Discrete Fourier coefficients of a periodic signal amplitude harmonic simulation in the computer

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Abstract. Periodic signal has practical applications in many occasions, the periodic signal met Dirichlet's condition can expanse in Fourier series.Usually series expansion coefficients indicates the amplitude of the signal harmonics. Only the mathematical expectation for the known periodic signals using Fourier coefficients that is not easy to solve, usually calculated by computer. This paper discrete Fourier coefficient to analyze the amplitude of harmonic component and Matlab numerical analysis of the amplitude of harmonic component in one circuit is introduced. The experimental results show good agreement with the numerical analysis.

Introduction

Fourier Transform (FT) is a signal from the time domain into the frequency domain transform of the form.[3] We hope to achieve signal spectral analysis or other work on the computer. Computer signal requirements are: in the time domain and frequency domain should be discrete, and should be limited long. The Fourier Transform (FT) can only handle a continuous signal, Discrete Fourier Transform (DFT) is to be born this need. Periodic signal may be represented as a DC component and a sum of the harmonic components. Usually in the calculation cycle signal harmonic amplitude, Only when satisfied Dirichlet conditions in order to represent the Fourier coefficients harmonic amplitude. Moreover, only those periodic signal having analytic expressions have analytical solutions, for the periodic signal only have mathematical expectation (RMS) of is generally represented by Fourier coefficients, but not easy to solve.

In this paper, using the discrete Fourier coefficients of harmonic amplitude analysis method, and use this method to perform actual circuit harmonic amplitude Matlab numerical analysis. Experimental results validate the numerical analysis conclusions.

Principle of periodic signal harmonic amplitude discrete Fourier analysis

Has the analytical expression of periodic signal harmonic amplitude. When a periodic signal only satisfy Dirichlet conditions in order to expand the Fourier series.

Typically,the periodic signal with the analytical expression Fourier coefficients of said harmonic component amplitude. Concrete expansion process is as follows: the periodic signal is provided f (t), its period T, $\Omega_0 = 2\pi F_0 = 2\pi / T_0$ is discrete spectral lines between two adjacent angular frequency interval, k is the harmonic number. It can be expanded as:

$$f(t) = A_0 / 2 + \sum_{k=1}^{\infty} A_k \cos(k\Omega_0 t + \varphi_k) \tag{1}$$

Generally, $A_k \cos(k\Omega_0 t + \varphi_k)$ called k harmonics, A_k is k's harmonic amplitude, φ_k is its initial phase angle.

Trigonometric form of the Fourier series meaning more clear, but the operation inconvenient, so often exponential form of Fourier series. That is:

$$f(t) = \frac{1}{2} \sum_{k=-\infty}^{\infty} A_k e^{j\varphi_k} e^{jk\Omega_0 t} = \sum_{k=-\infty}^{\infty} F_k e^{jk\Omega_0 t} \tag{2}$$

Where, F_k is the exponential Fourier series of complex coefficient, which is calculated

$$F_k(jk\Omega_0) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)e^{-jk\Omega_0 t}dt, k = 0, \pm 1, \pm 2, ... \qquad (3)$$

Fourier complex coefficients of the relationship between the Fourier coefficients:

$$F_n = \frac{1}{2}A_n e^{j\varphi_n} = \frac{1}{2}(a_n - jb_n) \qquad (4)$$

From (7) shows: the harmonic amplitudes

$$A_k = 2\left|F_k\right| \qquad (5)$$

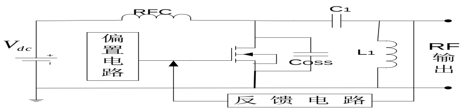
According to (6), then (5) can be grouped as:

$$f(t) = \sum_{k=-\infty}^{\infty} F_k(jk\Omega_0)e^{jk\Omega_0 t} \qquad (6)$$

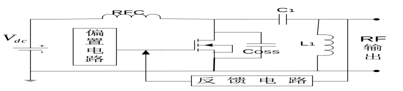
In summary, the use of (8) to solve a periodic signal amplitude harmonic having an analytical expression is reasonable.

