Study on the Parameters of Cole-Cole Model

Xiaojian Wang, Fujiang Mo, Jian Zhang, Ganggang Zhao

College of Electrical and information engineering, Jiangsu University, Zhenjiang, 212000, China

Keywords: Paper-oil insulation; Cole-Cole model; frequency spectroscopy method (FDS), complex dielectric permittivity.

Abstract. The paper presents ways for determining parameters of the characteristic impedance $Z_a(s)$, the value of time constant τ, ε' and ε'', through formula transformation and frequency spectroscopy method (FDS).Finding out the values of coefficient α with Cole-Cole model. Building double-relaxation model to fit complex dielectric permittivity ε in different temperature and moisture level, and finding the influence of them on the parameters of the Cole-Cole model.

Introduction

With the power grid capacity increasing, the demand of the reliability and safety of power supplying from power system are becoming more and stricter .Power transformer play an important role in power transmission. The study of transformer insulation is essential. Presently, many methods about the study of transformer paper-oil insulation had been duplicated. Return Voltage Method (RVM), Polarization and Depolarization Current (PDC) and Frequency Dielectric Spectroscopy (FDS), which are all based on dielectric response theory having been used for many years.

Cole-Cole model

Numerous models of transformer paper-oil insulation had been built. For example: X-Y model, Debye model and Cole-Cole model. Actually, dielectrics rarely have one dominant relaxation time and have the effect of each particle. Neglecting the impact and in a single relaxation time. It uses angular frequency (ω) to express the complex dielectric permit of preceptor. The long polymer chain molecular containing long chain particles, the absorption maxima of which may be significantly lower than the results from Debye model.Where ε_0 is the electric permittivity, ε_∞ is the material electric permittivity for frequency f→∞, ω is the angular frequency, τ is the relation time, and α is the coefficient(0< α <1)

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1+(j\omega\tau)^{1-\alpha}} \tag{1}$$

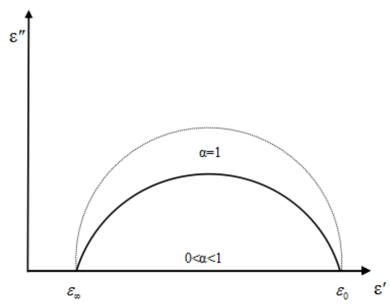

Fig.1 Cole-Cole graph according to (1).

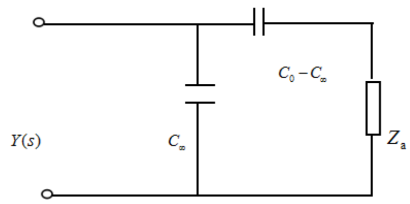

Fig.2 Electric equivalent circuit of the dielectric built on the basis of the Cole-Cole model.

$$Y(s)=sC_\infty+\frac{(C_0-C_\infty)s}{1+(s\tau)^{1-\alpha}}=sC_\infty+\frac{1}{Z(s)} \tag{2}$$

$$Z_a(s)=\frac{(s\tau)^\alpha+s\tau}{(C_0-C_\infty)(s\tau)^\alpha s}=\frac{1}{(C_0-C_\infty)s}+Z_a(s) \tag{3}$$

$$Z_a(s)=\frac{\tau}{(C_0-C_\infty)(s\tau)^\alpha} \tag{4}$$

The characteristic impedance describes relaxation properties of a dielectric. Since it is usually in the fractional power ($0<\alpha<1$), determining the element values of the equivalent circuit of impedance is possible only through approximation.

Parameters solving

In order to determine the value of accurately, it is necessary to know the parameters of the Cole-Cole model (C_0, C_∞, τ, α).Usually geometric capacity C_0 is most often given by a formula for the flat vacuum capacitor capacity, it can be given by using (5).

$$C_0=\frac{2\pi\varepsilon_0 \mathrm{h}}{\log\frac{r_{HV}}{r_{LV}}} \tag{5}$$

Where h is the winding height, r_{HV} is the external radius of main insulation, and r_{LV} is the internal radius of the main insulation.

