Target Tracking by Adaptive Waveform Design of Generalized Frequency Modulation

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***Abstract*-A tracking algorithm by adaptive waveform design of generalized frequency modulation (GFM) is researched in this paper. According to the tracker's dynamical requirements, the transmitted waveform for the next time step is adaptively designed based on GFM pulses as a sample waveform, in order to minimize the predicted tracking mean squared error (MSE). With the assumption of high SNR and the linear dynamical state and observation models, the adaptive design of GFM waveform is implemented by minimizing the predicted MSE with the waveform dependent Cramer-Rao Lower Bound (CRLB) and Kalman filtering. Simulation results show that under the same SNR conditions, the tracking MSE of the algorithm is lower than that of the tracking algorithms based on linear frequency modulation with the fixed or adaptive parameters.**

Keywords-generalized frequency modulation; target tracking; kalman filtering; adaptive waveform design

I. INTRODUCTION

If we design the transmitted waveforms of radar to make it match the statistics of target and environment, then we can improve the performance of radar systems. The technology of radar waveform-agile was born based on this idea. Waveform-agility can provide performance improvements such as reduced target tracking error, improved target detection, higher target identification accuracy. In recent years, the technology of waveform-agile for target tracking has become a hotspot in radar signal processing area [1].

In the process of target tracking, the environment is always varying with time and so the tracker needs to obtain the dynamic information of the target and the environment. If we can design the transmitted waveform for the next time step according to the requirements of the tracker, we can meet the requirements of the tracker better, and then we can improve the performance of target tracking. In addition, the limitation of computation capability also inspires the motivation to use adaptive waveform design. The data rate of modern radar is obvious higher than ever, the resulted bottle-neck of data processing seriously affects system performance, and thus it is necessary to collect the information matched to the target only. For example, to use waveforms that can only increase Doppler resolution does not provide any information in identifying an object that is known to be stationary. Therefore the two points become the actual causes to apply waveform agility for tracking [2-3] to radar.

In [4], the authors proposed a method that optimizes the transmitted waveform based on the wide sense stationary-uncorrelated scattering target impulse response (TIR), which is a suboptimal method of online waveform design. Compared with traditional linear frequency modulation (LFM) waveform, with this method we can obtain a lower mean squared error (MSE), but it is only suitable for solving the problem of TIR estimate in Gauss noise background. In terms of adaptive LFM waveform design for target tracking, the authors inferred the Cramer-Rao Lower Bound (CRLB) of delay and Doppler with ambiguity function in [5], and they also proposed an adaptive LFM waveform design algorithm with a Kalman filter. Compared with an algorithm based on LFM waveform of fixed parameters, we can obtain a lower tracking MSE with this method. However it adopts a LFM waveform as a sample for adaptive design other than a nonlinear frequency modulation waveform, which is better for accurate measurement of the range and range rate of radar targets.

There are many advantages with nonlinear frequency-modulated waveform, such as lower delay-Doppler coupling, which can improve the tracking performance of radar evidently [6]. Moreover, nonlinear frequency-modulation is helpful to improve the performance of low probability intercept (LPI). So in this paper we study the tracking algorithm based on a generalized frequency-modulated (GFM) waveform by simulation in order to verify the performance of the algorithm. In the algorithm, the transmitted waveform is adaptively designed according to the requirement of tracker, in order to minimize the predicted MSE of target tracking. With the assumption of the linear models of target dynamics and observation, and a high SNR condition, we use CRLB and a Kalman filter to realize adaptive design of GFM waveform by minimizing the predicted MSE of tracking.

II. GFM WAVEFORM AND CRLB

A. GFM Waveform Structure

GFM signal can be formulated as

$$s_T(t) = \sqrt{2}\,\mathrm{Re}[\sqrt{E_T}\,\tilde{s}(t)\exp(j2\pi f_c t)] \quad (1)$$

where f_c is the carrier frequency and E_T is the energy of the transmitted pulse. The complex envelope $\tilde{s}(t)$ is defined as

$$\tilde{s}(t) = (\frac{1}{\pi\lambda^2})^{\frac{1}{4}}\exp(-\frac{t^2}{2\lambda^2})\exp(j2\pi b\xi(t)) \quad (2)$$

where λ is the duration of the envelope, b is frequency modulation (FM) rate, and $\xi(t)$ is the chirp phase function.

TABLE I. THE PHASE FUNCTION AND BAND WIDTH OF GFM WAVEFORM.

