Bifurcation control in a small-world network model via TDFC

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Abstract. In this paper, we analyze Hopf bifurcation phenomenon in a nonlinear small-world network model with time delay and fixing probability.By choosing time-delay as a bifurcation parameter, we obtain that the model exhibits of Hopf bifurcation when time delay passes through a critical value. In order to control the undesirable Hopf bifurcation, a time-delayed feedback control strategy is proposed. By choosing feedback parameter properly, it shows that the onset of Hopf bifurcation has been postponed without changing the original equilibrium point of the system.Finally, numerical simulations are given to verify the theoretical results.

Introduction

There are many complex systems can be describe as network in nature. Although the scale of network is huge, there is a relatively short path between any two notes which indicates that this network has the performance of small-world and aggregation [1]. This so called as small-world network has attracted plenty of attention since the work of Watts and Strogatz which consists of a rewired regular lattice with a very small fraction of long-range connections [2]. In order to mimic the phenomenon of nonlinear interactions and response in real life networks and consider the impact of nonlinear elements, Yang introduced nonlinear effects on the linear spreading models with time-delays and studied the fractional dimension of such time-delayed linear models [3-4]. On the other hand,Watts added linkages between pairs of randomly chosen nodes with a very small probability p [5]. For the sake of clarifying the relation between the network evolution with different probability p and dynamical behaviors. Li proposed a model which describes the effect of the new link-adding probability p on the stability and bifurcation behaviors of disease spreading in N–W model with one dimension case [6].

During the past decades, the research of bifurcation control has more and more deeply. Bifurcation control refers to design a controller to reduce some existing bifurcation dynamics of a given nonlinear system so that achieving some desirable dynamical behaviors[7]. The topical aim of bifurcation control includes delay the onset of an inherent bifurcation, lead into a new bifurcation in a more suitable parameter values, control some numbers of limit cycle from the bifurcation, change the parameter value of an existing bifurcation point,etc.[8].

Various methods of bifurcation control can be found in recent studies, which has theoretical significance to improve the stability and reliability of the system [9-10]. In [11], nonlinear feedback controller for Hopf bifurcation was considered by Yagnoobi and Abed.Chen [12] developed a dynamic state feedback control law incorporating a washout filter to control Hopf bifurcation in the Lorenz system. Zhao [13] applied a delayed feedback control strategy to control a Hopf bifurcation by using measure parameter as bifurcation parameter. In this paper, time-delayed feedback control strategy will be extended to consider control of Hopf bifurcations in a delayed systems.

In this paper,we choose time-delay as a bifurcation parameter,and a time-delayed feedback control strategy is applied to control the Hopf bifurcation in a small-world network model considering fixing probability p. As to the strategy can achieve Hopf bifurcation control with the same value of the equilibrium point in the original system, the nature of the uncontrolled system can be retained completely.By using this strategy, we will investigate that we increase the critical value of time-delay and postpone the onset of undesirable Hopf bifurcation, besides, we extend the stable range in parameter space and enhance the performance of system.

The remainder of this paper is organized as follows. In Section 2, the results of the stability and Hopf bifurcation of the original small-world network model are briefly reviewed. In Section 3,we apply a time-delayed feedback control strategy to original model, and then study the existence of the Hopf bifurcation of this model. To verify the theoretic analysis, numerical simulations are given in Section 4. Finally, Section 5 concludes with some discussions.

Existence of Hopf bifurcation in uncontrolled System

In this section,we consider a time delay Hopf bifurcation in a small-world network model.The uncontrolled system can be formulated as follows:

$$\frac{dV(t)}{dt}=1+2pV(t-\delta)-\mu(1+2p)V^2(t-\delta) \qquad (1)$$

where V is the total influenced volume, μ is a measure of nonlinear interactions in the network, and p is the probability of add linkages between pairs of randomly chosen nodes. The model contains regular lattices with $p=0$, small-world networks with $0<p<<1$ and random networks with $p=1$.

Let V^* be an equilibrium point of system (1).It then satisfies

$$V^*=\frac{p+\sqrt{p^2+\mu(1+2p)}}{\mu(1+2p)} \qquad (2)$$

For convenience,the results of system (1) are summarized as follows.The analysis of specific details for the system (1) has been described in[10].

