Minimum Deviation Models for Multiple Attribute Decision Making in Intuitionistic Fuzzy Setting

Guiwu Wei* and Xiaofei Zhao

Institute of Decision Sciences, Chongqing University of Arts and Sciences
Yongchuan 402160, China

**Corresponding author,E-mail:weiguiwu@163.com*

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Abstract

With respect to intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives and incomplete weight information, a method based the minimum deviation is proposed. Firstly, some operational laws of intuitionistic fuzzy numbers, score function and accuracy function of intuitionistic fuzzy numbers are introduced. Then, to reflect the decision maker's preference information, an optimization model based on the minimum deviation method, by which the attribute weights can be determined, is established. For the special situations where the information about attribute weights is completely unknown, we establish another optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. We utilize the intuitionistic fuzzy weighted averaging (IFWA) operator to aggregate the intuitionistic fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. The method can sufficiently utilize the objective information, and meet decision makers' subjective preference, can also be easily performed on computer. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Key Words: *Multiple attribute decision-making; Intuitionistic fuzzy numbers; Intuitionistic fuzzy weighted averaging (IFWA) operator; Weight information, Preference*

1 Introduction

Atanassov [1-3] introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set [4]. The intuitionistic fuzzy set has received more and more attention since its appearance [5-49]. Gau and Buehrer [5] introduced the concept of vague set. But Bustince and Burillo [6] showed that vague sets are intuitionistic fuzzy sets. Chen and Tan [7] presented new techniques for handling multiple attribute fuzzy decision making problems based on vague set theory. And then Hong and Choi [8] provided another technique for handling multiple attribute fuzzy decision making problems based on vague set theory, they provided new functions to measure the degree of accuracy in the grades of membership of each alternative with respect to a set of attribute. However, they assumed that the degree of importance to each attribute is constant. Szmidt and Kacprzyk [9-12] considered the use of intuitionistic fuzzy sets for building soft decision-making models with imprecise information, and proposed two solution concepts about the intuitionistic fuzzy core and the consensus winner for group decision making using intuitionistic fuzzy sets. Szmidt and Kacprzyk [13] proposed a non-probabilistic type of entropy measure for intuitionistic fuzzy sets. Szmidt and Kacprzyk [14] discussed distances between intuitionistic fuzzy sets. Bustince [15] presented different theorems for building intuitionistic fuzzy relations on a set with predetermined properties. Li and Cheng [16] studied similarity measures of intuitionistic fuzzy sets and their application to pattern recognitions. Szmidt and Kacprzyk [17] proposed some solution concepts in group decision making with intuitionistic fuzzy preference relations, such as intuitionistic fuzzy core and consensus winner, etc. Szmidt and Kacprzyk [18] investigated the consensus-reaching process in group decision making based on individual intuitionistic fuzzy preference relations. Atanassov et al. [19] provided an algorithm for solving the multi-person multi-attribute decision making problems, in which the attribute weights are given as exact numerical values and the attribute values are expressed in intuitionistic fuzzy numbers. Li [20] investigated multiple attribute decision making with intuitionistic fuzzy information and constructed several linear programming models to generate optimal weights for attribute. Lin [21] presented a new method for handling multiple attribute fuzzy decision making problems, where the characteristics of the alternatives are represented by intuitionistic fuzzy sets. The proposed method allows the degrees of satisfiability and non-satisfiability of each alternative with respect to a set of attribute to be represented by intuitionistic fuzzy sets, respectively. Furthermore, the proposed method allows the decision-maker to assign the degree of membership and the degree of non-membership of the attribute to the fuzzy concept "importance." Xu [22] investigate the group decision making problems in which all the information provided by the decision makers is expressed as intuitionistic fuzzy decision matrices where each of the elements is characterized by intuitionistic fuzzy number, and the information about attribute weights is partially known, which may be constructed by various forms. Li [23] extended the linear programming techniques for multidimensional analysis of preference (LINMAP) to develop a new methodology for solving multiattribute decision making problems under Atanassov's intuitionistic fuzzy (IF) environments. Xu and Yager [24] developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, and the intuitionistic fuzzy hybrid geometric (IFHG) operator and gave an application of the IFHG operator to multiple attribute group decision making with intuitionistic fuzzy information. Xu [25] developed some arithmetic aggregation operators, such as the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, and the intuitionistic fuzzy hybrid aggregation (IFHA) operator. Xu [26] investigated the intuitionistic fuzzy MADM with the information about attribute weights is incompletely known or completely unknown, a method based on the ideal solution was proposed. Liu and Wang [27] developed an evaluation function for the decision making problem to measure the degrees to which alternatives satisfy and do not satisfy the decision maker's requirement. Then, they proposed the intuitionistic fuzzy point operators, and defined a series of new score functions for the MADM problems based on intuitionistic fuzzy point operators and evaluation function.