Waveform	**Phase Function** $\xi(t)$	**Band Width** B
LFM	t^2	bT_s
HFM	$\ln(T_s + \lvert t \rvert)$	$b/(3T_s)$
EFM	$e^{\lvert t \rvert}$	$b \cdot \lvert 1 - e^{T_s/2} \rvert$

The transmitted GFM waveform includes LFM, hyperbolic frequency modulation (HFM) and exponential frequency modulation (EFM), with the phase function $\xi(t)$, band width B and pulse width T_s as shown in Table I. We can obtain different time frequency characteristics by changing the phase function. Let $\theta_k = [\lambda_k, b_k]^T$ denote the parameter vector of GFM waveform at time k, where $\lambda_k = T_s/\alpha$, and $\alpha = 7.4338$.

B. GFM Waveform Ambiguity Function and CRLB

The delay and Doppler resolution properties of arbitrary waveform are decided by the ambiguity function. For radar, GFM signal meet the narrowband condition, and its narrowband ambiguity function is defined as

$$AF_{\tilde{s}}(\tau, v) = \int_{-\infty}^{\infty} \tilde{s}(t + \frac{\tau}{2})\tilde{s}^*(t - \frac{\tau}{2})\exp(-j2\pi vt)dt \qquad (3)$$

where τ is the delay of target echo, and v is Doppler shift. We can get the CRLB of Doppler shift and delay with the narrowband ambiguity function. By calculating the second partial derivative of ambiguity function with formulation (3), and set $\tau = 0, v = 0$, we obtain the elements of Fisher information matrix [6]. Using η to represent SNR, we can formulate the Fisher information matrix of GFM as

$$J = \eta \begin{bmatrix} \frac{1}{2\lambda^2} + g(\xi) & 2\pi f(\xi) \\ 2\pi f(\xi) & (2\pi)^2 \frac{\lambda^2}{2} \end{bmatrix} \qquad (4)$$

where $g(\xi)$ and $f(\xi)$ is defined as

$$g(\xi) = (2\pi b)^2 \int_{-\infty}^{\infty} \frac{1}{\lambda\sqrt{\pi}} \exp(-\frac{t^2}{\lambda^2})[\xi'(t)]^2 dt \qquad (5)$$

$$f(\xi) = 2\pi b \int_{-\infty}^{\infty} \frac{t}{\lambda\sqrt{\pi}} \exp(-\frac{t^2}{\lambda^2})\xi'(t)dt \qquad (6)$$

and $\xi'(t) = d\xi(t)/dt$. For the variance of estimate error of the delay and Doppler $[\tau, v]^T$, the CRLB is the inverse of the Fisher information matrix.

III. DESIGNING ALGORITHM OF ADAPTIVE GFM WAVEFORM

A. System Models

The target dynamics and measurements are modeled as linear equations. The target state vector at time k is given by $X_k = [x_k \quad y_k \quad \dot{x} \quad \dot{y}]$, where x_k and y_k are the target positions in the 2-dimensional plane, and $\dot{x}$ and $\dot{y}$ are the velocities of the target at time *k*. Then the equation of target states can be given by

$$X_k = FX_{k-1} + W_k \qquad (7)$$

where F is the transition matrix, and W_k is stationary white noise process with covariance matrices $Q(k)$.

The observation vector of a target at time k is given by $Z_k = [x_k \quad y_k]$, and then its observation equation can be formulated as

$$Z_k = HX_k + V_k \qquad (8)$$

where H the observation matrix, V_k is white noise vector with covariance matrices $R(\theta_k)$, and θ_k is the parameters vector of the waveform at time *k*. It is obvious that the parameter of waveform affects the covariance matrix of observation noise. Thus we can reduce measurement error of delay and Doppler shift in target tracking by dynamically designing the parameters of transmitted waveform. It is the essential cause that we can improve the performance of target tracking with waveform agility in radar.

B. Cost Function and Waveform Design Algorithm

With the observation data at time 1: *k*-1, the purpose of the algorithm is to design a set of optimal waveform parameters adaptively, which can be used to minimize the predicted MSE of tracking at time *k*. So the cost function is defined as the covariance matrix of predicted MSE at time *k*.

$$J(\theta_k) = E_{X_k, Z_k | Z_{1:k-1}}\{(X_k - \hat{X}_k)^T (X_k - \hat{X}_k)\} \qquad (9)$$

where $E\{\cdot\}$ is an expectation over predicted states and observations, $\hat{X}_k$ is the estimate of X_k given the sequence of observations values. We can minimize the cost function $J(\theta_k)$ at time *k* by designing the waveform parameter adaptively. The optimal value of waveform parameter θ_k is given by

$$\tilde{\theta}_k = \arg\min_{\theta_k} \mathrm{Tr}\{J(\theta_k)\} \qquad (10)$$

where $\mathrm{Tr}\{\cdot\}$ denotes the trace of a matrix. For a given θ_k, we can use Kalman filtering algorithm to calculate the covariance matrix in formulation (9).