Theorem 1 For the system (1),when $\delta_0=-\pi/2b_1,\omega_0=-b_1$ we can get the following results in:

(1)When $\delta<\delta_0$, the equilibrium point of the system (1) is locally asymptotically stable;

(2)When $\delta=\delta_0$,a Hopf bifurcation occurs;

(3)When $\delta>\delta_0$,the equilibrium point of the system (1) is unstable.

Hybrid control of bifurcation

Previous studies have suggested that when system loses its stability and a Hopf bifurcation occurs,it is easy to cause the information transmission congestion, and even lead to the collapse of the system.Therefore, we need to design a effective controller to control the Hopf bifurcation,and expand the stable interval of equilibrium point.Thus,we add time-delayed feedback controller,the controlled system can be described as

$$\frac{dV(t)}{dt}=1+2pV(t-\delta)-\mu(1+2p)V^2(t-\delta)+\alpha(V(t-\delta)-V(t)) \qquad (3)$$

where α is a feedback parameter and $0<\alpha<1$.When $\alpha=0$ the controlled system (3) reduces to the original system (1).It is obvious that the equilibrium point of controlled system is equal to the original system.So,we set $v(t)=V(t)-V^*$,inserting it into Eq.(3) and linearizing at the equilibrium point and by Taylor expansion,we have

$$\dot{v}(t)=-\alpha v(t)+b_1v(t-\delta) \qquad (4)$$

where $b_1=\alpha-2\sqrt{p^2+\mu(1+2p)}$,and the characteristic equation of Eq.(4) is

$$\lambda+\alpha-b_1e^{-\lambda\delta}=0 \qquad (5)$$

If the characteristic Eq.(4) has pure imaginary roots $\lambda=\pm i\omega,\omega>0$,inserting then into the Eq.(5) and separating the real and imaginary parts

$$\begin{cases}\alpha + b_1\cos\omega\delta = 0\\ \omega + b_1\sin\omega\delta = 0\end{cases} \tag{6}$$

Therefore

$$\omega_0 = \sqrt{b_1^{\,2} - \alpha^2}, \delta_0 = \frac{1}{\omega_0}\arccos\left(-\frac{\alpha}{b_1}\right) \tag{7}$$

Then we consider whether Eq.(5) has roots with positive real part when $\delta = \delta_0$ Let $k + i\omega$ be a root of Eq.(5),and $k > 0, \omega > 0$,we get

$$\begin{cases}k - \alpha - b_1 e^{-k\delta}\cos\omega\delta = 0\\ \omega + b_1 e^{-k\delta}\sin\omega\delta = 0\end{cases} \tag{8}$$

From Eq.(8),we know

$$\frac{(2n+1)\pi}{2} < \omega\delta < \frac{(2n+3)\pi}{2}, n = 0,2,4,\cdots, \omega\delta < \frac{(2n+1)\pi}{2}, n = 0,2,4,\cdots \tag{9}$$

Therefore,Eq.(4) may have roots with positive real part except for $n = 0$ .In order to find a Hopf bifurcation point,we consider transversality condition is

$$\left.\frac{d(\mathrm{Re}\,\lambda)}{d\delta}\right|_{\delta=\delta_0} > 0, \quad \frac{d\lambda}{d\delta} = -\frac{b_1\lambda e^{-\lambda\delta}}{1 + b_1\lambda e^{-\lambda\delta}} \tag{10}$$

Hence,let $\lambda = \pm i\omega$,we have

$$\frac{d\lambda}{d\delta} = -\frac{b_1(k + i\omega)e^{-k\delta}(\cos(\omega\delta) - i\sin(\omega\delta))}{1 + b_1\delta e^{-k\delta}(\cos(\omega\delta) - i\sin(\omega\delta))} \tag{11}$$

As we know,when $\delta = \delta_0, \sin\omega_0\delta_0 = -\omega_0 / b_1$,we obtain

$$\mathrm{Re}\left(\frac{d\lambda}{d\delta}\right)_{\delta=\delta_0} = \frac{\omega_0^{\,2}}{(1 - \alpha\delta_0)^2 + (\omega_0\delta_0)^2} > 0$$

Thus,we know that the Eq.(4) has at least a root with positive real parts when $\delta > \delta_0$ the equilibrium point of Eq.(3) is unstable and there must be a bifurcation.

Based on the above analysis ,we can get the following theorem.

