A New Similarity Measure of Intuitionistic Fuzzy Sets and Application to Pattern Recognition

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***Abstract*—This paper addresses the issue of similarity measures of intuitionistic fuzzy sets (IFSs). Many measures of similarity between IFSs proposed before did not consider the abstention group influence, which may lead to counter-intuitive results in some cases. In this paper, first, we analyze the limitations of the existing similarity measures. Then, a new similarity measure of intuitionistic fuzzy sets is proposed and several numeric examples are given to demonstrate the validity of the proposed measure. Finally, the proposed similarity measure is applied to pattern recognition and medical diagnosis.**

Keywords-intuitionistic fuzzy sets; similarity measure; abstention group influence; pattern recognition; medical diagnosis

I. INTRODUCTION

Since Zadeh [1] introduced fuzzy sets (FSs) theory, several generalized forms have been proposed, among which intuitionistic fuzzy sets (IFSs), proposed by Atanassov [2-6], have been found to be highly useful to describe the imprecise or uncertain information. Gau and Buehrer [7] proposed the notion of vague sets, which coincides with that of IFSs, as pointed out by Bustince and Burillo [8]. IFSs have been widely used for pattern recognition and decision making, where the decision information is often imprecise due to time pressure or lack of data. Li and Cheng [9] and Mitchell [10] applied similarity measures of IFSs to perform classification. Wang and Xin [11] introduced some distance measures for pattern recognitions. Vlachos and Sergiadis [12] proposed discrimination information measure for medical diagnosis. Khatibi and Montazer [13] examined the capabilities of FSs and IFSs in the medical pattern recognition with uncertainty by using five similarity measures. A comparison of the distance and similarity measures from the pattern recognition point of view was presented by Papakostas, Hatzimichailidis and Kaburlasos [14].

The similarity measures of IFSs are used to estimate the similarity degree between two IFSs. Many researchers, [9, 15, 16, 18], have shown great interest in the similarity measure theory of IFSs. However, the existing measures did not consider the abstention group influence and may lead to counter-intuitive results in some cases. In this paper, we take into account the abstention group and propose a new similarity measure based on groups-voting model. Compared with the similarity measures in [9, 15, 16, 18], the proposed measure can overcome some drawbacks of counter-intuition. Several numeric examples are given to demonstrate the validity of the proposed measure. Additionally, applications to pattern recognition and medical diagnosis are also presented.

II. A BRIEF INTRODUCTION OF INTUITIONISTIC FUZZY SETS

Definition 2.1 ([16]). Let X be a universe of discourse. An intuitionistic fuzzy set A in X is an object with the form: $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$, where $\mu_A: X\to[0,1]$, $\nu_A: X\to[0,1]$, with the condition $0\le\mu_A(x)+\nu_A(x)\le 1$, $\forall x\in X$.

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote the degree of membership and non-membership of x to A, respectively. For each IFS A in X, we call : $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$, the intuitionistic index of x in A, which denotes the hesitancy degree of x to A. For convenience, $IFS(X)$ denote the set of all the IFSs in X, and let $A,B\in IFS(X)$, then two of their relations and operations are defined as ([2]): (1) $A\subseteq B$ if and only if $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$, for each $x\in X$; (2) $A=B$ if and only if $A\subseteq B$ and $A\supseteq B$; (3) $A^C=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$.

III. SIMILARITY MEASURES BETWEEN IFSS

The similarity measure indicates the closeness of IFSs, and the larger similarity measure corresponds to the closer degree of two IFSs. However, the existing similarity measures can lead to count-intuitive results in some cases or can't distinguish which two IFSs are closer, which is demonstrated by some numeric examples. And then a new similarity measure is proposed.

