The Summary of Decision Making Problems in incomplete Soft Set

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Abstract. In this paper, three approaches to deal with missing data in soft set are analyzed, Yan Zou[1] predicated the incomplete data with the average probability, the decision value of an object with incomplete information is calculated by weighted-average of all possible choice values of the objects. Zhi Kong[2]simplifies the approach and present the simplified probability to directly instead of the incomplete information. In paper[3],Deng introduces notions of complete distance between two objects and relative dominance degree between two parameters. An object-parameter method is proposed to predict unknown data in incomplete fuzzy soft set. In the current paper, a comparison is made to illustrate the advantages and disadvantages in all the approaches, a counterexample is given to show that in some cases the data provided by this approach is not guaranteed to be positive.

Introduction

The concept of soft set originated from the work of D. Molodtsov [4]. It is a parameterized family of subsets of a universe of discourse and is a powerful mathematical model in dealing with data sets with fuzziness and uncertainty. In a broad sense, a soft set can be represented by an information system or an information table intuitively. In the past, the theory of soft sets has been extensively explored[5,6,7,8]. Compared with other mathematical tools, such as probability theory, fuzzy set theory and rough set theory, a soft set model doesn't refer to prerequisite knowledge of the raw data sets. Soft sets can be generalized to some other structures such as with interval-valued fuzzy sets [9], interval-valued intuitionistic fuzzy soft set [10], the combination of soft sets with fuzzy sets [11], with rough set [12], with vague sets [13] still yield quite another structure.

Decision making in the field of a soft set is one of the most important tasks. An optimal choice for objects can be computed by summing the values of objects on all parameters or attributes. Once some entries of an information table are unknown, it is impossible to get any information to decision problems. In such case, the soft sets are called incomplete soft sets. If we delete all objects with missing entries, then the information is not complete. Zou and Xiao [1] presented a weighted-average method for soft sets. The missing values are predicted by weighted-average of all possible choice values of the object and the weight of each possible choice value is decided by the distribution of other objects. The weighted-average method can only predict the sum of values of every object instead of all parameters, each unknown entry in information tables cannot be estimated. The average-probability method can replace all unknown entry with the mean of the known data in one column, the predicted values of all unknown entries in one parameter column are all equal, this is not reasonable in reality. In this paper[3], Deng designs a novel method, called object-parameter method which predicts individual unknown entry in soft sets and in fuzzy soft sets. This method considers all information with objects and parameters, but in some situations, the data may not be positive.

The rest of this paper is arranged as follows. Section 2 recalls some fundamental concepts from soft set theory. Two classical methods of predicting unknown data for soft sets and fuzzy soft sets are reviewed in Section 3. A modified approach in response to Deng's method is given in Section 4.

Basic Concepts about soft sets

This section recalls some basic notions on soft sets.

Definition 2.1. A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U, i.e., $F:E \rightarrow P(U)$, where $P(U)$ is the power set of U.

The soft set is a parameterized family of subsets of the set U. Every set $F(e)$, $e \in E$, from this family may be considered as the set of e-elements of the soft set (F, E), or as the e-approximate elements of the soft set.

Example 2.1. Let universe $U = \{h_1, h_2, h_3, h_4\}$ be a set of houses, a set of parameters $E = \{e_1, e_2, e_3, e_4\}$ be a set of status of houses which stand for the parameters "beautiful", "cheap", "in green surroundings", and "in good location" respectively. The mapping F be a mapping of E into the set of all subsets of the set U. Now consider a soft set (F, E) that describes the "attractiveness of houses for purchase", where

$$F(e_1) = \{h_1, h_3, h_4\}, F(e_2) = \{h_1, h_2\}, F(e_3) = \{h_1, h_3\}, \text{ and } F(e_4) = \{h_2, h_3, h_4\}.$$

A two-dimensional table is used to represent the soft set { F, E }. Table I is the tabular form of the soft set { F, E }. If $h_i \in F(e_j)$, then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries (see Table I).