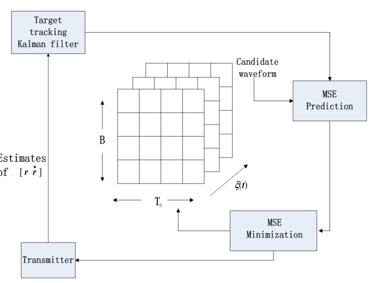

FIGURE I. BLOCK DIAGRAM OF THE TRACKING ALGORITHM.

C. Relationship Between the Estimation Errors and The Cost Function

Target range r and range rate $\dot{r}$ can be expressed by delay τ and Doppler shift v respectively, and thus we have $r = c\tau/2$ and $\dot{r} = cv/2f_c$, where c is the velocity of propagation, f_c is the carrier frequency. Assuming that the target state which needs to be predicted defined as $\varphi = [r \quad \dot{r}]$, then we have

$$\varphi = diag(c/2, c/2f_c) \times [\tau, v]^T \qquad (11)$$

According to the definition of CRLB, the CRLB of target range and range rate can be expressed as

$$E[(\hat{\varphi} - \varphi)(\hat{\varphi} - \varphi)^T] \geq diag(c/2, c/2f_c) \times J^{-1} \times diag(c/2, c/2f_c)^T \qquad (12)$$

So the covariance matrix of observation noise vector is $R(\theta_k) = E[(\hat{\varphi} - \varphi)(\hat{\varphi} - \varphi)^T]$. With the different phase functions, band width *B* and pulse width T_s of transmitted waveform, we can change the covariance matrix of observation noise $R(\theta_k)$, and then affect the predicted covariance matrix of tracking error $J(\theta_k)$. In this way we can obtain the optimal waveform parameter $\tilde{\theta}_k$ according to (10), therefore the adaptive waveform design process is realized. The basic flow of the algorithm is shown in Fig. 1.

IV. Simulation and Analysis

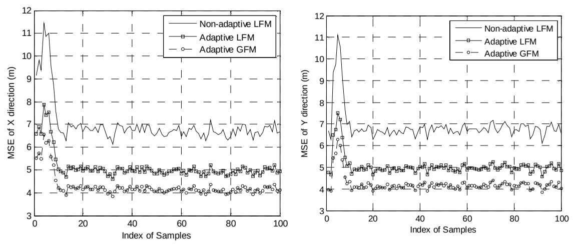

FIGURE II. TARGET TRACKING MSE OF X AND Y DIRECTION.

For simulation we assume that the target dynamics are modeled by a linear constant velocity model, and the radar is located at the origin of the coordinate on the 2-dimensional plane. The target is moving away from radar with a constant velocity. The parameters of target dynamics and measurement models are defined as follows, the transition matrix is expressed as $F = [1, t, 0, 0; 0, 1, 0, 0; 0, 0, 1, t; 0, 0, 0, 1]$, and the observation matrix is $H = [1, 0, 0, 0; 0, 0, 1, 0]$. The initial position of the target in X and Y direction is (20m, 0m), and the range rates are all 100m/s. The echo SNR is assumed to be 5dB. We use the mean of 500 Monte Carlo simulations in order to estimate the smoothed mean squared error. The parameters of transmitted waveforms are shown in Table II. Fig. 2 show the mean squared error of the range from X and Y directions.

According to simulation results, the tracking algorithms with adaptive waveform parameters (including the one with adaptive LFM and the other with adaptive GFM) have a better performance than the non-adaptive algorithm. It indicates that the involving of adaptive waveform design in tracking can reduce the tracking error. In addition, the tracking error of adaptive GFM is less than adaptive LFM according to Fig.2. The main cause is that GFM waveform has a lower range and Doppler coupling characteristic than that of a LFM waveform. Since the adoption of GFM can meet the requirement of tracker better, the tracking algorithm with adaptive GFM has a better performance.

TABLE I .THE PARAMETER OF TRANSMITTED WAVEFORMS.

Waveform	**Non-adaptive**	**Adaptive LFM**	**Adaptive GFM**
Pulse width	2 ms	1—3 ms	1—3 ms
Band width	5 kHz	1—10 kHz	1—10 kHz

V. Conclusion

In this paper we studied an adaptive GFM waveform design algorithm for target tracking, instead of adaptive LFM waveform design algorithm proposed in [5]. Firstly we introduced the GFM waveform structure, and formulated the CRLB of Doppler shift and time delay according to ambiguity function. Then we defined the target dynamics and observation models, and presented an adaptive waveform design algorithm to minimize the predicted tracking MSE. Finally we verified the performance of this algorithm by simulation. Simulation results show that adaptive waveform design for target tracking can reduce target tracking error, and the performance of adaptive GFM waveform design algorithm is better than that with adaptive LFM waveform design. Thus we conclude that the tracking algorithm by adaptive waveform design of GFM waveform can improve the performance of target tracking effectively.

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