A. Analysis on Existing Similarity Measures

Definition 3.1 ([9]). A real-valued function S:IFS $(X)\times$ IFS $(X)\to[0,1]$ is called a similarity measure on IFS (X), if it satisfies the axiomatic requirements: (1) $0\le S(A,B)\le 1$; (2) $S(A,B)=1$ if $A=B$; (3) $S(A,B)=S(B,A)$; (4) $S(A,C)\le S(A,B)$ and $S(A,C)\le S(B,C)$ if $A\subseteq B\subseteq C$.

For two IFSs A and B, Li and Cheng [9] proposed the following similarity measure between IFS A and IFS B. For each IFS A, let $\varphi_A(x_i)=\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}$, where $x\in X=\{x_1,x_2,\cdots,x_n\}$.

Then

$$S_1(A,B)=1-\frac{1}{\sqrt[p]{n}}\left(\sum_{i=1}^{n}\left(\varphi_A(x_i)-\varphi_B(x_i)\right)^p\right)^{1/p} \quad (1)$$

If let $p=1$, it is reduced to the following formula:

$$S_2(A,B)=1-\frac{1}{n}\sum_{i=1}^{n}\left|\varphi_A(x_i)-\varphi_B(x_i)\right| \quad (2)$$

Example 3.1. Let $A=\{\langle x,0.1,0.2\rangle\}$, $B=\{\langle x,0.4,0.4\rangle\}$ and $C=\{\langle x,0.2,0.2\rangle\}$ be three IFSs. Intuitively, we can see that IFS A is much more similar to IFS C than to IFS B.

However, if we use the Eq. (2) to calculate the similarity measures, then $S_2(A,B)=S_2(A,C)=0.95$, which is not reasonable. For two IFSs A and B, Xu et al. [15, 18] defined similarity measures between A and B as follow:

$$S_3(A,B)=1-\left[\frac{1}{2n}\sum_{j=1}^{n}\left(\left|\mu_A(x_j)-\mu_B(x_j)\right|^\alpha+\left|\nu_A(x_j)-\nu_B(x_j)\right|^\alpha+\left|\pi_A(x_j)-\pi_B(x_j)\right|^\alpha\right)\right]^{1/\alpha}$$

(3)

Where $\alpha>0$.

$$S_4(A,B)=\left[\sum_{j=1}^{n}\left(\min\{\mu_A(x_j),\mu_B(x_j)\}+\min\{\nu_A(x_j),\nu_B(x_j)\}+\min\{\pi_A(x_j),\pi_B(x_j)\}\right)\right]\Big/\left[\sum_{j=1}^{n}\left(\max\{\mu_A(x_j),\mu_B(x_j)\}+\max\{\nu_A(x_j),\nu_B(x_j)\}+\max\{\pi_A(x_j),\pi_B(x_j)\}\right)\right] \quad (4)$$

If let $\alpha\to+\infty$ and $\alpha=1$ in Eq. (3), then it is reduced to the following formulas, respectively:

$$S_5(A,B)=1-\max_j\left\{\left|\mu_A(x_j)-\mu_B(x_j)\right|,\left|\nu_A(x_j)-\nu_B(x_j)\right|,\left|\pi_A(x_j)-\pi_B(x_j)\right|\right\}$$

(5)

$$S_6(A,B)=1-\frac{1}{2n}\sum_{j=1}^{n}\left(\left|\mu_A(x_j)-\mu_B(x_j)\right|+\left|\nu_A(x_j)-\nu_B(x_j)\right|+\left|\pi_A(x_j)-\pi_B(x_j)\right|\right)$$

(6)

Example 3.2. Let $A=\{\langle x,0.3,0.5\rangle\}$, $B=\{\langle x,0.4,0.4\rangle\}$ and $C=\{\langle x,0.4,0.5\rangle\}$ be three IFSs. Intuitively, we can see that IFS A is much more similar to IFS C than to IFS B.

However, if we use the Eqs. (4), (5) and (6) to calculate the similarity measures, then we have the results: by (4) we have $S_4(A,B)=S_4(A,C)=0.8182$; by (5) we have $S_5(A,B)=S_5(A,C)=0.9$; by (6) we have $S_6(A,B)=S_6(A,C)=0.9$.

