L1-L2 hybrid noise model to image super-resolution

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Abstract. L1-L2 hybrid noise model (HNM) method is proposed in this paper for image/video super-resolution. This method has the advantages of both L1 norm minimization (i.e. edge preservation) and L2 norm minimization (i.e. smoothing characterization). In view of noise distribution changing and selecting L1 norm minimization or L2 norm minimization, we propose an efficient adaptive membership degree (AMD) method, which get the ideal result but the proposed AMD method can reduce the number of iterations and save much computational cost. Experimental results indicate that the proposed method is of higher peak signal to noise ratio (PSNR) and structural similarity (SSIM). And it has better reconstructed effect in edge and smoothing part.

Introduction

Super-resolution reconstruction, using prior information of the target, the information of a single image and the complementary information among multi images and extracting the high frequency information of low resolution image, with time bandwidth for spatial resolution, is used to make the reconstructed result more close to the ideal and original image. It's widely applied to image/video process, such as remote sensing image, medical image, video surveillance and high-definition television etc. The current researches concentrate mainly on spatial domain which include Iterative back projection (IBP) [1], Projection on to convex sets (POCS)[2] , Maximum a posteriori (MAP) [3]-[5]and Maximum likelihood (ML), etc. Where, Iterative back projection (IBP) method has small compute and fast convergence rate, but we are difficult to use prior information and the final result of super-resolution isn't unique. Currently, the majority methods which are used widely include Projection on to convex sets (POCS) and Maximum a posteriori (MAP). We can use the prior information conveniently, but we must consider its calculations and slow convergence rate.

In generally, the LR frames need to be registered previously via motion estimation, the point spread function (PSF) estimation and photometric correction, etc [6]-[9].A mathematical model is then used to bridge the LR observations with the unknown HR scene. Finally, Maximum likelihood (ML) estimation techniques are often used to reconstruct the HR scene [7] [9]. Most of the existed ML estimators assume that the noise in the LR observation model is Gaussian distribution [10] [11]. However, for many real world image sequences, we can find the Laplace distribution is more accurate to model that the impulsive noise (such as the salt and pepper noise) is inside them.

Mostly, the noise submits to Gaussian distribution or Laplace distribution. In order to suppress them preferably in the SR image reconstruction process, we propose L1-L2 hybrid noise model which includes the advantages of the ML estimation with Gaussian additive and Laplace impulsive noise in this paper. Moreover, we propose an efficient adaptive membership degree (AMD) method, which is able to terminate the iteration efficiently and save much computational cost. The experiments indicate that the proposed method can effectively reconstruct the HR images from the LR images, preserving image details and suppressing noise well.

Section 2 introduces the image model, which include the deduce process of observation model, motion model and PSF. Section 3 describes our method in detail. Section 4 presents experiment results and section 5 concludes the paper.

Discussed problems

Observation model

In this paper, we use the following linear observation model for LR image formation7-8, as follows:

$$y^k = D_k H_k F_k x + N_k, k = 1, ..., K. \quad (1)$$

Where K is the number of the LR images, x is the HR image, and y^k is the k^{th} LR images. The matrices D_k and H_k represent the sensor spatial sampling and the system PSF, respectively. F_k is the geometric motion operator between y^k and x ,and N_k is the system noise.

Motion model

In this paper, we use the planar projective motion model (8DoF)[8] to capture the geometric motion operator . The planar projective motion model (8DoF) is mostly suitable for motion model that planar or approximate planar objects are captured from a variety of angles. For small image regions and short time sequences, the planar projective (8DoF) motion model can often approximate adequately the true underlying motion when the overall scene is not completely described.

The point-spread function (PSF)

The PSF often be called the blurring which caused by camera optics and the spatial integration performed by a CCD sensor. As in many literatures [8], the PSF is approximated by a simple parametric function which focuses on each low-resolution pixel: the two most common are an isotropic 2D Gaussian with a variance σ_{PSF}^2 .

We apply the planar projective (8DoF) motion model to handle the original image Lena and then $\sigma_{PSF}^2 = 0.4$, down-sampled it with a factor of two, Gaussian noise with a variance $\sigma^2 = 0.08$ to generate four low images as follows.