Suppose

$$U = \{h_1, h_2, \ldots\ldots, h_n\}, E = \{e_1, e_2, \ldots\ldots, e_m\},$$

(F, E) is a soft set with tabular representation. Define, $f_E(h_i) = \sum_j \{h_{ij}\}$,where h_{ij} are the entries in soft set table.

TABLE I. THE TABULAR REPRESENTATION OF { H, A }

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
h_1	1	0	1	0	1	1	1	*	1	0
h_2	0	1	1	*	0	1	0	1	1	1
h_3	1	1	0	1	1	1	1	0	1	1
h_4	1	*	0	1	0	0	0	0	1	1
h_5	1	0	1	0	1	1	*	0	*	0
h_6	1	0	1	*	0	1	1	*	1	1
h_7	1	1	0	1	1	1	0	0	1	0
h_8	*	1	1	1	0	0	0	1	1	0
h_9	0	0	0	1	1	0	1	1	0	0
h_{10}	0	1	0	1	1	1	*	0	1	1
h_{11}	1	0	1	1	1	1	0	0	*	1
h_{12}	0	0	0	1	1	0	0	0	1	0

The following table shows the two approaches of average-probability and weighted-average respectively.

TABLE II. THE TABULAR REPRESENTATION OF { H, A }

U	average-probability	weighted-average
h_1	6.5	6.3
h_2	6.5	6.5
h_3	7	7
h_4	5.5	5.45
h_5	6	6.3
h_6	7	6.8
h_7	6	6
h_8	3.5	3.64
h_9	5	5
h_{10}	7.5	7.4
h_{11}	6.5	6.9
h_{12}	2	2

Proposed Approaches

In Deng's approach, let (F,E) be a fuzzy soft set over U, for $h_i, h_j \in U$ and $e_k \in E$, the relative distance from h_i to h_j with respect to e_k is denoted by

$$d_{ij,k} = \frac{h_{ik} - h_{jk}}{\sum_{l \in U_k} | h_{lk} - h_{jk} |}$$

d_{ij} is the complete distance between the values of objects h_i and h_j on all the parameters. The complete distance d_{ij} can be positive or negative, it is defined as

$$d_{ij} = \frac{\sum_{k=1}^{n} d_{ij,k}}{| \{ k \mid (i \in U_k) \wedge (j \in U_k) \} |}$$

the unknown entry h_{jl} is evaluated according to the information from the relationship between the values of objects on a certain parameter by

$$h_{jl}^{object} = \frac{\sum_{i \in U_l} (h_{il} - d_{ij})}{| U_l |}$$

But in some cases, h_{jl}^{object} may be negative.

TABLE III. THE TABULAR REPRESENTATION OF { H, A }

U	e_1	e_2	e_3
h_1	0.3	0.2	0.1
h_2	0.2	0.1	*
h_3	0.3	0.2	0.2

TABLE IV. THE TABULAR REPRESENTATION OF { H, A }

U	e_1	e_2	e_3
h_1	0.3	0.2	0.1
h_2	0.2	0.1	-0.35(0.35)
h_3	0.3	0.2	0.2

$$d_{12,1} = \frac{h_{11} - h_{21}}{| h_{11} - h_{21} | + | h_{31} - h_{21} |} = \frac{0.3 - 0.2}{| 0.3 - 0.2 | + | 0.2 - 0.1 |} = 0.5$$

$$d_{12,2} = \frac{h_{12} - h_{22}}{| h_{12} - h_{22} | + | h_{32} - h_{22} |} = \frac{0.2 - 0.1}{| 0.2 - 0.1 | + | 0.2 - 0.1 |} = 0.5$$

$$d_{32,1} = \frac{h_{31} - h_{21}}{| h_{31} - h_{21} | + | h_{11} - h_{21} |} = \frac{0.3 - 0.2}{| 0.3 - 0.2 | + | 0.3 - 0.2 |} = 0.5$$