Capacity C_∞ can be measured at a high frequency of measurement voltage. Also the value of time constant τ and parameter α are necessary to determine the characteristic impedance $Z_a(s)$.

$$\varepsilon'=\varepsilon_\infty+\frac{\varepsilon_0-\varepsilon_\infty}{1+\omega^2\tau^2},\quad \varepsilon''=\frac{(\varepsilon_0-\varepsilon_\infty)\,\omega\tau}{1+\omega^2\tau^2} \tag{6}$$

When the value of frequency is high enough, from Debye equation (6), we can see that the value of ε' approach to ε_∞, and the value of ε'' approach to 0 ($\omega\to\infty$).Formula (7) and (8) can be obtained from Debye equation.

$$\left[\varepsilon'-\frac{1}{2}\left(\varepsilon_0+\varepsilon_\infty\right)\right]^2+(\varepsilon'')^2=\frac{1}{4}(\varepsilon_0-\varepsilon_\infty)^2 \tag{7}$$

$$\varepsilon'=-\omega\tau\varepsilon''+\varepsilon_0,\quad \varepsilon'=\frac{\varepsilon'}{\omega\tau}+\varepsilon_\infty \tag{8}$$

The value of time constant τ , ε_0 and ε_∞ can be determined by the formulas(8).Capacity ε_0 and ε_∞ are determined by the intersection of the linear approximation with the axis $\omega\varepsilon''$ and $\frac{\varepsilon'}{\omega}$.Coefficient τ is equal to the slope of this approximation. From (9), (10) and (11), and Fig.3, the value of parameter α can be determined.

$$\mathrm{u}=\sqrt{\left(\varepsilon_1'-\varepsilon_\infty\right)^2+\varepsilon_1''^2},\, v=\sqrt{\left(\varepsilon_0-\varepsilon_1\right)^2+\varepsilon_1''^2} \tag{9}$$

$$\left|\mathrm{Arg}(v)\right|+\left|Arg(u)\right|=(1-\alpha)\frac{\pi}{2} \tag{10}$$

$$\alpha=1-\frac{\pi}{2}\left(\varphi_1+\varphi_2\right)=1-\frac{2}{\pi}(\mathrm{tg}^{-1}\frac{\varepsilon''}{\varepsilon_0-\varepsilon_1'}+tg^{-1}\frac{\varepsilon_1''}{\varepsilon_1'-\varepsilon_\infty}) \tag{11}$$

After four parameters (ε_0, ε_∞, α, τ) are all determined, the value of characteristic impedance $Z_a(s)$ can be solved.

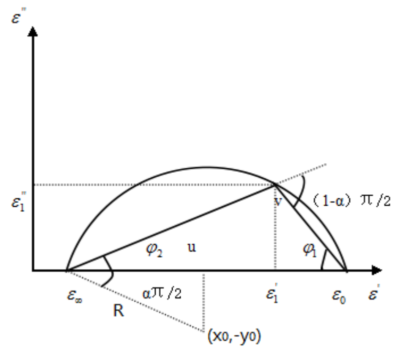

Fig 3.Way of determining the coefficient α of the Cole-Cole model from the complex permittivity characteristics' is the circle radius and (x_0, y_0) is the circumventer coordinates.

Study results

The increasing of complex dielectric permittivity of paper-oil samples in low frequency are mostly based on the formula $A\omega^{-n}$, other parts can be studied with building Cole-Cole model. Setting up double-relaxation model to analysis the influence of temperature and moisture level on complex dielectric permittivity and frequent.

$$\varepsilon' = \varepsilon_\infty + A\omega^{-n} + \mathrm{Re}\left[\frac{\Delta\varepsilon_1}{1+(j\omega\tau_1)^{(1-\alpha_1)}} + \frac{\Delta\varepsilon_2}{(1+j\omega\tau_1)^{(1-\alpha_2)}}\right] \quad (12)$$

$$\varepsilon'' = \frac{\sigma_{dc}}{\varepsilon_0\omega} + A\omega^{-n}\cot((1-n)\frac{\pi}{2}) + \mathrm{Im}\left[\frac{\Delta\varepsilon_1}{1+(j\omega\tau_1)^{(1-\alpha_1)}} + \frac{\Delta\varepsilon_2}{(1+jw\tau)^{(1-\alpha_2)}}\right] \quad (13)$$

The parameters in (12) and (13) are determined with least squares fitting (experiment data in Table 1). The value of ε_∞ is difficult to measure, and it is often valued as the extreme of complex dielectric permittivity.

