Simulation on Trajectory for the Increasing Quality of Cable Throwing in the Air

Jianhua WU[1, a], Runqing QIU[1, b], Xin ZHAO[2, c]

[1]Department of Mechanical Engineering, Tianjin university of Technology, Tianjin, 300384, China

[2]School of Maritime, Tianjin university of Technology, Tianjin, 300384, China

[a]email: jhwu@tjut.edu.cn, [b]email:qrunqing@163.com, [c]email:uglyroom@tom.com

Keywords: Cable-throwing; Trajectory; Simulation

Abstract. The cable is indispensable appliance when ship berth alongside. With the increasing of ship's tonnage, the distance of ship berth is also increasing. The distance and approaches of seaman cable-throwing have failed to meet the demand, so designing appliances of cable throwing is necessary. To solve this problem, the trajectory calculation and analysis need to be done in the process of throwing-cable. The paper established an increasing quality of cable throwing model in the air based on the theory of Runge-Kutta Equation and its trajectory is simulated by MATLAB. It was lay the theoretical foundation for the precise design of the cable-throwing appliance.

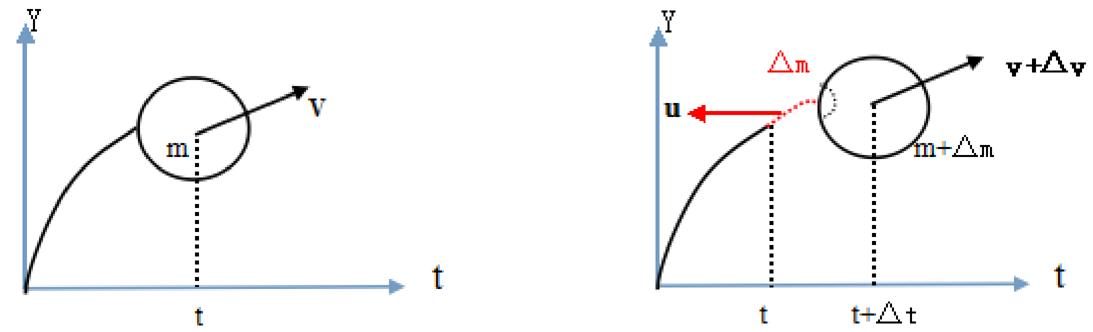
Introduction

Cable-throwing is defined as the cable thrown to dock or others ship's seeker from the ship. It is widely used in mooring boats, ship berthing alongside and other operations between ships or ship and coast [1]. The operation of cable-throwing is a common way which completed by seaman and seldom with the help of equipment. Due to the restriction of human factors, artificial cable-throwing casting distance is limited. So artificial cable-throwing for berthing of small vessels can meet the need, but for large cargo ships, manpower alone will be not satisfied the requirement since the distance.

Nowadays, most of the vessels are equipped with cable-throwing guns. Because the cable-throwing guns are mainly driven by compressed air or gunfire [2], the guns are not very safety. To realize the need for accurate throwing the heaving ball and with cable, it is necessary to calculate the trajectory of space for establishing a more precise physical model. The motion of cable-throwing also includes shedding and dragging cable type. This paper mainly studies the dragging cable type and takes considered into the cable quality as a process variable as well as air resistance to build a physical model. Model is gotten by Runge-Kutta method, the trajectory is simulated by MATLAB.

Building Physical Model Process of Cable-throwing-cable

The projectile of Cable-throwing gun is composed by a heaving ball and a hollow tube which internal cable wound on it. After the cable ball launched, the cable that wrapping in wall begin constantly dragging. At the initial time t, the system which consist of heaving ball and with cable total mass is m=1.2kg,the initial velocity $v=v_0$ =40m/s, the angle with the horizontal directionθ_0=+ 30°; Afterwards Δt, cable has a small mass increased Δm ($\Delta m>0$). The quality system of particles into (1.2+Δm) kg, velocity becomes (40+Δv) m/s($\Delta v<0$). Dragging of the cable of the absolute velocity is setting as u. The cable is assumed to no relative speed between the ground when it dragged on, i.e. u = 0. Speed direction of each time is shown in Figure 1.



