# Semi down sampling wavelet transform and its application in image watermarking

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**Abstract.** At watermarking is developed, The construction proposed in this paper is based on a no subsampled pyramid (NSP) transform and wavelet transform (WT). The result of no subsampled transform is a flexible multiscaleand shift-invariant image decomposition that can be efficiently implemented via the à torus algorithm, the corresponding frame elements for à torus algorithm are regular, symmetric, and the frame is close to a tight one that can implement linear phase filter. Shift-invariance is useful in eliminating the Gibbs phenomenon and increasing the invisibility of watermark. The subsampled wavelet transform obtains a small size low-pass sub band which is embedded watermark, so the watermark can sufficiently spread in image after reconstruction, thereby the robustness and the invisibility of watermark will be improved. We assess the performance of the SSWT in image watermarking, the SSWT compares favourably to other existing methods in the literature on both robustness and invisibility of watermark.

## Introduction

In order to remove the Gibbs phenomenon of Wavelet transform, M. J. Shensi [8] proposed No subsampled wavelet transform. No subsampled transform has the has the property of translation invariance, it can effectively remove Gibbs phenomenon in conversion, which is useful for image watermarking technology, since it can effectively improve the transparency of the watermark. But each sub-band No subsampled transform has the same size, when the watermark is embedded into one or some sub-band images reconstructed by the inverse transform, the watermark cannot get better diffusion, which is not good for the robustness of the watermark and transparency.

Based on the above discussion, the paper developed a new semi-down sampling wavelet transform (SSWT). First, non-down sampling an image pyramid (NSP) filter, resulting in a low frequency sub-band and a series of high-frequency sub-band; non-filtered samples obtained at low frequency sub-band wavelet transform was then used to obtain a further smaller than the original image low frequency sub band and a series of high frequency sub bands. In this low frequency sub band we reconstruct image by watermark embedding algorithm, so the watermark can better diffuse watermarked image, which ensuring the robustness of the watermark and high transparency.

## No subsampled pyramid wavelet transform

## NonSubsampled pyramid decomposition

No subsampled pyramid get a translation invariant filter structure, which is a similar to sub band decomposition method of the Laplacian pyramid decomposition and can be implemented through two channels No subsampled two-dimensional filter banks, Figure 1 illustrates three non-sampling pyramid decomposition. There is only J+1 redundancy after the results of such decomposition, where J stands for exploded series, but after three wavelet transform decomposition have 3J+1 redundancy.





The J stage of the ideal low-pass filter in the pass band to  $\operatorname{support}[-(\pi/2^{j}),(\pi/2^{j})]^{2}$ , for the corresponding ideal high-pass filter passband support is the complement of the low pass filter support, that is  $[-(\pi/2^{j-1}),(\pi/2^{j-1})]^{2} \setminus [-(\pi/2^{j}),(\pi/2^{j})]^{2}$ . Subsequent levels through the sampling filters filter to get the first stage, so no additional filter design can get multi-scale decomposition.

## Nonsubsampled pyramid filter bank (NSPFB)

Figure 2 shows the non-subsampled pyramid filter banks, wherein  $H_0(z)$  stands for the low-pass and  $H_1(z)$  high-pass decomposition filter; and, respectively,  $G_0(z)$  stands for low-pass and  $G_1(z)$  stands for high-pass reconstruction filter. Since in the case of a finite impulse response, No subsampled filtering for high-dimensional transform is easier to implement, so the main discussions is about FIR. For the average two-channel no subsampled pyramid filter banks, perfect reconstruction filter satisfies the following conditions are Besought identity:



Fig. 2: NSPFB filter group (1)

## $H_0(Z)\square G_0(Z) + H_1(Z)\square G_1(Z) = 1$

## Frame Analysis of NSPFB

Here's analysis and synthesis framework for system operators to illustrate NSPFB.

Definition: A family of vectors  $\{f_i\}_{i \in N}$  in Hilbert spaces called a framework. If there are two constants A > 0, B > 0, such that:

$$A || f ||^{2} \le \sum_{i \in \mathcal{N}} |\langle f, f_{i} \rangle|^{2} \le B || f ||^{2} , \quad \forall f \in H$$
(2)

When A = B, the framework is called the tight frame, when (2) is satisfied, the most tightly integer constants A, B are called framework boundary.

In figure 2, vector groups  $\{h_0[\square-n], h_1[\square-n]\}_{n \in \mathbb{Z}^2}$  is a frame of  $l_2(\mathbb{Z}^2)$ . If there exist  $0 < A < B < \infty$  which satisfy:

$$A \le \underbrace{|h_0(e^{j\omega})|^2 + |h_0(e^j)|^2}_{t(e^{j\omega})} \le B.$$
(3)

Thus, NSPFB frame boundary is calculated as follows:

$$A = ess. \inf_{\omega \in [-\pi,\pi]^2} t(e^{j\omega}), \quad B = ess. \sup_{\omega \in [-\pi,\pi]^2} t(e^{j\omega})$$
(4)

Here *ess*.inf and *ess*.sup stands for nature low infimum and nature high infimum. From equation (3), as long as  $t(e^{j\omega})$  is almost constant everywhere, then the frame is tight. For, this means that such conditions  $H_0(Z)H_0(Z^{-1}) + H_1(Z)H_1(Z^{-1}) = c$ . can be met only when the filter is a linear phase filter.

