU
DC2 (k=4)	20	3.05 $\lambda = 2\times10^{-1}$	3.52 $\lambda = 2\times10^{-3}, \sigma = 0.12$
	30	6.41 $\lambda = 2\times10^{-2}$	8.03 $\lambda = 4\times10^{-4}, \sigma = 0.06$
	40	12.75 $\lambda = 4\times10^{-3}$	15.10 $\lambda = 7\times10^{-5}, \sigma = 0.02$

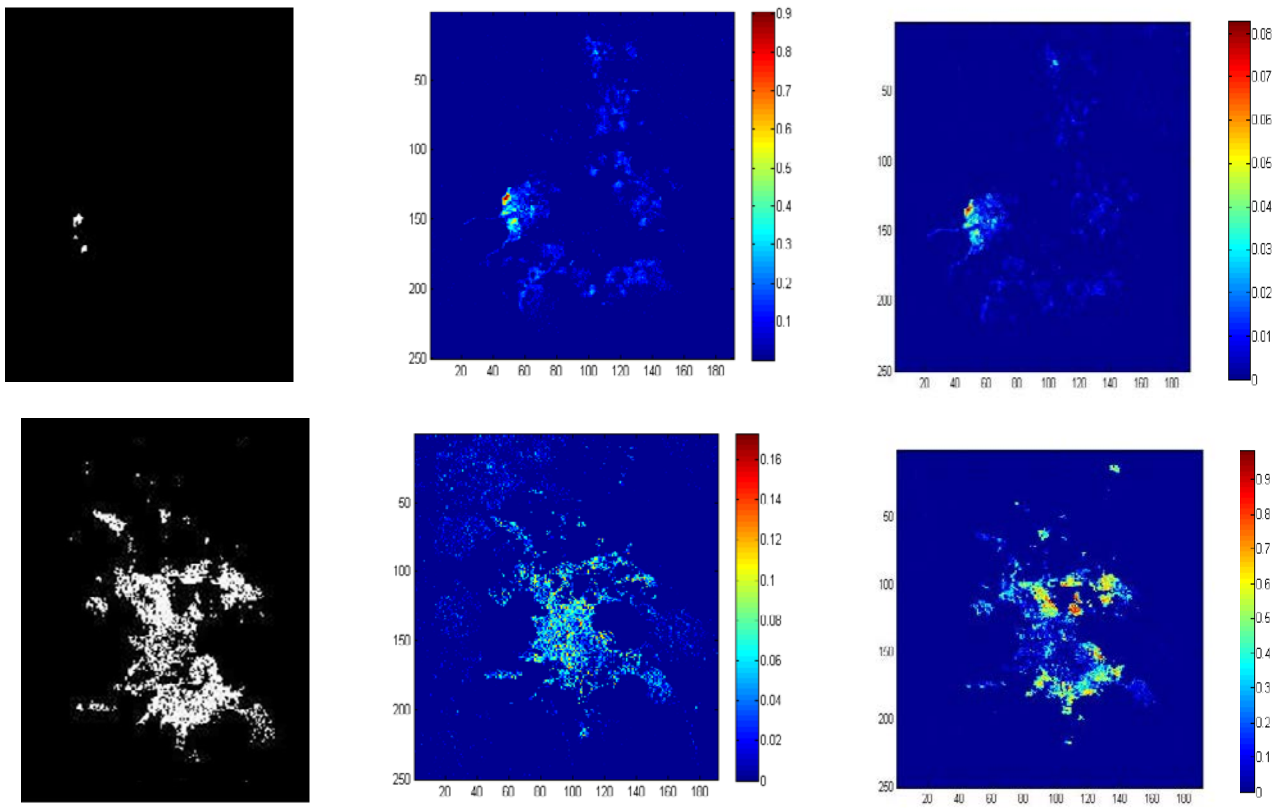

Fig.1. Estimated abundance fractions with the different methods for the Cuprite data

As the smoothed Approximation L_0 model behave much better than L_1 model, we only display the results obtained by these two models. As shown in Fig 1, the varying degrees of unmixing accuracy for the two typical minerals, Buddingtonite and Chalcedony. Compared with SUnSAL, the ASL_0SU algorithm is closer the classification maps produced by the SUGS Tetracorder algorithm. However, generally speaking, we can conclude that our algorithm outperforms the SUnSAL algorithm.

Conclusions

To improve the accuracy of spectral unmixing, we consider using the L_0 norm to replace the L_1 norm to measure the SNR, and propose a new smooth function to approximate the L_0 norm. we also have used an effective method basde on variable splitting and augmented Lagrangian algorithm to solve the approximate L_0 norm problem. Experimental results on both the synthetic data and real data gives sparser and more accuate of our new models.

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