In the process of intuitionistic fuzzy MADM with preference information on alternatives, sometimes, the attribute values and preference values on alternatives take the form of intuitionistic fuzzy numbers, and the information about attribute weights is incompletely known or completely unknown because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain. All of the above methods, however, will be unsuitable for dealing with such situations. Therefore, it is necessary to pay attention to this issue. The aim of this paper is to develop a method, based on the minimum deviation method, to overcome this limitation.

The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to intuitionistic fuzzy sets. In Section 3 we introduce intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives, in which the information about attribute weights is incompletely known, and the attribute values and preference values on alternatives take the form of intuitionistic fuzzy numbers. To determine the attribute weights, an optimization model based on the minimum deviation method, by which the attribute weights can be determined, is established. For the special situations where the information about attribute weights is completely unknown, we establish another optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. We utilize the intuitionistic fuzzy weighted averaging

(IFWA) operator to aggregate the intuitionistic fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. In Section 4, an illustrative example is pointed out. In Section 5 we conclude the paper and give some remarks.

2 Preliminaries

In the following, we introduce some basic concepts related to intuitionistic fuzzy sets.

Definition 1 Let X to be a universe of discourse, then a fuzzy set is defined as:

$$A = \left\{ \left\langle x, \mu_A(x) \right\rangle \middle| x \in X \right\} \tag{1}$$

which is characterized by a membership function $\mu_A : X \to [0,1]$, where $\mu_A(x)$ denotes the degree of membership of the element x to the set A [3].

Atanassov extended the fuzzy set to the IFS, shown as follows:

Definition 2 An IFS A in X is given by

$$A = \left\{ \left\langle x, \mu_A(x), \nu_A(x) \right\rangle \middle| x \in X \right\} \tag{2}$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, with the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \ \forall\, x \in X$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non- membership degree of the element x to the set A [1,2].

Definition 3 For each IFS A in X, if

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \ \forall\, x \in X . \tag{3}$$

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A [1,2].

Definition 4 Let $\tilde{a} = (\mu, \nu)$ be an intuitionistic fuzzy number, a score function S of an intuitionistic fuzzy value can be represented as follows [7]:

$$S(\tilde{a}) = \mu - \nu , \ \ S(\tilde{a}) \in [-1,1]. \tag{4}$$

Definition 5 Let $\tilde{a} = (\mu, \nu)$ be an intuitionistic fuzzy number, an accuracy function H of an intuitionistic fuzzy value can be represented as follows [8]:

$$H(\tilde{a}) = \mu + \nu , \ \ H(\tilde{a}) \in [0,1] . \tag{5}$$

to evaluate the degree of accuracy of the intuitionistic fuzzy value $\tilde{a} = (\mu, \nu)$, where $H(\tilde{a}) \in [0,1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the intuitionistic fuzzy value $\tilde{a}$.

As presented above, the score function S and the accuracy function H are, respectively, defined as the difference and the sum of the membership function $\tilde{\mu}_A(x)$ and the non-membership function $\tilde{\nu}_A(x)$. Xu [24] showed that the relation between the score function S and the accuracy function H is similar to the relation between mean and variance in statistics. Based on the score function S and the accuracy function H, in the following, Xu[24] give an order relation between two intuitionistic fuzzy values, which is defined as follows:

Definition 6 Let $\tilde{a}_1 = (\mu_1, \nu_1)$ and $\tilde{a}_2 = (\mu_2, \nu_2)$ be two intuitionistic fuzzy values, $s(\tilde{a}_1) = \mu_1 - \nu_1$ and $s(\tilde{a}_2) = \mu_2 - \nu_2$ be the scores of $\tilde{a}$ and $\tilde{b}$, respectively, and let $H(\tilde{a}_1) = \mu_1 + \nu_1$ and $H(\tilde{a}_2) = \mu_2 + \nu_2$ be the accuracy degrees of $\tilde{a}$ and $\tilde{b}$, respectively, then if $S(\tilde{a}) < S(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a} < \tilde{b}$; if $S(\tilde{a}) = S(\tilde{b})$, then

(1) if $H(\tilde{a}) = H(\tilde{b})$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a} = \tilde{b}$; (2) if $H(\tilde{a}) < H(\tilde{b})$, $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a} < \tilde{b}$ [24].