Theorem 2 For the controlled system (3),when $\delta_0 = 1/\omega_0 \arccos(-\alpha / b_1), \omega_0 = \sqrt{b_1^2 - \alpha^2}$, we can easily obtain

(1)When $\delta < \delta_0$, the equilibrium point of the controlled system (3) is locally asymptotically stable;

(2)When $\delta = \delta_0$,the controlled system (3) exists a Hopf bifurcation at equilibrium point.

(3)When $\delta > \delta_0$,the equilibrium point of the controlled system (3) is asymptotically unstable.

Numerical simulations

In this section, we illustrate the effectiveness of time feedback control by numerical simulation.For comparison,we choose the system parameters as used in [6],with $p = 0.1$, $\mu = 0.8$.

Firstly,we choose $\alpha = 0$,which means the system is the original model.From calculating we can get $\delta_0 = 0.7854, \omega_0 = 2$ .The dynamical behavior of the uncontrolled model (1) is illustrated in Fig.1 and Fig.2.It is shown that when $\delta < \delta_0$,trajectories converge to the equilibrium point, when δ passes through δ_0 ,the equilibrium point loses stability and a Hopf bifurcation occurs.

Then,we consider the influence of controlled model for Hopf bifurcation.We choose $\alpha = 0.5$,and we get $\delta_0 = 1.05, \omega_0 = 1.393$ .It is shown that the critical value τ_0 increases.The dynamical behavior of the controlled model (3) is illustrated in Fig.3 and Fig.4.We can see that by adjusting the control parameter, limit cycle in the uncontrolled system becomes an asymptotically stable equilibrium point in the controlled system.

Finally,the bifurcation diagram of original system and controlled system are obtained in Fig.5

and Fig.6,respectively.It is shown that the onset of Hopf bifurcation is postponed, and the stable range in parameter space is extended.

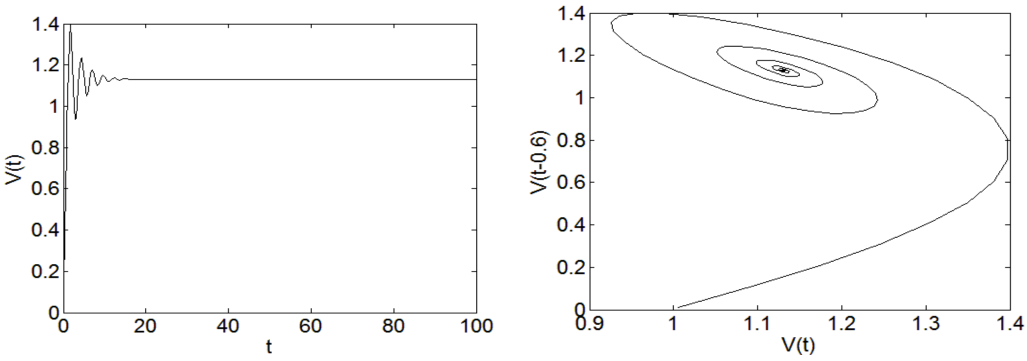

Fig. 1 Waveform graph and phase portrait of uncontrolled system with $\delta = 0.6$

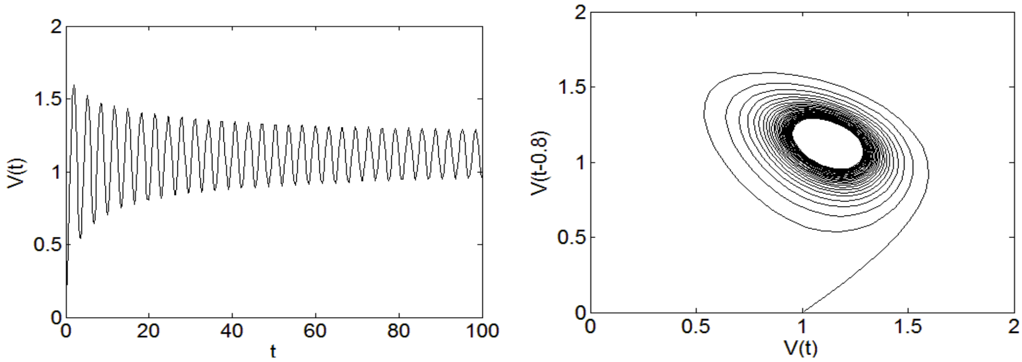

Fig. 2 Waveform graph and phase portrait of uncontrolled system with $\delta = 0.8$

