Non-probabilistic Mechanism Reliability Analysis with Interval Dimension and Clearance Variables

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***Abstract*—In traditional mechanism reliability analysis, dimension and clearance variables are typically assumed to be random variables with a given distribution. This treatment may lead to large errors when statistic information of uncertainties are not precisely known. This work intends to remedy this problem. A non-probabilistic reliability model, which based on the interval set, is introduced to measure the reliability of mechanism. The n-dimensional case of the reliability measure is defined as the ratio of the hypervolume of the safe region to the whole hypervolume of variation of uncertain variables. Only the tolerance bounds of dimension variables and radius of joint clearances are required in this approach. The proposed method is applied to reliability analysis of a four-bar mechanism. In the absence of enough information on uncertainty, the presented non-probabilistic theory may give a feasible assessment of the mechanism reliability.**

Keywords-reliability; Non-probabilistic; tolerance; joint clearance

I. INTRODUCTION

Uncertain factors in mechanism systems, such as manufacturing errors, joint clearances, deformations of structural components affect the positional and directional control of the motion significantly [1-3]. To maintain the motion accuracy, the mechanism reliability has become a very important concept in practical engineering [4]. The mechanism reliability is defined as the probability of the output member's position and/or falling within a specified range from the desired position and/or orientation [5]. Most of the existing mechanism reliability analysis methodologies are based on probabilistic model, which regard all the uncertain parameters as random variables with a typical distribution [6-8]. However, it is difficult to obtain probability densities and distribution of all uncertain factors, and we can only provide the approximate results, which would produce large errors in probabilistic mechanism reliability analysis. In the real engineering applications, the uncertain dimension variables are bounded within their tolerance range, and the clearance variables are bounded within their clearance circles. Without knowing the enough statistic information of these uncertain parameters, a better way is to consider the uncertainties as intervals [9, 10].

Non-probabilistic reliability models are announced that would give a more feasible assessment of the structural safety than probabilistic reliability models in the absence of enough information on uncertainty [11]. For structural system reliability design, several non-probabilistic reliability models are proposed based on both convex set model and interval arithmetic during last twenty years, and how to measure the 'non-probabilistic reliability' become a focus of research [12-14]. To the authors' knowledge, the non-probabilistic mechanism reliability model has not yet been studied.

In this work, the uncertain dimension and clearance variables are described by intervals. An interval reliability model, which is based on the ratio of the volume of the safe region to the total volume of the region for structural reliability analysis, is introduced to evaluate the reliability of a mechanism system. Finally, by comparing advanced model with the traditional probabilistic reliability analysis model, illuminating interval reliability analysis model is effective.

II. UNCERTAIN DIMENSION AND CLEARANCE VARIABLES

In this section, we discuss the basic uncertain variables in mechanism design. At first, let the major mechanism dimension variables, such as the link lengths of a four bar linkage, be $\mathbf{L} = \left(L_1, L_2, \cdots, L_n\right)$. Because of the manufacturing errors, the dimension variables are uncertain. Suppose all the uncertain parameters are interval variables, they can be rewritten as

$$\underline{\mathbf{L}} \le \mathbf{L} \le \overline{\mathbf{L}} \text{ or } \underline{L}_i \le L_i \le \overline{L}_i, \quad i = 1, 2, \cdots, n \tag{1}$$

where $\underline{\mathbf{L}} = \left(\underline{L}_i\right)$ and $\overline{\mathbf{L}} = \left(\overline{L}_i\right)$ denote the lower and upper bound of uncertain dimension variables, respectively.

Using the interval mathematics, the inequality can be denoted as

$$\mathbf{L} \in \mathbf{L}^I \text{ or } L_i \in L_i^I, \quad i = 1, 2, \cdots, n \tag{2}$$

The uncertain dimension variables are often normalized as

$$\delta_{\mathbf{L}} = \frac{\mathbf{L} - \mathbf{L}^c}{\mathbf{L}^r} \text{ or } \delta_{L_i} = \frac{L_i - L_i^c}{L_i^r} \tag{3}$$

where L_i^c are the mean of interval, L_i^r are the radius of interval, $\delta_{L_i} \in \left[-1, 1\right]$ are the standardized interval variables.