$$d_{32,2} = \frac{h_{32} - h_{22}}{| h_{32} - h_{22} | + | h_{12} - h_{22} |} = \frac{0.2 - 0.1}{| 0.2 - 0.1 | + | 0.2 - 0.1 |} = 0.5$$

$$d_{12} = \frac{1}{2}(d_{12,1} + d_{12,2}) = 0.5; d_{32} = \frac{1}{2}(d_{32,1} + d_{32,2}) = 0.5$$

$$h_{23}^{object} = \frac{\sum_{i \in U_3} (h_{i3} - d_{i2})}{| U_3 |} = \frac{(h_{13} - d_{12}) + (h_{33} - d_{32})}{| U_3 |} = -0.35$$

As $h_{23}^{object} < 0$, therefore, the algorithm should be altered, it is clear that $| d_{ij,k} | \leq 1$, and $| d_{ij} | \leq 1$, so it is sure that $| h_{jl}^{object} | \leq 1$, but it is not necessarily to be positive. So we can replace the data by its abstract value instead. The relationship between values of an object on all parameters is also very important in predicting unknown data in an incomplete fuzzy soft set. In Deng's approach, let (F,E) be a fuzzy soft set over U, for $h_i \in U$ and $e_k, e_l \in E$, the degree of e_k being relatively dominant to e_l regarding h_i is defined by

$$r_{i,kl} = \frac{h_{ik} - h_{il}}{h_{ik} + h_{il}}$$

The degree of e_k being definitely dominant to e_l is defined by

$$c_{kl}=\frac{\sum_{i\in U_k\cap U_l} r_{i,kl}}{|U_k\cap U_l|}$$

the degree of average dominance of e_k to e_l is characterized by

$$v_{kl}=\frac{c_{kl}}{\sum_{\{q|U_q\cap U_l\neq\varnothing\}}|c_{ql}|}$$

The unknown entry h_{jl} is evaluated according to the information from the relationship between the parameters regarding objects h_j by

$$h_{jl}^{parameter}=\frac{\sum_{k\in E_j}(h_{jk}-v_{kl})}{|E_j|}$$

From the above table,

$$r_{1,23}=\frac{h_{12}-h_{13}}{h_{12}+h_{13}}=1/3; r_{3,23}=\frac{h_{32}-h_{33}}{h_{32}+h_{33}}=0; r_{1,13}=\frac{h_{11}-h_{13}}{h_{11}+h_{13}}=0.5; r_{3,13}=\frac{h_{31}-h_{33}}{h_{31}+h_{33}}=0.2;$$

$$c_{23}=\frac{1}{2}(r_{1,23}+r_{3,23})=1/6; c_{13}=\frac{1}{2}(r_{1,13}+r_{3,13})=0.35; v_{13}=\frac{c_{13}}{|c_{23}|+|c_{13}|}=0.6774; v_{23}=\frac{c_{23}}{|c_{13}|+|c_{23}|}=0.3226;$$

$$h_{23}^{parameter}=\frac{\sum_{k\in E_j}(h_{2k}-v_{k3})}{|E_j|}=\frac{(h_{21}-v_{13})+(h_{22}-v_{23})}{|E_j|}=-0.35$$

As $h_{23}^{parameter}<0,$ therefore, the algorithm should be changed, as $|r_{ij,k}|\leq 1,$ and $|v_{ij}|\leq 1,$ so it is sure that $|h_{jl}^{parameter}|\leq 1,$ but it is not necessarily to be positive. In both cases, we can replace the data with its abstract value, that is

$$h_{jl}^{object}=\frac{|\sum_{i\in U_l}(h_{il}-d_{ij})|}{|U_l|}; h_{jl}^{parameter}=\frac{|\sum_{k\in E_j}(h_{jk}-v_{kl})|}{|E_j|}$$

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