(a) Time t

（b） After Δt

Fig.1. The diagram of speed direction of each time

In the initial time t, the total momentum of the ball and with cable:

$$P_1 = \mathrm{m} \cdot \mathrm{v}_0 = \mathrm{m} \times 40 \tag{1}$$

At time t + Δt, the total momentum of the particle system becomes as follows:

$$\begin{aligned} P_2 &= (\mathrm{m} + \Delta \mathrm{m})(\mathrm{v}_0 + \Delta v) + \Delta \mathrm{m}(-u) \\ &= (\mathrm{m} + \Delta \mathrm{m})(40 + \Delta \mathrm{v}) + \Delta \mathrm{m} \cdot 0 \\ &= m \times 40 + \mathrm{m}\Delta \mathrm{v} + 40\Delta \mathrm{m} + \Delta \mathrm{m}\Delta \mathrm{v} \end{aligned} \tag{2}$$

According Momentum Theorem: Impulse objects' force equals change in momentum of the object [3]:

$$F \cdot \mathrm{dt} \;=\; \mathrm{d}P \tag{3}$$

From (1), (2) and (3), (4) could be obtained :

$$\begin{cases} dP = P_2 - P_1 = F \cdot \mathrm{dt} \\ (\mathrm{m} \times 40 + \mathrm{m}\Delta \mathrm{v} + 40\Delta \mathrm{m} + \Delta \mathrm{m}\Delta \mathrm{v}) + \mathrm{m} \times 40 = F \cdot \mathrm{dt} \\ \mathrm{m}\Delta \mathrm{v} + 40\Delta \mathrm{m} + \Delta \mathrm{m}\Delta \mathrm{v} = F \cdot \mathrm{dt} \end{cases} \tag{4}$$

Because the time interval Δt is extremely small, the impact of second-order over small amount of momentum can be neglected, (4) could be the follow:

$$F = \mathrm{m}\frac{\Delta \mathrm{v}}{\Delta \mathrm{t}} + 40\frac{\Delta m}{\Delta t} \tag{5}$$

When (5) Δt approaches to 0 and (5) change into:

$$F \;=\; \mathrm{m}\,\frac{\mathrm{dv}}{dt} \;+\; 40\,\frac{dm}{dt} \tag{6}$$

$\mathrm{m} = \mathrm{m}_0 + \int_0^t \rho_{\mathrm{c}} \mathrm{v} dt$, wherein the linear density of the cable (kg /m), is selected cable diameter d = 6mm,ρ_c=0.018kg/m, substituted into (6), having the formula:

$$F = (1.2 + 0.018\int_0^t \mathrm{vdt})\frac{\mathrm{dv}}{dt} + \mathrm{v}(0.018 \times v) \tag{7}$$

Where F is the resultant force acting on the variable mass system of particles. Formula (7) is the heaving motion physics equations of the ball and with cable constantly shedding process. Combined with Newton's Second Law, in the natural coordinate system above is expressed as:

$$\vec{F} = \mathrm{m}\vec{a} = \mathrm{m}\ (\frac{v^2}{p}\vec{\eta} + \frac{dv}{dt}\vec{\tau}) \tag{8}$$

Projecting of the ball and cable system along with the direction of motion and perpendicular to the direction of the velocity, as shown in Figure 2, and (9) is obtained:

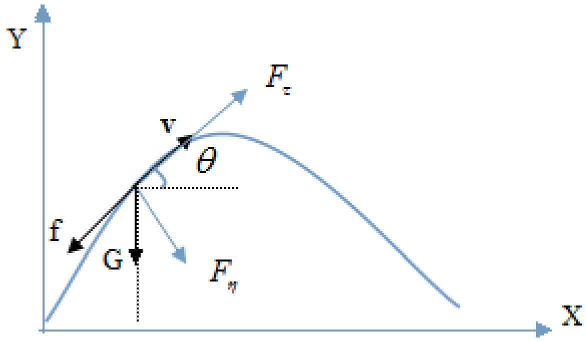


Figure.2. Force analysis

$$\begin{cases} F_\tau = (1.2 + 0.018\int_0^t \mathrm{v}dt)\dfrac{dv}{dt} + v(0.018 \times v) \\ F_\eta = (1.2 + 0.018\int_0^t \mathrm{v}dt)\dfrac{v^2}{\rho} \\ \dfrac{1}{\rho} = \left|\dfrac{d\theta}{ds}\right| = -\mathrm{v}\dfrac{d\theta}{dt} \end{cases} \tag{9}$$

Direction **τ** is parallel with the speed direction, and predetermined that consistent with the velocity is positive. Direction **η** is perpendicular with the velocity direction and downward is positive.1/ρ is radius of curvature of the trajectory of the system. θ is refer to direction between instantaneous velocity v and angle of the horizontal.