Fig 1.use 8DoF method to low resolution image

Methodology

Formulation

The noise N_k in Eq. (1) is often assumed to be Gaussian or Laplace distributed, and ML estimation can be calculated by the L_p norm minimization criterion[7] for the HR image $\hat{x}$

$$\hat{x} = \arg\min_x \left\{ \sum_{k=1}^{N} \| y^k - D_k H_k F_k x \|_p^p \right\}, p = 1, 2. \quad (2)$$

Where $p = 1$ corresponds to Laplace distribution, and $p = 2$ corresponds to Gaussian distribution.

Since the super-solution is an ill-posed problem, so the solution of Eq. (2) is not unique. In order to get a stable solution, we use Bilinear Total Variation (BTV) regularization method.

Using the probability density functions (PDF) of Gaussian, we can get the ML estimation of σ_L and m_L are calculated as

Hybrid Noise Model (HNM)

$$\hat{\sigma}_L, \hat{m}_L = \underset{\sigma_L, m_L}{ArgMax}(P_L(\underline{\mathrm{V}})) = \underset{\sigma_L, m_L}{ArgMax}(\ln P_L(\underline{\mathrm{V}})) \quad (3)$$

The same scheme can be used to estimate the Gaussian model parameters. We use the following generalized likelihood ration test (GLRT) [9], to decide between the two hypotheses about the noise model:

$$\frac{P_G(\underline{V};\hat{\sigma}_G,\hat{m}_G)}{P_L(\underline{V};\hat{\sigma}_L,\hat{m}_L)}>1 \tag{4}$$

According to the equations, we define $\nu \overset{\Delta}{=} \hat{\sigma}_L / \hat{\sigma}_G$

$$\nu \overset{\Delta}{=} \frac{\hat{\sigma}_L}{\hat{\sigma}_G} > \left(\frac{\pi}{2e}\right)^{\frac{1}{2}} \approx 0.7602 \tag{5}$$

If the ratio $\nu > 0.7602$, the noise trends to submit to Gaussian distribution. Conversely, noise submits to Laplace distribution.

Often, the L_2 – norm for Gaussian noise model has a good performance of smoothing image, while the L_1 – norm for Laplace noise model can well preserve the details of image, such as edge.

In order to adaptively balance between the image smoothing and edge preserving, we define the following membership function $H(\nu)$ for the Laplace distribution which is about ratio ν (observe the red line in Fig.2).We can find a phenomenon which $\nu = 0.7602$, then two noise distribution tend to equation. i.e. $H_{0.7602} = 0.5$ .In addition, we use scale weight parameter α from Eq. (3) to constraint H_0 as follows

$$H_0 = \begin{cases} \alpha + 0.5, 0 \le \alpha < 0.5 \\ 1.5 - \alpha, 0.5 \le \alpha \le 1 \end{cases} \tag{6}$$

So

$$H(\nu) = \frac{0.5 - H_0}{0.7602}\nu + H_0 \tag{7}$$

Obviously, the membership function corresponding to the Gaussian model is $1 - H(\nu)$ (observe the blue line in Fig.2).

So the objective function of the data term with the combination of additive Gaussian noise model and impulsive Laplace noise model is as follows

$$F_e = (1 - H(\nu))\sum_{k=1}^{K}\frac{1}{2\sigma_G^2}\|r_k\|_2^2 + H(\nu)\sum_{k=1}^{K}\frac{1}{2\sigma_L}\|r_k\|_1^1 \tag{8}$$

Where, $r_k = Y^k - D_k H_k F_k X$.

In order to get a stable solution and have good edge preservation, the Bilinear Total Variation(BTV) is used as the regularization term in this paper and the Eqs (3), (4) generate the total objective function is as follows

$$\begin{aligned} \hat{X}_{map} &= \arg\min_X \{(1 - H(\nu))\sum_{k=1}^{K}\frac{1}{2\sigma_G^2}\|r_k\|_2^2 \\ &+ H(\nu)\sum_{k=1}^{K}\frac{1}{2\sigma_L}\|r_k\|_1^1 + \lambda\sum_{l=0}^{P}\sum_{m=0}^{P}\alpha^{m+l}\|X - S_x^l S_y^m X\|_1^1\} \end{aligned} \tag{9}$$

Where λ is regularization parameter that weights the first term against the second term?