Obviously, these results are not reasonable.

In addition, Wei et al. [16] gave another similarity measure of IFSs based on entropy theory, as follows.

$$S_7(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{1-\min\{\mu_i,\nu_i\}}{1+\max\{\mu_i,\nu_i\}} \quad (7)$$

where $\mu_i=\left|\mu_A(x_i)-\mu_B(x_i)\right|$, $\nu_i=\left|\nu_A(x_i)-\nu_B(x_i)\right|$.

Example 3.3. Let $A=\{\langle x,0.5,0.5\rangle\}$, $B=\{\langle x,0.4,0.6\rangle\}$ and $C=\{\langle x,0.4,0.4\rangle\}$ be three IFSs. If we use the Eq. (7) to calculate the similarity measures, then we have $S_7(A,B)=S_7(A,C)=0.8182$. Obviously, we can not distinguish which one IFS A is closer to.

B. A New Similarity Measure

Now we explain the problem in example 3.3 in the view of groups voting. For an alternative $A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle \mid x_i\in X\}$, symbols $\mu_A(x_i)$, $\nu_A(x_i)$, $\pi_A(x_i)$ denote shares of the supporters, the dissenters and the abstention group, respectively. Then the shares of the supporters and the dissenters to alternative A are 50% and 50%, respectively. To alternative B, the corresponding shares are 40% and 60%. To alternative C, the shares of the supporters, the dissenters and the abstention group are 40%, 40% and 20%, respectively. It is obvious that A is more similar to C than to B.

In the following, we consider the abstention group influence and give a new definition for a similarity measure between IFSs A and B as follows.

Definition 3.2. Let $X=\{x_1,x_2,\cdots,x_n\}$ be a finite universe of discourse. For each IFS $A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle \mid x_i\in X\}$, let $\mu'_A(x_i)=\mu_A(x_i)+\frac{1+\mu_A(x_i)-\nu_A(x_i)}{2}\pi_A(x_i)$, $\nu'_A(x_i)=\nu_A(x_i)$. Then a similarity measure between A and B is given by:

$$S(A,B)=1-\frac{1}{n^{1/\alpha}}\left[\sum_{j=1}^{n}\left|\mu'_A(x_i)-\mu'_B(x_i)\right|^\alpha+\left|\nu'_A(x_i)-\nu'_B(x_i)\right|^\alpha\right]^{1/\alpha}\quad(\alpha>0) \quad (8)$$

In Eq. (8), $\frac{1+\mu_A(x_i)-\nu_A(x_i)}{2}$ denotes the possibility that some people from the abstention group tend to cast votes. Firstly, we suppose the possibilities that people from the abstention group tend to cast votes and some are dissenters are the same, i.e. we weigh $\pi_A(x_i)$ with 1/2, then we modify the weigh by $\frac{\mu_A(x_i)-\nu_A(x_i)}{2}$. In the view of group voting, if shares of the supporters and the dissenters are 80% and 10%, respectively, then the majority of the abstention group will tend to cast votes, which indicates that the more the supporters are, the larger proportion that people from the abstention group tend to cast votes.

Next, the examples 3.1-3.3 are again considered and used to demonstrate the feasibility and validity of the new similarity measure.

If let $\alpha=1$, then the Eq. (8) is reduced to the following formula:

$$S^1(A,B)=1-\frac{1}{n}\left[\sum_{j=1}^{n}\left|\mu'_A(x_i)-\mu'_B(x_i)\right|+\left|\nu'_A(x_i)-\nu'_B(x_i)\right|\right] \quad (9)$$

where $\mu_A^{'}(x_i)=\mu_A(x_i)+\frac{1+\mu_A(x_i)-\nu_A(x_i)}{2}\pi_A(x_i)$, $\nu_A^{'}(x_i)=\nu_A(x_i)$.