$$\begin{cases} -G\sin\theta - f_\tau = F_\tau \\ G\cos\theta = F_\eta \end{cases} \tag{10}$$

Since the actual projectile motion velocity is much lower than the low flight speed of cannonball, resistance should be considered which not be neglected, that is fr=kv. Substituting into (10) equation:

$$\begin{cases} -(1.2+0.018\int_0^t vdt)g\sin\theta - kv = (1.2+0.018\int_0^t vdt)\dfrac{dv}{dt} + v(0.018\times v) \\ \dfrac{d\theta}{dt} = \dfrac{-g\cos\theta}{v} \end{cases} \tag{11}$$

k is the air resistance coefficient, the motion of the system's resistance to meet the Viscous Resistance Stokes Law [4], k=－6πη$_a$r. The radius of the ball r=30(mm), η$_a$ for air viscosity coefficient (pa*s). From the table of all kinds of material viscosity coefficient numerical values [5], air coefficient of viscosity at 20°C,ηa=1.82×10^{-5}(pa * s).

And combine two kinematic equations:

$$\begin{cases} \dfrac{dx}{dt} = v\cos\theta \\ \dfrac{dy}{dt} = v\sin\theta \end{cases} \tag{12}$$

Consisting of ordinary differential equations:

$$\begin{cases} \dfrac{dv}{dt} = -9.8\times\sin\theta - \dfrac{6\pi\times1.82\times10^{-5}\times0.03}{1.2+0.018\int_0^t vdt}v - \dfrac{0.018}{1.2+0.018\int_0^t vdt}v^2 \\ \dfrac{d\theta}{dt} = \dfrac{-9.8\times\cos\theta}{v} \\ \dfrac{dx}{dt} = v\cos\theta \\ \dfrac{dy}{dt} = v\sin\theta \end{cases} \tag{13}$$

Research the Solution of Heaving Ball and with Cable's Physics Model

The analytic method of heaving ball and with cable

For (13) where the above group of ordinary differential equations, which form more complex to solve using conventional separation of variables, it can only use an approximate analytic method to solve it. Runge-Kutta method that is a solution of the nonlinear problem and very effective, it indirectly using Taylor formula of thought to construct high-precision numerical methods which at each node in the interpolation using Taylor series expansion of the derivative of the function with n points on a linear function of the value of the combined instead of f, then follow the Taylor formula to start [6]:

$$\begin{cases} k_1 = f(t_n, u_n), \\ k_2 = f(t_n + c_2 h, u_n + a_{21} h k_1), \\ k_3 = f(t_n + c_3 h, u_n + a_{31} h k_1 + a_{32} h k_2), \\ \quad\vdots \\ u_{n+1} = u_n + h\sum_{i-1}^{s} w_i k_i \end{cases} \tag{14}$$

Ordinary differential equation (14) could be solved by Runge-Kutta method in the software of MATLAB.

Writing MATLAB program and solving

MATLAB provides the numerical solution of ordinary differential equation—Runge-Kutta method, it is generally invoked format:[X,Y]=ode45 (@f, [x0 , xn], y0). X and Y are the two variables. X corresponds to the argument X in the range of solving x0 to xn, f is a solving function, y0 is the initial value of function [7].

Transform (13) to meet the format of MATLAB programming, set Y=[$\int_0^t vdt$,v,θ, x, y], the equation becomes:

$$\frac{dY}{dt} = \begin{bmatrix} Y_2 \\ -9.8\sin(Y_3) - \dfrac{6\pi\times1.82\times10^{-5}\times0.03}{1.2+0.018(Y_1)}Y_2 - \dfrac{0.018}{1.2+0.018(Y_1)}Y_2{}^2 \\ \dfrac{-9.8\cos(Y_3)}{Y_2} \\ Y_2\cos(Y_3) \\ Y_2\sin(Y_3) \end{bmatrix} \tag{15}$$