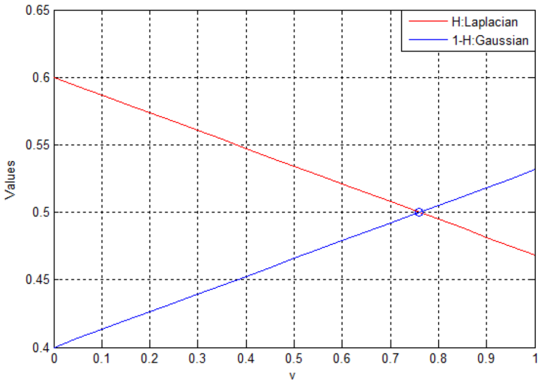


Fig 2.Menbership functions of Gaussian model (blue line) and Laplace model (red line)

Adaptive Membership Degree

Generally, the convergence is assumed to be reached by setting a maximum iteration number $N_{\max}$ and comparing the values of the objective function between two successive iterations. However, this method is not general and difficult to choose proper threshold sometimes. By the experiments, we found it is unnecessary to stop until $N_{\max}$ is reached, since the reconstructed SR image can be good enough before $N_{\max}$ is reached.

The pseudo code of this method is as follows.

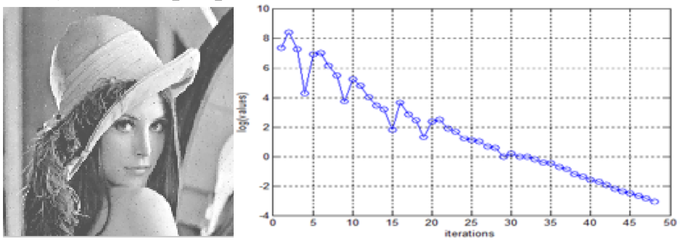
Table1 Adaptive membership degree method

Adaptive Membership Degree (AMD)
While iteration times< $N_{\max}$, do
Compute the absolute value variance of ration ν and scaled weight parameter α during two successive iterations, i.e.,
$\eta = \lvert \nu - \alpha \rvert$
If η is successively less than ε more than two times
End iteration.
Else, continue until the iterations reach $N_{\max}$.

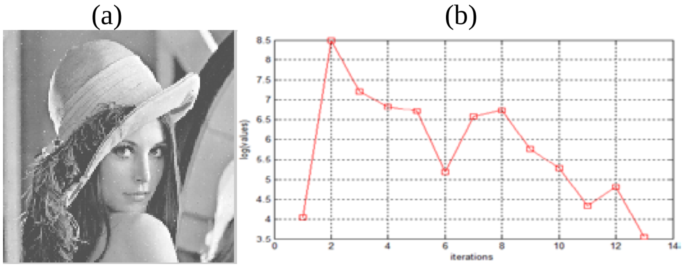
Experimental Results

In this paper, we name the L_1-norm with BTV regularized method L1BTV [7], while the L_2-norm with BTV regularized method L2BTV [8]. We apply the mixed image to do the experiments and add Gaussian noise and salt and pepper noise respectively and we assume the PSF and motion operator unknown. In order to verify the Hybrid Noise Method (HNM) effectively, we will compare the proposed method with L1BTV, L2BTV. In order to estimate the objective property of the proposed method, we apply Peak signal to noise ratio (PSNR) and Structural similarity (SSIM) [6] to the standard of the reconstruct image quality.

In the first experiment, we verified the adaptive membership degree (AMD) method by comparing it with scaled conjugate gradient (SCG) algorithm. We added the salt and pepper noise to the Lena (512×512) image with intensity of 0.1 and recover it using the L1BTV method. Fig.3 (b) shows the difference of the objective function (in Eq. (9)) between two successive iterations. Fig.3 (d) shows AMD method iteration process and iterations end when the iteration number is 13. The reconstructed SR image by AMD is shown in Fig.3(c), while the result by L1BTV is shown in Fig3 (a) when $N_{\max}$ is reached. By the experiment, we can find the SSIM (Structural similarity) of two results are the same (0.9885) but the proposed AMD method can save much computational cost.