Example 3.1'. Let $A=\{\langle x,0.1,0.2\rangle\}$, $B=\{\langle x,0.4,0.4\rangle\}$ and $C=\{\langle x,0.2,0.2\rangle\}$ be three IFSs. Now we calculate the similarity measures $S(A,B)$ and $S(A,C)$ by Eq. (9). Then $S^1(A,B)=0.715$, $S^1(A,C)=0.915$, which indicates that IFS A is much more similar to IFS C than to IFS B and it is consistent with intuition.

Example 3.2'. Let $A=\{\langle x,0.3,0.5\rangle\}$, $B=\{\langle x,0.4,0.4\rangle\}$ and $C=\{\langle x,0.4,0.5\rangle\}$ be three IFSs. By Eq. (9), we get $S^1(A,B)=0.78$, $S^1(A,C)=0.935$, which indicates that IFS A is much more similar to IFS C than to IFS B and it is consistent with intuition.

Example 3.3'. Let $A=\{\langle x,0.5,0.5\rangle\}$, $B=\{\langle x,0.4,0.6\rangle\}$ and $C=\{\langle x,0.4,0.4\rangle\}$ be three IFSs. By Eq. (9), we get $S^1(A,B)=0.8$, $S^1(A,C)=0.9$, which indicates that IFS A is much more similar to IFS C than to IFS B and it is consistent with the analysis above.

IV. Application

IFSs are a suitable tool to cope with imperfect information. In this section we present applications in the context of pattern recognition and medical diagnosis.

A. *Pattern Recognition*

We apply the similarity measure defined by (9) to solve some pattern recognition problems with intuitionistic fuzzy information, which involves the following steps:

Step 1: Suppose that there exist m patterns represented by IFSs $A_i=\{\langle x_j,\mu_{A_i}(x_j),\nu_{A_i}(x_j)\rangle \mid x_j\in X\}$ $(i=1,2,\cdots,m)$ in the feature space $X=\{x_1,x_2,\cdots,x_m\}$, and suppose that there is a sample to be recognized, which is represented by an IFS $B=\{\langle x_j,\mu_B(x_j),\nu_B(x_j)\rangle \mid x_j\in X\}$.

Step 2: Calculate the similarity measure $S(A_i,B)$ between A_i and B by Eq. (9).

Step 3: Select the largest one, denoted by $S(A_{i0},B)$, from $S(A_i,B)(i=1,2,\cdots,m)$.

Then the sample B belongs to the pattern A_{i0}.

Now we consider an example of a pattern recognition problem on the classification of hybrid mineral.

Example 4.1. We consider the same data as in [11]. Assume that there are five kinds of mineral fields and an unknown hybrid mineral, which are featured by the content of six minerals. We can express the five kinds of typical hybrid mineral by five IFSs C_1, C_2, C_3, C_4, C_5 in the feature space $X=\{x_1,...,x_6\}$ (see the following Table 1). Given another kind of hybrid mineral

$$B=\{\langle x_1,0.629,0,303\rangle,\langle x_2,0,524,0.356\rangle,\langle x_3,0.210,0.689\rangle,\langle x_4,0.218,0.753\rangle,\langle x_5,0.069,0.876\rangle,\langle x_6,0.658,0.256\rangle\}$$

Our purpose is to distinguish which field the unknown pattern B belongs to.

Using above steps and calculating the similarity degree $S(A_i,B)$ between A_i and B by (9), we can get: $S(C_1,B)=0.5514$, $S(C_2,B)=0.4733, S(C_3,B)=0.2184$, $S(C_4,B)=0.6647, S(C_5,B)=0.4662$.