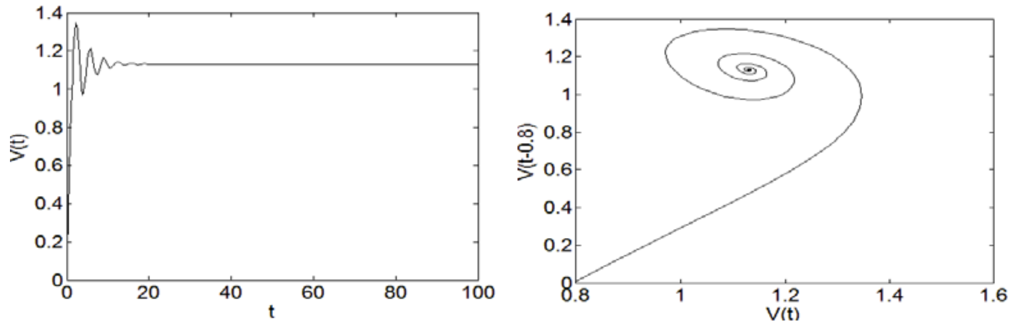

Fig. 3 Waveform graph and phase portrait of controlled system with $\delta = 0.8, \alpha = 0.5$

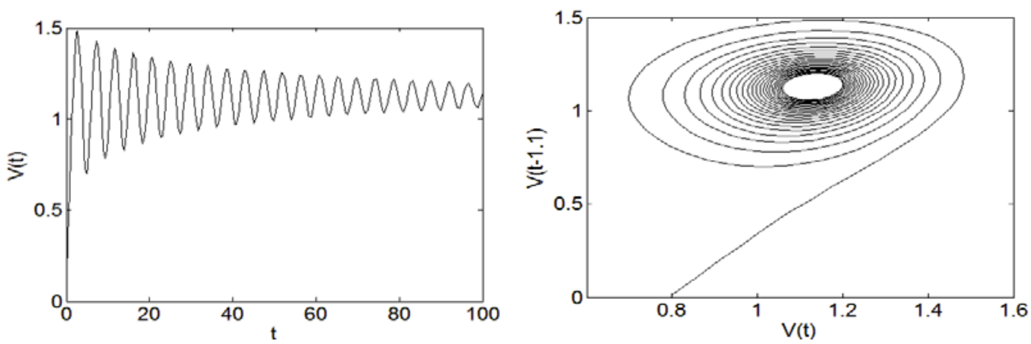

Fig. 4 Waveform graph and phase portrait of controlled system with $\delta = 1.1, \alpha = 0.5$

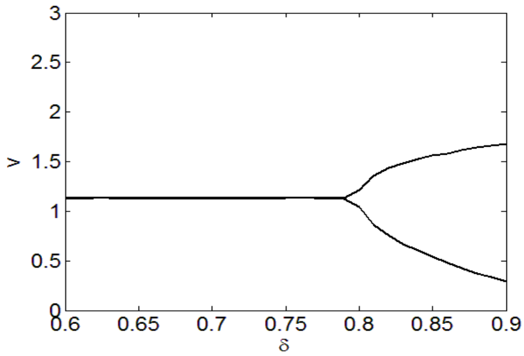

Fig. 5 Bifurcation diagram of uncontrolled system with $\alpha = 0, \delta_0 = 0.79$

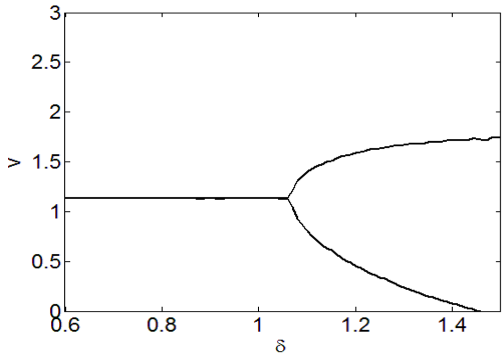

Fig. 6 Bifurcation diagram of controlled system with $\alpha = 0.5, \delta_0 = 1.05$

Conclusion

In this paper,we study the stability analysis and control of Hopf bifurcation for a small-world network model.In order to enhance the stability of system,time-delayed feedback control has been proposed.By selecting appropriate feedback parameter,this method can effectively postpone the onset of Hopf bifurcation so that achieving some desirable dynamical behaviors.Numerical results have been presented to verify the validity of the theoretical analysis.

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