The interval model of clearance in a revolute joint between links i and j is shown in Figure 1, where O_i is the certer of the hole of link i and O_j in the center of the pin of link j. Then the vector $\overline{O_i O_j}$ can be used to express the joint clearance. The magnitude r_{ij} denotes the radius of the clearance circle, and the coordinates x_{ij} and y_{ij} are called clearance variables.

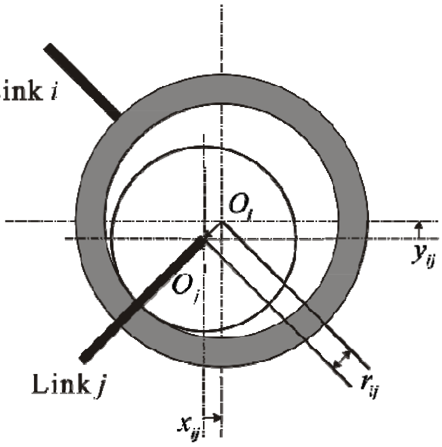

FIGURE I. INTERVAL MODEL OF A REVOLUTE JOINT CLEARANCE

In the present analysis, x_{ij} and y_{ij} are assumed to be interval numbers, and r_{ij} is subject to the constraint $r_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2} \le R_{ij}$, where R_{ij} is the maximum magnitude of r_{ij}. The clearance variable can be rewritten as $C_j = \left(x_{ij}, y_{ij}\right)$ for simplification, and we can obtain the interval clearance variables as $\mathbf{C} \in \left[\underline{\mathbf{C}}, \overline{\mathbf{C}}\right]$.

III. NON-PROBABILISTIC MECHANISM RELIABILITY MODEL

In this section, we will discuss the measurement of non-probabilistic mechanism reliability based on the motion error. The performance function of mechanism reliability can be expressed as

$$M = g\left(\mathbf{L}, \mathbf{C}\right) = \left|Y_{act}\left(\mathbf{L}, \mathbf{C}\right) - Y_{req}\right| - \varepsilon \tag{4}$$

where Y_{act} and Y_{req} are the actual motion output and required motion output, respectively, ε is the allowable motion error. We consider that a failure occurs when M is greater than 0.

The basic theorem of non-probabilistic reliability model based on the ratio of the volume was first introduced by Wang [15]. According to the number of uncertain variables, the reliability measure would be different.

In mechanism design, more than three uncertain variables should be considered ordinarily. For the sake of convenience, we consider the case of three-dimension, for example, $L_1 \in \left[\underline{L}_1, \overline{L}_1\right]$, $L_2 \in \left[\underline{L}_2, \overline{L}_2\right]$ and $C_1 \in \left[\underline{C}_1, \overline{C}_1\right]$. Using Eq. (3), we can get the standardized interval variables of L_1, L_2 and C_1

$$\delta_{L_1} = \frac{L_1 - L_1^c}{L_1^r},\ \delta_{L_2} = \frac{L_2 - L_2^c}{L_2^r}\ and\ \delta_{C_1} = \frac{C_1 - C_1^c}{C_1^r} \tag{5}$$

We can obtain the failure plane of standardized variables space

$$M\left(\delta_{L_1}, \delta_{L_2}, \delta_{C_1}\right) = 0 \tag{6}$$

And then, we can obtain three-dimensional nonlinear standardized interval variables interference model, as the Figure 2.