Because NSPFB are redundant, so there is an infinite number of inverse, among these inverse, pseudo-inverse under the least squares sense is optimal. Given a framework of the analysis filter, then the corresponding frame is given by the synthesis filter  $G_i(z) = H_i(z)/t(z)$ , i = 0,1. Such synthesis filtering framework is dual framework for analysis filtering, its frame boundaries are  $B^{-1}$ ,  $A^{-1}$ ,

respectively. If the frame is not tight, even analysis filtering is synthesized pseudo-inverse filtering will also be an infinite impulse response.

Obtained from the above discussion we can conclude that: (1) linear phase filter and tight frames are mutually exclusive, and (2) If the pseudo-inverse is wanted, but if the frame is not tight, the synthesis filter will be IIR. The result is both linear phase and synthesis filter corresponding to the pseudo-inverse of NSPFB system will not be found. But we can get one with a pseudo-inverse filtering by approximation methods. For a given number of filters, when the frame tends to tighten more, you will get a better approximation of the pseudo-inverse of FIR.

## SSWT filter bank structure

NSP can transform the image into multistate decomposition, but also has translational invariance feature, translation invariance in the image analysis is a very important property. But each sub-band due to sampling spend NSP transformation, so that after conversion are the same size as the original image, which is not conducive to the application of image watermarking. In order to improve the transparency of the watermark and robustness, we need a sub-band size of decreasing transformation, making the watermark signal embedded into a small low-frequency sub-band, so that the watermark signal by the inverse transform can be more diffused in the host image, which is beneficial while improving transparency and watermark robustness. Another image after converting only filtered through a part of the NSP strong edges, before the watermark embedding, it is necessary to further filter out low frequency sub-band portion of the weak edge, which is also very beneficial to improve the transparency of the watermark and robustness. Through the above analysis, we designed a new filtering method, which is to transform the image using NSP, and then the lowfrequency sub-band wavelet transform, we call this transformation is semi down sampling wavelet transform (SSWT). After NSP decomposition, each sub-band of an image will have the same size. We can flexibly choose the number of stages NSP transform and wavelet transform, according to the specific application of the series. Because the wavelet filter bank satisfy Besought identity:

 $\tilde{G}(z)\Box G(z) + \tilde{H}(z)\Box H(z) = 1$ 

Therefore, the wavelet transform can be completely reconstructed; 2.2 and 2.3 analysis shows that under certain conditions, NSP transformation can be completely reconstructed either. So that the proposed half-sampling wavelet filter can be completely reconstructed. Figure 3 shows the SSWT filter bank design.

(5)



Fig. 3: SSWT filter bank design

#### The image watermarking algorithm based on SSWT

To illustrate the potential of SSWT, we studied its application in image watermarking, we propose a new watermarking algorithm:

(1) NSP transformation of the host image to obtain a low-frequency sub band and a series of high-frequency sub bands;

(2) Performed wavelet transform in the low frequency sub-band obtained in (1), resulting in a smaller range of low frequency sub-band and high-frequency sub bands;

(3) Design a watermark which has the same size of that in the low frequency, and embedding the watermark into the low frequency sub-band according to certain rules;

(4) perform an inverse wavelet transform;

(5) Perform NSP inverse transform to obtain the watermarked image.

## The watermark embedding [10]

The low frequency band  $LL_i$  above-mentioned in (2) is  $N \times N$  coefficient matrix, denoted by A.Select a  $N \times N$  size of the binary image as a watermark, denoted by B. Embedding the

watermark information into A, i.e., sum two  $N \times N$  matrix, where the element which has watermark information will have value 1 or 0. The following is the specific algorithm:

*if*  $A(i, j) \ge 0$  and B(i, j) = 1, then  $A(i, j) = A(i, j) - rem(A(i, j), S) + T_1$ ;

*if*  $A(i, j) \ge 0$  and B(i, j) = 0, then  $A(i, j) = A(i, j) - rem(A(i, j), S) + T_2$ ;

*if* A(i, j) < 0 and B(i, j) = 1, then  $A(i, j) = A(i, j) + rem(A(i, j), S) - T_1$ ;

*if* A(i, j) < 0 and B(i, j) = 0, then  $A(i, j) = A(i, j) + rem(A(i, j), S) - T_2$ .

Here,  $T_1, T_2$  are the threshold value of the watermark embedding, S is watermark embedding strength factor. To enlarge the value as much as possible while keeping watermark invisible,  $T_1, T_2$  can be assigned 3S/4 and S/4.