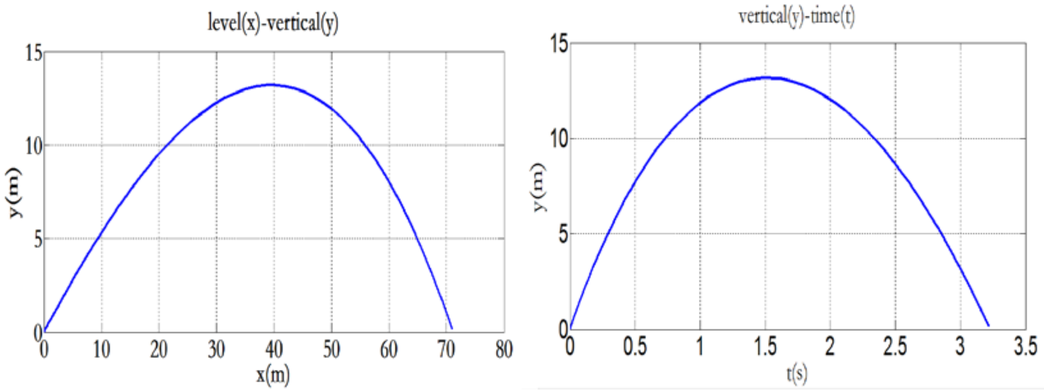
Equation (15) is function f. Known by the original data: Y0= [0; 40; pi/6; 0; 0]. So: [X,Y]=ode45 (@f, [0, 3.2], [0; 40; pi/6; 0; 0]).

The following is part of the numerical analysis of the data:

t	Y(1)	Y(2)	Y(3)	Y(4)	Y(5)
0	0	40	0.52359877	0	0
5.02E-06	0.000200951	39.99985	0.52359771	0.00017402	0.00010047
0.00078371	0.031339471	39.97736	0.52343243	0.02714208	0.01566747
0.01569803	0.624392861	39.55249	0.52024593	0.54126031	0.31129303
0.17811930	6.708748813	35.53814	0.4829570	5.87367911	3.24051442
0.57811930	19.45714876	28.76863	0.37066491	17.4489276	8.56641677
1.21811930	35.71435461	22.75187	0.13103144	33.1161551	12.7583545
1.61811930	44.38991234	20.82011	-0.0494286	41.771683	13.1358416
2.01811930	52.51184077	19.95024	-0.2401928	49.7974380	11.9729239
2.65811930	65.28958117	20.28369	-0.5299850	61.5799145	7.14643172
3.03358948	73.06278253	21.18918	-0.6761386	67.9661112	2.72697734
3.20000000	76.63103801	21.70570	-0.7340421	70.6820593	0.41342663

Simulate the trajectory of heaving ball and with cable system in air

Through the way of curve fitting over drawing heaving ball and with cable system displacement movement graph by the numerical solutions obtained above, shown in Figure 3:



(a) The level and vertical curves (b) The vertical and time curves

Figure.3. Trajectory of Cable-throwing-cable

It can be obtained from the Figure 3.(a), in the early speed of 40 m/s, under 30° with the horizontal direction of the initial conditions , the cable-throwing could be as far as dumped 71 meters , a maximum height of about 13.1 meters. Figure.3.(b) show the Cable-throwing could be moved as long as 3.2 seconds.

Conclusion

Cable-throwing is working in the air where the resistance could not be neglected. According to the form of movement, cable-throwing is divided into the two kinds. They are dragging and slacking movement. The way of dragging would lead to an increasing the quality of the cable, and the slacking rope would result in a decreasing the quality of cable. In the paper, firstly building up differential equations based on the Newton's Second Law which applied in the variable mass. Then the trajectory for the increasing quality of cable throwing in the air is simulated by MATLAB. It is laid the theoretical foundation for the precise design of the dragging cable-throwing gun.

References:

[1] Yuguang Gong, XinWang. Sailor’s technology [M]. Dalian Maritime University Press, 2007,12.132-134.

[2] Wenbin GU, Ming LU, Jianqing LIU, Qin-xingDONG，et al. Simulation and experimental research on line throwing rocket with flight [J].Science Direct, 2014(10).

[3] Aiqin WANG. Theoretical mechanics [M]. Northwestern University Press, 2009,01. 187-188.

[4] Yuqing Zhou, Zhaoning Ye, Zong-han Wu. Ball in air resistance calculation and analysis[J].Physics and Engineering, 2002(1).

[5] Anshen QI, Chanying DU. Basic mechanics [M]. Higher education press, 2012,12. 361-363.

[6] Jun Qin. Runge Kutta method in solving differential equation model of the application [D].Anhui University, 2010.

[7] Shuxiang Wan. MATLAB mathematical problem solving [M].Harbin Map Publishing House, 2007.193-195.