TABLE I. THE DATA OF EXAMPLE 4.1.

	x_1	x_2	x_3	x_4	x_5	x_6
C_1	(0.74,0.13)	(0.03,0.82)	(0.19,0.63)	(0.49,0.36)	(0.02,0.63)	(0.74,0.13)
C_2	(0.12,0.67)	(0.03,0.83)	(0.05,0.80)	(0.14,0.65)	(0.02,0.82)	(0.39,0.65)
C_3	(0.45,0.39)	(0.66,0.30)	(1.00,0.00)	(1.00,0.00)	(1.00,0.00)	(1.00,0.00)
C_4	(0.28,0.72)	(0.52,0.37)	(0.47,0.42)	(0.30,0.66)	(0.19,0.81)	(0.74,0.12)
C_5	(0.33,0.45)	(1.00,0.00)	(0.18,0.73)	(0.16,0.77)	(0.05,0.90)	(0.68,0.26)

Clearly, the similarity degree $S(C_4,B)$ between C_4 and B is the largest one. Hence hybrid mineral B should be produced by the mineral field C_4, which is different from the result in [11] that hybrid mineral B is produced by the mineral field C_5. However, analyzing the feature of hybrid minerals C_4, C_5 and the unknown pattern B, we can see that the unknown pattern B is much more similar to the C_4 than to C_5. So the result in this paper is more consistent with reality.

B. *Medical Diagnosis*

The theory of IFSs has been utilized to perform medical diagnosis in [12, 16, 17]. Here, we give an example to show how to solve medical diagnosis problem with intuitionistic fuzzy information by our similarity measure defined in Eq. (9).

Example 4.2. We consider the same data as in [12, 17]. Assume that there are a set of diagnoses D, a set of symptoms S and a set of patients P, where

$$D=\{Viral\ \ fever, Malaria, Typhoid, Stomach\ \ problem, Chest\ \ pain\},$$

$$S=\{Temperature, Headache, Sotmach\ pain, Cough, Chest\ \ pain\}$$

$$P=\{Al, Bob, Joe, Ted\}.$$

Table 2 presents the characteristic symptoms for the considered diagnoses, and Table 3 gives the symptoms for each patient. Each element of the tables is given in the form of a pair of numbers corresponding to the membership μ and non-membership ν , respectively. One needs to find a proper diagnosis for each patient $p_i, i=1,2,3,4$

TABLE II. SYMPTOMS CHARACTERISTIC FOR THE CONSIDERED DIAGNOSES.

	Viral fever	**Malaria**	**Typhoid**	**Stomach problem**	**Chest problem**
Temperature	(0.4,0.0)	(0.7,0.0)	(0.3,0.3)	(0.1,0.7)	(0.1,0.8)
Headache	(0.3,0.5)	(0.2,0.6)	(0.6,0.1)	(0.2,0.4)	(0.0,0.8)
Stomach pain	(0.1,0.7)	(0.0,0.9)	(0.2,0.7)	(0.8,0.0)	(0.2,0.8)
Cough	(0.4,0.3)	(0.7,0.0)	(0.2,0.6)	(0.2,0.7)	(0.2,0.8)
Chest pain	(0.1,0.7)	(0.1,0.8)	(0.1,0.9)	(0.2,0.7)	(0.8,0.1)

TABLE III. SYMPTOMS CHARACTERISTIC FOR THE CONSIDERED PATIENTS.

	Temperature	Headache	Stomach pain	Cough	Chest pain
Al	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Bob	(0.0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Joe	(0.8,0.1)	(0.8,0.1)	(0.0,0.6)	(0.2,0.7)	(0.0,0.5)
Ted	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

Now we show how to utilize the proposed similarity measure (9) to derive a proper diagnosis for each patient $p_i, i=1,2,3,4$. We first calculate the similarity degree $S(p_i, d_k)$ between symptoms of each patient $p_i, i=1,2,3,4$ and the set of symptoms that are characteristic for each diagnosis $d_k \in D, k=1,2,3,4,5$. From Eq. (9), we have

$$S(p_i,d_k)=1-\frac{1}{n}\left[\sum_{j=1}^{n}\left|\mu'_{p_i}(x_i)-\mu'_{d_k}(x_i)\right|+\left|\nu'_{p_i}(x_i)-\nu'_{d_k}(x_i)\right|\right] \qquad (10)$$

where

$$\mu'(x_i)=\mu(x_i)+\frac{1+\mu(x_i)-\nu(x_i)}{2}\pi(x_i),\ \nu'(x_i)=\nu(x_i) \qquad (11)$$