Definition 7 Let $a_j = (\mu_j, \nu_j)(j = 1, 2, \cdots, n)$ be a collection of intuitionistic fuzzy values, and let IFWA: $Q^n \to Q$, if

$$\mathrm{IFWA}_{\omega}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)=\sum_{j=1}^{n}\omega_j\tilde{a}_j =\left(1-\prod_{j=1}^{n}\left(1-\mu_j\right)^{\omega_j},\prod_{j=1}^{n}\nu_j^{\omega_j}\right) \tag{6}$$

where $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ be the weight vector of $\tilde{a}_j\left(j=1,2,\cdots,n\right)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, then IFWA is called the intuitionistic fuzzy weighted averaging (IFWA) operator [25].

Definition 8 Let $\tilde{a}_1=\left(\mu_1,\nu_1\right)$ and $\tilde{a}_2=\left(\mu_2,\nu_2\right)$ be two intuitionistic fuzzy numbers, then the normalized Hamming distance between $\tilde{a}_1=\left(\mu_1,\nu_1\right)$ and $\tilde{a}_2=\left(\mu_2,\nu_2\right)$ is defined as follows [26]:

$$d\left(\tilde{a}_1,\tilde{a}_2\right)=\frac{1}{2}\left(\left|\mu_1-\mu_2\right|+\left|\nu_1-\nu_2\right|\right) \tag{7}$$

3 Minimum deviation models for multiple attribute decision making in intuitionistic fuzzy setting with preference information on alternatives

The following assumptions or notations are used to represent the intuitionistic fuzzy MADM problems with incomplete weight information:

(1) The alternatives are known. Let $A=\left\{A_1,A_2,\cdots,A_m\right\}$ be a discrete set of alternatives;

(2) The attributes are known. Let $G=\left\{G_1,G_2,\cdots,G_n\right\}$ be a set of attributes;

(3) The subjective preference information on alternatives is known, and let $\theta=\left(\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_m\right)$ be subjective preference value vector, $\tilde{\theta}_i=\left(\alpha_i,\beta_i\right)$ is intuitionistic fuzzy number, which is subjective preference value on alternative $A_i\left(i=1,2,\cdots,m\right)$.

(4)The information about attribute weights is incompletely known. Let $w=\left(w_1,w_2,\cdots,w_n\right)\in H$ be the weight vector of attributes, where $w_j\geq 0$, $j=1,2,\cdots,n$, $\sum_{j=1}^{n}w_j=1$, H is a set of the known weight information, which can be constructed by the following forms [50-53], for $i\neq j$: **Form 1.** A weak ranking: $w_i\geq w_j$; **Form 2.** A strict ranking: $w_i-w_j\geq\alpha_i$, $\alpha_i>0$; **Form 3.** A ranking of differences: $w_i-w_j\geq w_k-w_l$, for $j\neq k\neq l$;**Form 4.** A ranking with multiples: $w_i\geq\beta_i w_j$, $0\leq\beta_i\leq 1$; **Form 5.** An interval form: $\alpha_i\leq w_i\leq\alpha_i+\varepsilon_i, 0\leq\alpha_i<\alpha_i+\varepsilon_i\leq 1$.

Suppose that $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\mu_{ij},\nu_{ij}\right)_{m\times n}$ is the intuitionistic fuzzy decision matrix, where μ_{ij} indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, ν_{ij} indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $\mu_{ij}\subset\left[0,1\right]$, $\nu_{ij}\subset\left[0,1\right]$, $\mu_{ij}+\nu_{ij}\leq 1$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

Definition 9 Let $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\mu_{ij},\nu_{ij}\right)_{m\times n}$ be an intuitionistic fuzzy decision matrix, $\tilde{r}_i=\left(\tilde{r}_{i1},\tilde{r}_{i2},\cdots,\tilde{r}_{in}\right)$ be the vector of attribute values corresponding to the alternative A_i, $i=1,2,\cdots,m$, then we call

$$\tilde{r}_i=\left(\mu_{ij},\nu_{ij}\right)=\mathrm{IFWA}_w\left(\tilde{r}_{i1},\tilde{r}_{i2},\cdots,\tilde{r}_{in}\right) =\left(1-\prod_{j=1}^{n}\left(1-\mu_{ij}\right)^{w_j},\prod_{j=1}^{n}\nu_{ij}^{w_j}\right), \quad i=1,2,\cdots,m. \tag{8}$$

the overall value of the alternative A_i, where $w=\left(w_1,w_2,\cdots,w_n\right)^T$ is the weight vector of attributes.