Has the mathematical expectation of a periodic signal. For has the mathematical expectation of a periodic signal, use (8) analytic solution can not be obtained, generally use the computer calculates. However, since the calculation of the time domain function (8) is continuous, and is not suitable for a computer to realize, so that the time domain, frequency domain are discretized, sampled.

Set a period After discrete have N discrete points. Set time domain sampling period $T=1/f_S$, according to the frequency domain extension f_S cycle. The combined analysis of Section 3.1。 $N = \frac{\Omega_s}{\Omega_0} = \frac{T_0}{T}$, then:

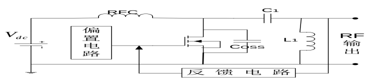


(7)



(8)

(7) that is the expression of the k-th harmonic coefficients discrete Fourier . Compare (5), (6), (8) can be obtained:

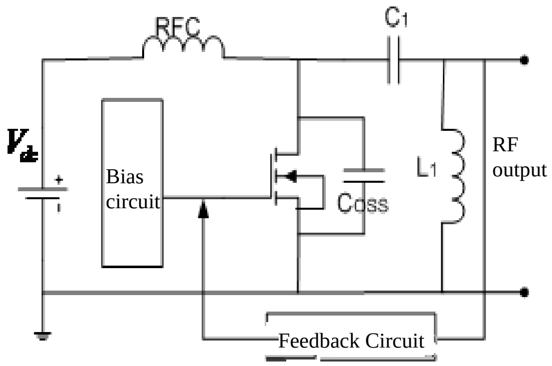


(9)

Then (9) is using a discrete Fourier Coefficients of amplitude formula.

Matlab numerical analysis of harmonic amplitude periodic signal instances

Under the circuit diagram of the the literature [2] , it is a class C oscillator consisting of a MOSFET, the principle is shown in Figure,the schematic diagram shown in Fig.1, in order to highlight the impact C_{OSS} oscillation harmonic with a resonant capacitor C_{OSS} [6],the output circuit simplified model shown in Fig.2 [6]. Where V_c is the voltage between the drain-source C_{OSS}, V_{dc} is DC voltage, V_0 is the output voltage. Since the nonlinear C_{OSS} makes V_0 contains harmonic components.



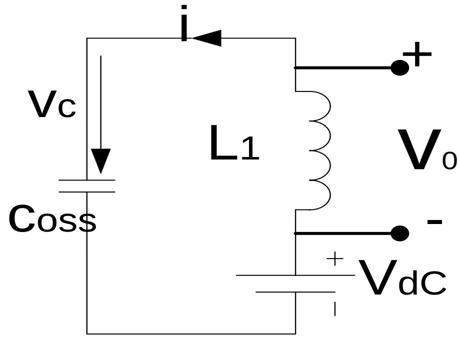


Fig.1 Schematic oscillator   Fig.2 A simplified model output circuit

When the Vdc = 200V, Vomin = -200V ,Matlab simulation program are as follows[1] [4] [5]:

```
clear all;
clc;
format long;
l=1.2926;
i0=0;
vdc=200;
u0=400;
N=737
if  u0<0.1
      c=3000;
   elseif u0<0.5
      c=10^(2.891-1.104*log10(u0)-0.518*(log10(u0))^2);
   else
      c=10^(3.004-0.559*log10(u0)+0.055*(log10(u0))^2);
   end;
  i(1)=i0+1/l*(vdc-u0)*0.0001;
  u(1)=u0+1/c*i(1)*100;
for  t=2:N
   i(t)=i(t-1)+1/l*(vdc-u(t-1))*0.0001;
   u(t)=u(t-1)+1/c*i(t)*100;
   if  u(t)<0.1
      c=3000;
   elseif u(t)<0.5
      c=10^(2.891-1.104*log10(u(t))-0.518*(log10(u(t)))^2);
   else
      c=10^(3.004-0.559*log10(u(t))+0.055*(log10(u(t)))^2);
   end;
end;
u=u-vdc;
U=fft(u);
U_abs=2*abs(U)/N;
% save pinpu U_abs
U1=U_abs(1:737);
fs=10^10;
f=fs*(0:736)/N;
stem(f,U1);
hold on
grid on
title('DTFT amplitude');
```