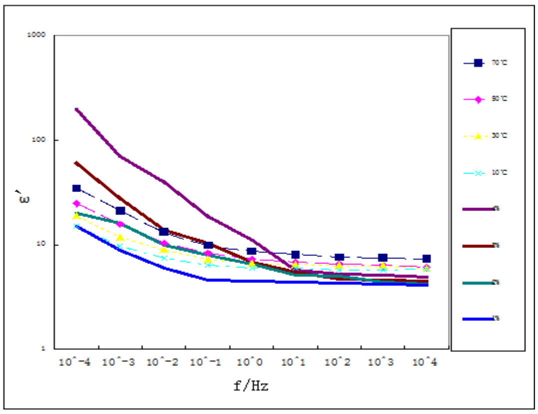

Fig.4.Fitting curve of ε'

Table 1.Parameters used for calculations in the equation.

Parameters	10℃ (2%)	30℃ (2%)	50℃ (2%)	70℃ (2%)	4%(50℃)	3%(50℃)	2%(50℃)	1%(50℃)
A	0.071	0.12	0.79	2.4	0.625	1.87	0.79	0.067
n	0.45	0.56	0.42	0.45	0.784	0.617	0.42	0.66
$\Delta\varepsilon_1$	3.8	8.1	0.12	1.29	14,8	0.1	0.12	1.78
τ_1	4500	200	19	0.24	0.79	0.003	19	29
α_1	0.36	0.13	0.734	0.33	0.49	0.81	0.734	0.43

$\Delta\varepsilon_2$	3.78	4.41	4.18	3.5	4.56	4.37	4.18	3.52
τ_2	3.1×10-10	4.4×10-11	3.3×10-12	1.4×10-12	1.9×10-11	7.2×10-11	3.3×10-12	2.5×10-12
α_2	0.75	0.73	0.76	0.74	0.75	0.76	0.76	0.751
σ_{dc}	7.81× 10-14	4.81×10-13	3.31×10-12	2.9× 10-11	1.2×10-11	7.10×10-12	3.31×10-12	3.20×10-13

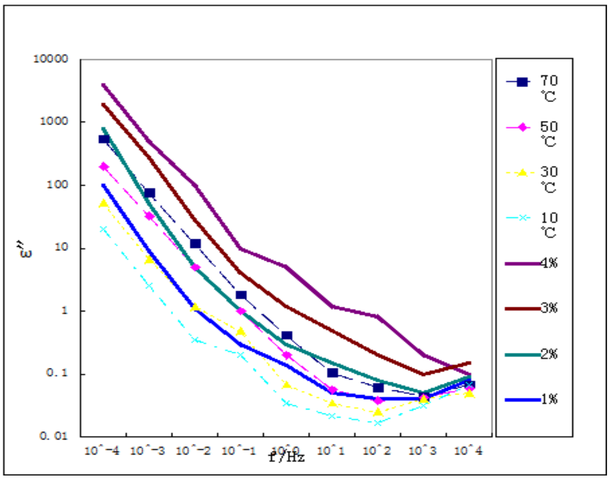

Fig.5.Fitting curve of ε''

As can be seen from Fig.4,Fig.5, increasing value of temperature causes significant increase of the value of ε' and ε''(Figure A),especially for the range of frequency(10^{-4}HZ-10^{-1}HZ).Analyzing the influence of moisture level, it also causes an obvious increasing of the value of ε' and ε'', for the range of frequency under 10HZ.

Both temperature and moisture level have an obvious effect on the value of ε' and ε'', and the impact on the value of ε'' is bigger. From (14), it is clear that the increase in temperature or the increasing of the moisture level will cause the reduction of the value of τ .

$$\tau = \frac{\varepsilon''}{(\varepsilon' - \varepsilon_\infty)\ \omega} \quad (14)$$

The influence of coefficient α on temperature and moisture level is inconspicuous. The value of α is depend on ε'_{max} and ε''_{max} .Both ε'_{max} and ε''_{max} are generally appear in low frequency part, and the value of them are huge. While the value of ε'_{min} is small, then the variation of the value of α is hard to see, as same as the moisture.