In the situation where the information about attribute weights is completely known, i.e., each attribute weight can be provided by the expert with crisp numerical value, we can weight each attribute value and aggregate all the weighted attribute values corresponding to each alternative into an overall one by using Eq. (8). Based on the overall attribute values $\tilde{r}_i$ of the alternatives $A_i\ (i=1,2,\cdots,m)$, we can rank all these alternatives and then select the most desirable one(s). The greater $\tilde{r}_i$, the better the alternative A_i will be.

Because of the complexity of objects, the fuzziness of thought, and the finiteness of knowledge, it' difficult for decision makers to derive the attribute weights, and sometimes, attribute weight information is incompletely known. In this situation, in order to reflect the decision maker's subjective preference and objective information, an optimization model is developed to get the attribute weight. However, there are some differences to the some extent between decision maker's subjective preference and objective information. For the more reasonable decision-making, to select attribute weight vector is to minimize total deviation between objective information and decision maker's subjective preference.

The minimum deviation method is selected here to compute the differences between decision maker's subjective preference and objective information. For the attribute $G_j \in G$, the deviation of alternative A_i to decision maker's subjective preference can be defined as follows:

$$D_{ij}(w) = d\left(\tilde{r}_{ij}, \tilde{\theta}_i\right) w_j, i=1,2,\cdots,m, j=1,2,\cdots,n.$$

Let

$$D_i(w) = \sum_{j=1}^{n} D_{ij}(w) = \sum_{j=1}^{n} d\left(\tilde{r}_{ij}, \tilde{\theta}_i\right) w_j, i=1,2,\cdots,m$$

Then $D_i(w)$ represent the deviation value of the alternatives A_i to decision maker's subjective preference value θ_i.

Based on the above analysis, we have to choose the weight vector w to minimize all deviation values for all the alternatives. To do so, we can construct a linear programming model as follows:

(M-1)

$$\begin{cases} \min D(w) = \sum_{i=1}^{m}\sum_{j=1}^{n} D_{ij}(w) = \sum_{i=1}^{m}\sum_{j=1}^{n} d\left(\tilde{r}_{ij}, \tilde{\theta}_i\right) w_j \\ \text{Subject to} \quad w \in H, \sum_{j=1}^{n} w_j = 1, w_j \geq 0, j=1,2,\cdots,n \end{cases}$$

where $d\left(\tilde{r}_{ij}, \tilde{\theta}_i\right) = \frac{1}{2}\left(\left|\mu_{ij} - \alpha_i\right| + \left|\nu_{ij} - \beta_i\right|\right)$.

By solving the model (M-1), we get the optimal solution $w = (w_1, w_2, \cdots, w_n)$, which can be used as the weight vector of attributes.

If the information about attribute weights is completely unknown, we can establish another programming model:

(M-2)

$$\begin{cases} \min D(w) = \sum_{i=1}^{m}\sum_{j=1}^{n} D_{ij}(w) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\left|\mu_{ij} - \alpha_i\right| + \left|\nu_{ij} - \beta_i\right|\right) w_j \\ s.t.\ \sum_{j=1}^{n} w_j^2 = 1, w_j \geq 0, j=1,2,\cdots,n \end{cases}$$

To solve this model, we construct the Lagrange function:

$$L(w,\lambda) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\left|\mu_{ij} - \alpha_i\right| + \left|\nu_{ij} - \beta_i\right|\right) w_j + \frac{\lambda}{4}\left(\sum_{j=1}^{n} w_j^2 - 1\right) \tag{9}$$

where λ is the Lagrange multiplier.

Differentiating Eq. (9) with respect to $w_j\ (j=1,2,\cdots,n)$ and λ, and setting these partial derivatives equal to zero, the following set of equations is obtained:

$$\begin{cases} \frac{\partial L}{\partial w_j} = \sum_{i=1}^{m}\left(\left|\mu_{ij} - \alpha_i\right| + \left|\nu_{ij} - \beta_i\right|\right) + \lambda w_j = 0 \\ \frac{\partial L}{\partial \lambda} = \sum_{j=1}^{n} w_j^2 - 1 = 0 \end{cases}$$