The simulation results with reference to Fig.3

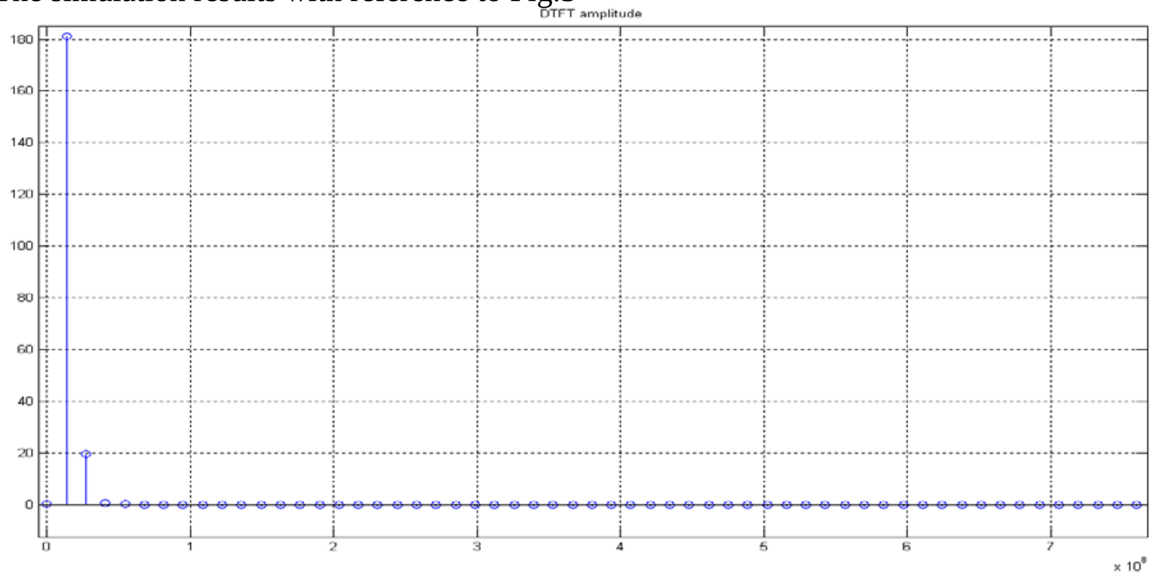
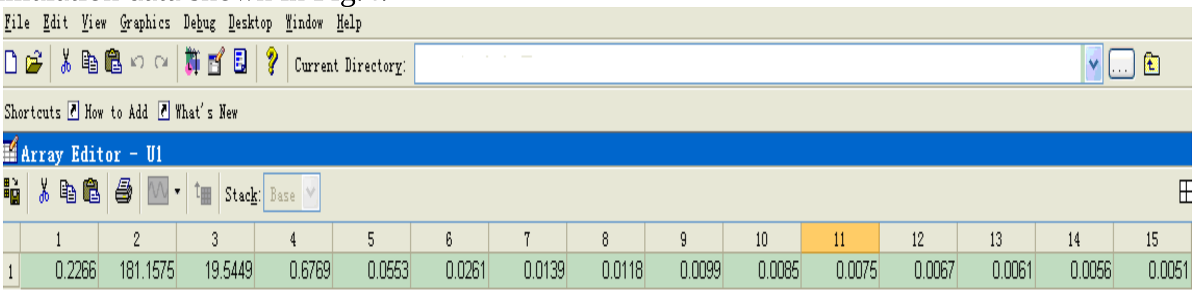


Fig.3 harmonic amplitude vs. Frequency

Simulation data shown in Fig.4:



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.2266	181.1575	19.5449	0.6769	0.0553	0.0261	0.0139	0.0118	0.0099	0.0085	0.0075	0.0067	0.0061	0.0056	0.0051

Fig.4 harmonic amplitudes

Fig.1 shows the analysis of: the input signal most of the energy concentrated in the fundamental and the second harmonic wave, with the increase in the number of the harmonic energy is gradually decreased.The analysis of the data (see Fig.4) and there is a strict correspondence between the physical, evidence of the second chapter of the only known mathematical expectation periodic signal analysis,at the same time demonstrates the high timeliness analysis of the formula (9).

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