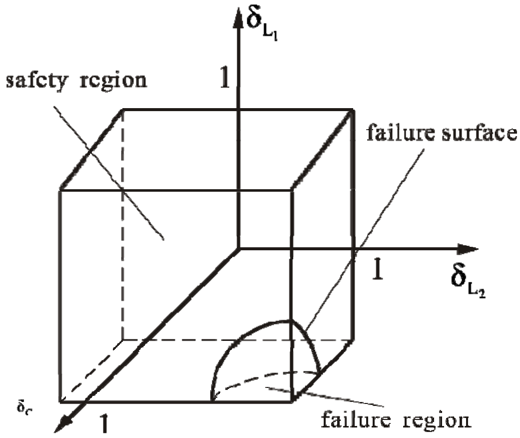

FIGURE II. THREE-DIMENSIONAL NONLINEAR STANDARDIZED INTERVAL VARIABLES INTERFERENCE MODEL

In this figure, standardized interval variables region is divided into two parts by the failure plane, i.e. failure region and safe region. We can define the measurement of non-probabilistic reliability as

$$K_r = \eta\left(M\left(\delta_{L_1}, \delta_{L_2}, \delta_{C_1}\right) < 0\right) = \frac{V_s}{V} \tag{7}$$

where V_s denotes the volume of safe region, V refers to the volume of standardized interval variables region. Similarly, the measurement of non-probabilistic failure can be defined as

$$K_f = \eta\left(M\left(\delta_{L_1}, \delta_{L_2}, \delta_{C_1}\right) > 0\right) = \frac{V_{iv}}{V} \tag{8}$$

where V_{iv} is the volume of failure region.

If the number of uncertain variables is more than three, the standardized interval variables region becomes a hypercube. The definition of measurement of non-probabilistic reliability is that the ratio of the hypervolume of the safe region to the whole hypervolume of variation of uncertain variables. By aforementioned definition, the mechanism reliability can be calculated, when the intervals of dimension and joint clearance variables are known.

IV. NUMERICAL EXAMPLES

In this section, the non-probabilistic reliability of a four-bar function generating mechanism with/ without joint clearances is studied. As shown in Figure 3, a function

$y=\log(x)$ should be realized by such a mechanism, where $1 \le x \le 2$. The ranges of the motion input and output angle are both $[0°,60°]$, i.e. $0° \le \theta \le 60°$, $0° \le \psi \le 60°$. The probabilistic reliability of this mechanism without clearance was studied by Du [5], and we choose the same dimension parameters for comparison conveniently. The mean value and tolerance of the length L_1, L_2, L_3 and L_4 are given in Table 1. The radii of the clearance circles for all the joints are 0.05 mm.

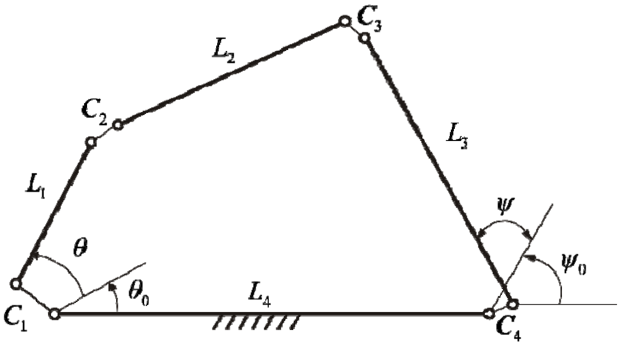

FIGURE III. FOUR-BAR MECHANISM

TABLE I. THE MEAN VALUE AND TOLERANCE OF DIMENSION VARIABLES

Variable	Mean (mm)	Tolerance (mm)
L_1	93.4	± 0.1
L_2	242.1	± 0.1
L_3	169.2	± 0.1
L_4	100	± 0.1

The required function between the motion output and motion input is derived as

$$Y_{req}(\theta)=\psi=\frac{60°}{\log(2)}\log\left(1+\frac{\theta-\theta_0}{60°}\right)+\psi_0 \qquad (9)$$

where θ_0 and ψ_0 are the initial input angle and initial output angle, respectively. In this study, we set $\theta_0=65.2°$ and $\psi_0=3.0°$. The allowable motion error is $\varepsilon=0.3°$.