By solving Eq. (10), we get a simple and exact formula for determining the attribute weights as follows:

$$w_j^* = \frac{\sum_{i=1}^{m}\left(\left|\mu_{ij} - \alpha_i\right| + \left|\nu_{ij} - \beta_i\right|\right)}{\sqrt{\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\left(\left|\mu_{ij} - \alpha_i\right| + \left|\nu_{ij} - \beta_i\right|\right)\right]^2}} \tag{10}$$

By normalizing $w_j^*\left(j=1,2,\cdots,n\right)$ be a unit, we have

$$w_j=\frac{\sum_{i=1}^{m}\left(\left|\mu_{ij}-\alpha_i\right|+\left|\nu_{ij}-\beta_i\right|\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m}\left(\left|\mu_{ij}-\alpha_i\right|+\left|\nu_{ij}-\beta_i\right|\right)} \tag{11}$$

Based on the above models, we develop a practical method for solving the MADM problems with preference information on alternatives, in which the information about attribute weights is incompletely known or completely unknown, and the attribute values and preference values on alternatives take the form of intuitionistic fuzzy information. The method involves the following steps:

Step 1. Let $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}$ be an intuitionistic fuzzy decision matrix, where $\tilde{r}_{ij}=\left(\mu_{ij},\nu_{ij}\right)$, which is an attribute value, given by an expert, for the alternative $A_i\in A$ with respect to the attribute $G_j\in G$, $w=\left(w_1,w_2,\cdots,w_n\right)$ be the weight vector of attributes, where $w_j\in\left[0,1\right]$, $j=1,2,\cdots,n$, H is a set of the known weight information, which can be constructed by the forms 1-5. let $\theta=\left(\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_m\right)$ be subjective preference value, $\tilde{\theta}_i=\left(\alpha_i,\beta_i\right)$ is intuitionistic fuzzy number, which is subjective preference value on alternative $A_i\left(i=1,2,\cdots,m\right)$.

Step 2. If the information about the attribute weights is partly known, then we solve the model (M-1) to obtain the attribute weights. If the information about the attribute weights is completely unknown, then we can obtain the attribute weights by using Eq. (11).

Step 3. Utilize the weight vector $w=\left(w_1,w_2,\cdots,w_n\right)$ and by Eq. (8), we obtain the overall values $\tilde{r}_i$ of the alternative $A_i\left(i=1,2,\cdots,m\right)$.

Step 4. calculate the scores $S\left(\tilde{r}_i\right)$ of the overall intuitionistic fuzzy preference value $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ to rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and then to select the best one(s) (if there is no difference between two scores $S\left(\tilde{r}_i\right)$ and $S\left(\tilde{r}_j\right)$, then we need to calculate the accuracy degrees $H\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_j\right)$ of the overall intuitionistic fuzzy preference value $\tilde{r}_i$ and $\tilde{r}_j$, respectively, and then rank the alternatives A_i and A_j in accordance with the accuracy degrees $H\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_j\right)$.

Step 5. Rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and select the best one(s) in accordance with $S\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$.

Step 6. End.

4 Illustrative Example

Let us suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from [54-60]). There is a panel with five possible alternatives to invest the money: ① A_1 is a car company; ② A_2 is a food company; ③ A_3 is a computer company; ④ A_4 is an arms company; ⑤ A_5 is a TV company. The investment company must take a decision according to the following four attributes: ① G_1 is the risk analysis; ② G_2 is the growth analysis; ③ G_3 is the social-political impact analysis; ④ G_4 is the environmental impact analysis. The five possible alternatives $A_i\left(i=1,2,\cdots,5\right)$ are to be evaluated using the intuitionistic fuzzy information by the decision maker under the above four attributes, as listed in the following matrix.

$$\tilde{R}=\begin{bmatrix}(0.4,0.5) & (0.3,0.6) & (0.6,0.3) & (0.7,0.2)\\ (0.3,0.7) & (0.7,0.2) & (0.7,0.2) & (0.4,0.5)\\ (0.4,0.6) & (0.5,0.1) & (0.5,0.3) & (0.6,0.3)\\ (0.4,0.3) & (0.6,0.3) & (0.3,0.4) & (0.2,0.6)\\ (0.2,0.6) & (0.7,0.3) & (0.7,0.1) & (0.5,0.3)\end{bmatrix}$$

Decision maker's subjective preference values on alternative $A_i\ (i=1,2,3,4,5)$ are as follows:

$$\tilde{\theta}_1=(0.3,0.5),\ \tilde{\theta}_2=(0.6,0.2),\tilde{\theta}_3=(0.5,0.4)$$
$$\tilde{\theta}_4=(0.7,0.2),\tilde{\theta}_5=(0.4,0.3)$$

Then, we utilize the approach developed to get the most desirable alternative(s).