Accordingly, we get Table 4 and Table 5 by (11). Each element of the tables is given in the form of a pair of numbers corresponding to the membership μ' and non-membership ν' , respectively. So we can obtain Table 6 that presents all the results for the considered patients.

TABLE IV. SYMPTOMS CHARACTERISTIC FOR THE CONSIDERED DIAGNOSES.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.82,0.0)	(0.955,0.0)	(0.5,0.3)	(0.14,0.7)	(0.115,0.8)
Headache	(0.38,0.5)	(0.26,0.6)	(0.825,0.1)	(0.36,0.4)	(0.02,0.8)
Stomach pain	(0.14,0.7)	(0.005,0.9)	(0.225,0.7)	(0.98,0.0)	(0.2,0.8)
Cough	(0.565,0.3)	(0.955,0.0)	(0.26,0.6)	(0.225,0.7)	(0.2,0.8)
Chest pain	(0.14,0.7)	(0.115,0.8)	(0.1,0.9)	(0.225,0.7)	(0.885,0.1)

TABLE V. SYMPTOMS CHARACTERISTIC FOR THE CONSIDERED PATIENTS.

	Temperature	Headache	Stomach pain	Cough	Chest pain
Al	(0.885,0.1)	(0.675,0.1)	(0.2,0.8)	(0.675,0.1)	(0.175,0.6)
Bob	(0.02,0.8)	(0.5,0.4)	(0.675,0.1)	(0.14,0.7)	(0.115,0.8)
Joe	(0.885,0.1)	(0.885,0.1)	(0.08,0.6)	(0.225,0.7)	(0.125,0.5)
Ted	(0.675,0.1)	(0.555,0.4)	(0.435,0.4)	(0.775,0.2)	(0.435,0.4)

TABLE VI. SIMILARITIES OF SYMPTOMS FOR EACH PATIENT TO THE CONSIDERED SET OF POSSIBLE DIAGNOSES

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Al	0.707	0.596	0.47	0.052	0.042
Bob	0.219	0.412	0.402	0.79	0.244
Joe	0.563	0.318	0.705	0.206	0.072
Ted	0.615	0.419	0.359	0.233	0.049

For example, we can get $S(p_1, d_1)$ by (10):

$$\begin{aligned}S(p_1,d_1)=1-\frac{1}{5}\Big[&|0.885-0.82|+|0.1-0.0|+|0.675-0.38|\\&+|0.1-0.5|+|0.2-0.14|+|0.8-0.7|+|0.675-0.565|\\&+|0.1-0.3|+|0.175-0.14|+|0.6-0.7|\Big]=0.707\end{aligned}$$

Then the proper diagnosis d_k for the patient p_i is derived according to the biggest numerical value from the obtained similarity measures in Table 6.

From Table 6, we can see Al suffers from Viral fever, Bob from Stomach problems, Joe from Typhoid and Ted from Viral Fever. These results are in agreement with the ones obtained by Vlachos and Sergiadis [12]. Compared with the results in [17], the diagnoses for Bob, Joe and Ted are the same, but the diagnosis for Al is different.

V. CONCLUSIONS

In this paper, we analyze the limitations of existing similarity measures. We also discuss the importance to search for a better similarity measure. Then, we take account of the abstention group influence and develop a new measure to address limitations of previous ones. The validity of our measure is illustrated through some numeric examples. Finally, we apply the newly proposed similarity measure to pattern recognition and medical diagnosis.

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