At first, the reliability of mechanism without clearance should be considered. The actual motion output angle without clearance is given by [16]

$$Y_{act}=\arctan\left(\frac{E}{D}\right)+\arccos\left(\frac{A}{B}\right)-\psi_0 \qquad (10)$$

where $D=L_4-L_1\cos(\theta+\theta_0)$, $E=-L_1\sin(\theta+\theta_0)$, $A=D^2+E^2+L_3^2-L_2^2$, and $B=-2L_3\sqrt{D^2+E^2}$.

Table 2 shows the probabilistic of failure without clearance over the range of $69.2° \le \theta+\theta_0 \le 78.2°$. We can conclude that the results of proposed model are all greater than the results of probabilistic reliability model. Namely, when the information of uncertain parameters is scanty, there exits some risk for the mechanism analysis and synthesis to use the probabilistic reliability model.

TABLE II. PROBABILITY OF FAILURE WITHOUT CLEARANCE

$\theta+\theta_0$	Non-probabilistic model	Probabilistic model in ref. [5]
69.2°	3.4501×10^{-5}	3.1372×10^{-5}
70.2°	5.3406×10^{-5}	2.9154×10^{-5}
71.2°	1.6075×10^{-4}	1.0179×10^{-4}
72.2°	3.150×10^{-4}	1.8623×10^{-4}
73.2°	3.5490×10^{-4}	2.0779×10^{-4}
74.2°	2.3426×10^{-4}	1.5096×10^{-4}
75.2°	1.3049×10^{-4}	7.2084×10^{-5}
76.2°	4.1578×10^{-5}	2.1733×10^{-5}
77.2°	1.5764×10^{-5}	3.7138×10^{-6}
78.2°	3.0477×10^{-6}	2.7564×10^{-7}

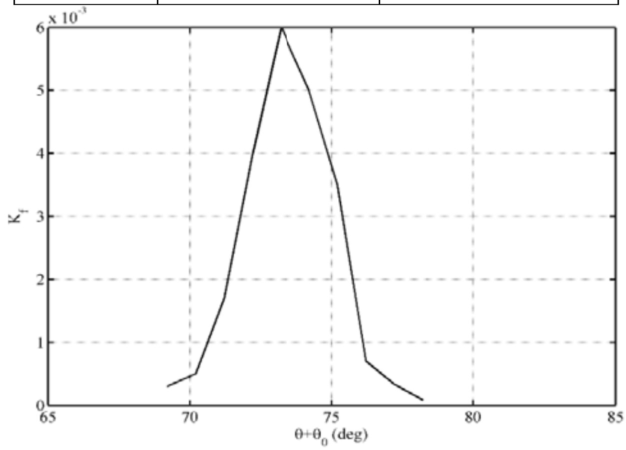

FIGURE IV. PROBABILITY OF FAILURE WITH CLEARANCE

When considering the clearances in joints 1 through 4, the possible rotation range of ψ can be obtained from the positions if $L_2 \in [L_2-4r, L_2+4r]$, where r is the radii of the clearance circles. Then the uncertain clearance variables can be translated into the dimension variable L_2. The probabilities of failure with clearances over the range of $69.2° \le \theta+\theta_0 \le 78.2°$ can be indicated in Figure 4. By comparing with the results in Table 2, the joints clearance would affect the mechanism reliability significantly.

V. CONCLUSIONS

In this study, we describe the uncertain dimension and clearance variables in mechanism as interval set, and a new non-probabilistic analysis model for mechanism reliability analysis is proposed. In this model, the measurement of reliability is considered as the ratio of the volume of safe region to the total volume of the region constructed by the basic interval variables. By comparing with the probabilistic reliability analysis model, only the tolerance bounds of dimension variables and the radius of joint clearance circle are required. This is convenient that the distribution of uncertainties in mechanism synthesis is usually unknown. The results of numerical examples suggested that the non-probabilistic reliability model of mechanism is a helpful supplement to the existing probabilistic model.

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