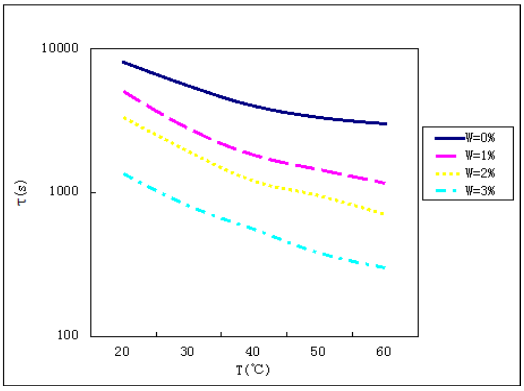

Fig 6.Influence of the moisture level of paper-oil insulation

On coefficient τ of Cole-Cole model.

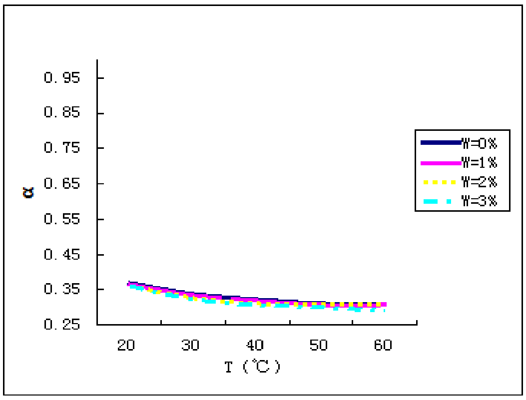

Fig 7.Influence of the moisture level of paper-oil insulation

On coefficient α of Cole-Cole model.

In Fig.6 and 7, it is obvious to see that an increasing of water in paper-oil insulation will short time constant τ of the Cole-Cole model, the increasing temperature also decreases the value of time constant τ .While neither water or temperature has an obvious influence on coefficient α .

Conclusions

In this paper, transforming the Debye equation and using linear function to obtain the value of time constant τ , ε_0 and ε_∞ .Determining the coefficient α in the Cole-Cole model. Setting up double-relaxation model to analysis the influence of temperature and moisture level on complex dielectric permittivity and frequency, then find out the influence of temperature and moisture level on parameter α and τ .The influence on τ is more obvious, and it can be used to evaluate the insulation of transformer paper-oil insulation performance.

References

[1]C. Ekanayake. Diagnosis of moisture in transformer insulation-application of frequency domain spectroscopy. Gothenburg,Sweden: Chalmers University of Technology. 2006.
[2]A. K. Jonscher. Dielectric relaxation in solids,2nd ed. London,UK, Chelsea.Dielectrics Press Limited, 1996.
[3]J. Subocz and M. Szrot, "Methods for evaluation of insulation quality using dielectric spectroscopy in frequency and time domains," (in Polish) Przegl ą d Elektrotechniczny , p. 214–216, Mar. 2008.
[4]D. Giselbrecht and T. Leibrfried, "Modeling of paper-oil insulation layers in frequency domain with Cole-Cole functions," presented at the IEEE Int. Symp. Elect. Insul., Toronto, ON, Canada, Jul. 11–14, 2006.
[5]S. Wolny and M. Zdanowski, "Analysis of recovery voltages param-eters of paper-oil insulation obtained from simulation investigations using the cole-cole model," IEEE Trans. Dielectr. Electr. Insul. , vol.16, no. 6, p. 1676–1680, Dec. 2009.
[6]P.J.Baird,H.Herman,G.C.Stevens,andP.N.Jarman,"Spectro-scopic measurement and analysis of water and oil in transformer insu-lating paper," IEEE Trans. Dielectr, Electr. Insul. , vol. 13, no. 1, p.293–308, Feb. 2006.
[7]Provencher H, Noirhomme B, David E. Influence of Temperature and Moisture on the Dielectric Response of paper-oil Insulation System[C]. 2008 Annual Report Conference on Electrical Insulation and Dielectric Phenomena (CEIDP), Quebec, Canada: IEEE, 2008: p.125 -128.
[8]Applications of the Cole-Cole Model for Determining Parameters of the Sabstitue Scheme of Paper-Oil Insulation", 3rd Intern. Conf. on Advances in Processing, Testi ng and Application of Dielectric Materials, Wroclaw, Poland, p. 209-212, 2007.