(a) (b)



(c) (d)

Fig 3.Comparisons for methods of SCG and AMD

In the second experiment, we evaluated the performance of our method based on the $L_1 - L_2$ hybrid noise model by removing the Gaussian and the salt and pepper noises. In order to verified the proposed method effectively by comparing it with L1BTV, L2BTV. We apply the planar projective motion model (8DoF) to handle the original image Lena and then σ_{PSF}^2 =0.4, down-sampled it with a factor of two to generate eight low images as follows.

(a) One of the LR image (b) HNM (c) L1BTV

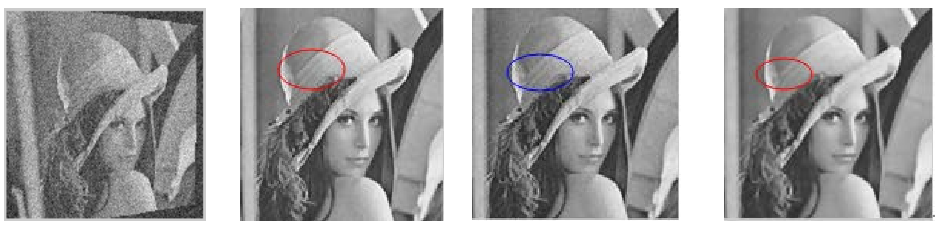
Fig 4.Comparisons between HNM and L1BTV on images with salt and pepper noise (Lena)

Fig.4 (a) shows one of the LR images which add the salt and pepper noise to the Lena (512×512) image with intensity of 0.1. Fig.4 (b) shows result of our method, and Fig.4(c) shows result of L1BTV. We can observe our method has a good reconstruct result than L1BTV.

(a) One of the LR image (b) HNM (c) L1BTV

Fig 5.Comparisons between HNM and L1BTV on images with salt and pepper noise (Boat)

Fig.5 (a) shows one of the LR images which add the salt and pepper noise to the Boat (512×512) image with intensity of 0.1. Fig.5 (b) shows result of our method, and Fig.5(c) shows result of L1BTV. We can observe our method has a good reconstruct result than L1BTV.



(a) One of the LR image (b) HNM (c) L1BTV (d) L2BTV

Fig 6.Comparisons between HNM, L1BTV and L2BTV on images with Gaussian noise (Lena)

Fig.6 (a) shows one of the LR images which add the Gaussian noise to the Lena (512×512) image with variance σ^2 is 0.08. Moreover, from table 2, we can observe our method to reconstruct image PSNR is 36.41, SSIM is 0.9856, while L1BTV method and L2BTV method to reconstruct image PSNR is only 34.35, 32.79 and SSIM is 0.9842,0.9836.

Table2 Comparison of PSNR and SSIM in the second experiment

Noise / Image			Gaussian			Salt&pepper		
			0.08	0.1	0.12	0.06	0.08	0.1
Lena	PSNR	HNM	36.41	34.64	38.02	48.77	49.35	49.88
		L1	34.35	33.69	37.64	41.58	41.90	37.64
		L2	32.79	32.43	35.09			
	SSIM	HNM	0.9856	0.9829	0.9809	0.9941	0.9940	0.9936
		L1	0.9842	0.9827	0.9799	0.9903	0.9890	0.9885
		L2	0.9836	0.9809	0.9796			
Boat	PSNR	HNM	38.90	35.94	37.32	37.98	37.06	38.58
		L1	37.92	34.13	36.64	29.52	31.89	36.39
		L2	37.54	33.94	35.90			
	SSIM	HNM	0.9838	0.9815	0.9800	0.9882	0.9832	0.9880
		L1	0.9816	0.9809	0.9772	0.9851	0.9834	0.9828
		L2	0.9809	0.9793	0.9760			

Conclusions

In this paper, we presented L1-L2 hybrid noise model (HNM) method to image super-solution, which combines the advantages of Gaussian model and Laplace model. The hybrid noise model integrates the Gaussian and Laplace models by their corresponding membership functions, which can preferably option to the noise intensity distribution during the iteration procedure. In addition, we proposed the adaptive membership degree (AMD) method, which can effectively and efficiently end the iteration. Comparisons with the L1BTV and L2BTV method on images with different noises demonstrated the superiority of the proposed method.

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