*rem*Operations similar to the mod operation, the only difference is that the calculation is rounded down, and rounding calculation is rounded to zero:

$$rem(x, y) = x - [x / y]y$$

$$mod(x, y) = x - \lfloor x / y \rfloor y$$
(6)
(7)

According to Mallet algorithm, with the increase of Wavelet decomposition series, the amplitude of the low frequency coefficients increases by a factor of approximately 2 in the growth. The watermark can be viewed as the superposition of a weak signal (watermark) under strong background, as long as the superposition of the signal is below the threshold contrast, the visual system will not feel the presence of the signal. According to Weber's law, the magnitude of the contrast threshold is proportional to the background signal, this means that with the increase in the number of stages of the wavelet decomposition, the watermark embedding strength increases approximately by a multiple of 2, so that the robustness of the watermark enhanced. Meanwhile, if the wavelet decomposition series gets bigger, watermark component can be better defused. So in watermarking algorithm, the watermark should improve the wavelet decomposition series as much as possible based on the information.

## Watermark extraction

After the implementation of step (1), (2) of the watermarked image, we can obtain a low frequency band of wavelet transform coefficient matrix  $LL_i$  (denoted as Y), the following formula can be used to detect the embedded watermark information:

*if*  $| rem(Y(i, j), S) | \ge (T_1 + T_2)/2$ , then Y(i, j) = 1;

*if*  $| rem(Y(i, j), S) | < (T_1 + T_2)/2$ , then Y(i, j) = 0.

Thus, the watermark information is restored out.

## **Experiment result**

We utilize Lena () standard to test above algorithms. Watermark uses a binary image. Figure 4 is an image of the watermark embedding and extraction of watermark. To illustrate the effectiveness of the algorithm proposed in this paper, we compare it with the traditional watermarking algorithm based on wavelet transform. We use normalized correlation coefficient (NC) and the peak signal to noise ratio (PSNR) to evaluate the robustness of the watermark and transparency.

#### Transparency

We decompose the original image using second degree NSP, and then get three low-frequency sub-band wavelet decomposition. We then embed this low-frequency sub-band with watermark algorithm. When S equals 13.8, the extracted NC will be 1 while S has to reach 16.1 if we want to get a same NC value. At this point the PSNR value of two algorithms is 60.0657 and 51.5563 respectively. With the increase of S, PSNR value will be smaller, that is to say watermarked image quality will degraded. Figure 5 is a comparison of PSNR value between SSWT algorithm and WT Algorithm. As can be seen from the figure, when taking the same S value, PSNR value based on the SSWT algorithm is significantly higher than that of WT algorithm, which shows a transparent watermark algorithm is significantly better than traditional watermarking algorithm based on wavelet transform.



Fig. 4: Watermarked image and the extracted watermark



Fig.5: comparison of PSNR

## Robustness

We have conducted a lot of comparative experiments to test the robustness of the algorithm of the watermark, the two algorithms was subjected to various forms of attack, including the addition of Gaussian noise, JPEG compression, median filtering, and zoom and rotating geometric attacks. In order to compare the merits of the two algorithms, this part of the experiment takes different S values of the two algorithms to make PSNR value roughly equal. In SSWT algorithm,  $_{PSNR = 47.5253(S = 60)}$  while in WT algorithm  $_{PSNR = 47.4134(S = 24)}$ .

Table 1 shows the performance comparison of two algorithms under anti-scaling and rotation geometric attacks. Through comparing the NC value of the two algorithms in Table 1 we can see, the proposed method in the fight against both geometric attacks and rotation attacks are significantly better than the WT-based algorithm, especially against the spin attack, no matter how many degrees of rotation images, NC value of this algorithm is always 1, indicating that the proposed method in the fight against geometric attacks has unparalleled advantages. Table 2 shows the performance under median filter, we can see our algorithm has a clear advantage over WT based algorithm.

Rotation	factor	0.5	0.75	0.9	1.1	1.5	2
WT(NC)	0.9598	0.9972	0.9947	0.9961	1.0000	0.9997	
SSWT(NC)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
Rotation	angle	-15	-10	5	25	35	45
WT(NC)	0.9879	0.9975	0.9969	0.9975	0.9975	0.9975	
SSWT(NC)	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	

Table 1: comparison of robustness under geometry (rotation and scaling)

Table 2: Comparison of robust under median filtering

Median	filtering	$2 \times 2$	3×3	$4 \times 4$	$5 \times 5$	
WT(NC)	0.6431	0.9238	0.6324	0.7881		
SSWT(NC)	0.7985	0.9961	0.7898	0.9202		

#### Conclusions

In this paper, we progress semi down sampling wavelet transform under the development of digital watermarking technology, and we propose a new image watermarking algorithm based on the transformation. We also conduct a lot of comparative experiments to compare its performance

with the traditional watermarking algorithm. Results show that this algorithm has obvious advantages over wavelet transform, thus confirming the effectiveness of the proposed algorithm. The semi-down sampling wavelet transform in No subsampled transform proposed in this paper has translation invariance, and it can also effectively remove Transform Gibbs phenomenon. Down sampling wavelet transform can ensure sufficient diffusion of the watermark information in the image, which is a good way to ensure high transparency and robustness of the watermark. In this paper, the development of the semi-down sampling wavelet transform applied to other image processing technology is our future research directions.

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