Case 1: The information about the attribute weights is partly known and the known weight information is given as follows:

$$H=\{0.20\le w_1\le 0.30, 0.10\le w_2\le 0.15, \\ 0.20\le w_3\le 0.28,\ 0.30\le w_4\le 0.35, \\ w_j\ge 0,\ j=1,2,3,4,\textstyle\sum_{j=1}^{4} w_j=1\}$$

Step 1. Utilize the model (M-1) to establish the following single-objective programming model:

$$\begin{cases}\min\ D(w)=1.05w_1+0.50w_2+0.90w_3+1.20w_4\\ s.t.\ \ w\in H\end{cases}$$

Solving this model, we get the weight vector of attributes:

$$w=(0.2700\ \ 0.1500\ \ 0.2800\ \ 0.3000\ )^T$$

Step 2. Utilize the weight vector w and by Eq. (8), we obtain the overall values $\tilde{r}_i$ of the alternatives $A_i\ (i=1,2,3,4,5)$.

$$\tilde{r}_1=(0.5548,0.3383),\tilde{r}_2=(0.5357,0.3692)$$
$$\tilde{r}_3=(0.5088,0.3068),\tilde{r}_4=(0.3574,0.4003)$$
$$\tilde{r}_5=(0.5443,0.2660)$$

Step 3. Calculate the scores $S(\tilde{r}_i)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i\ (i=1,2,3,4,5)$

$$S(\tilde{r}_1)=0.2164, S(\tilde{r}_2)=0.1665$$
$$S(\tilde{r}_3)=0.2020, S(\tilde{r}_4)=-0.0430$$
$$S(\tilde{r}_5)=0.2783$$

Step 4. Rank all the alternatives $A_i\ (i=1,2,3,4,5)$ in accordance with the scores $S(\tilde{r}_i)\ \ (i=1,2,\cdots,5)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i\ (i=1,2,3,4,5)$: $A_5\succ A_1\succ A_3\succ A_2\succ A_4$, and thus the most desirable alternative is A_5.

Case 2: If the information about the attribute weights is completely unknown, we utilize another approach developed to get the most desirable alternative(s).

Step 1. Utilize the Eq. (11) to get the weight vector of attributes:

$$w=(0.2877\ \ 0.1370\ \ 0.2465\ \ 0.3288\ )^T$$

Step 2. Utilize the weight vector w and by Eq. (8), we obtain the overall values $\tilde{r}_i$ of the alternatives $A_i\ (i=1,2,3,4,5)$.

$$\tilde{r}_1=(0.5585,0.3344),\tilde{r}_2=(0.5192,0.3876)$$
$$\tilde{r}_3=(0.5103,0.3150),\tilde{r}_4=(0.3519,0.4045)$$
$$\tilde{r}_5=(0.5294,0.2793)$$

Step 3. Calculate the scores $S(\tilde{r}_i)(i=1,2,3,4,5)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i\ (i=1,2,3,4,5)$

$$S(\tilde{r}_1)=0.2241, S(\tilde{r}_2)=0.1316$$
$$S(\tilde{r}_3)=0.1953, S(\tilde{r}_4)=-0.0525$$
$$S(\tilde{r}_5)=0.2501$$

Step 4. Rank all the alternatives $A_i\ (i=1,2,3,4,5)$ in accordance with the scores $S(\tilde{r}_i)$: $A_5\succ A_1\succ A_3\succ A_2\succ A_4$, and thus the most desirable alternative is A_5.

5 Conclusion

In this paper, we have investigated intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives, in which the information about attribute weights is incompletely known, and the attribute values and preference values on alternatives take the form of intuitionistic fuzzy numbers. To determine the attribute weights, an optimization model based on the minimum deviation method, by which make use of the subjective information provided by the decision maker and the known objective information, is established to derive the attribute weights. We utilize the intuitionistic fuzzy weighted averaging (IFWA) operator to aggregate the intuitionistic fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Thus, the resultant ranking of alternative reflects both the objective information and the decision maker's subjective considerations. Finally, an illustrative example is given. Furthermore, we can also extend the developed models and procedures to deal with the MADM with interval-valued intuitionistic fuzzy information. In future research, our work will focus on the application of intuitionistic fuzzy multiple attribute decision making in the fields such as investment, personnel examination, medical diagnosis, and military system efficiency evaluation.

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