The Design of the Multi-user MIMO Nonlinear Precoding System

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***Abstract*—This article mainly studied the non-linear multi-user MIMO system precoding technique in view of the precoding technology. On the basis of reviewing and summarizing predecessors' work, combining with the actual situation, it studied nonlinear precoding in TH (Tomlinson - Harashima) precoding. And it also proposed TH precoding scheme based on the quantitative channel state information (CSI) to achieve the average sum rate. And it proved that the nonlinear TH projected can achieve performance better than the linear force based on perfect CSI and quantitative CSI zero (ZF) precoding.**

Keywords: MIMO; multi-users; nonlinear precoding

I. INTRODUCTION

In order to reduce the interference between the parallel data flow, we can use precoding matching transmission channel both in the transmitter data flow (precoding) and the processing of received signal (equality). Therefore, common widely used linear precoding scheme has low complexity based on forced to zero (ZF) and minimum mean square error (MMSE) criterion. Despite the very low complexity, linear precoding scheme also has capacity loss problems. And both in the transmitter or receiver, nonlinear precoding processing can provide alternative methods which can improve the performance of linear precoding. The implementation method of high complexity in practice includes linear precoding combined with decision feedback equalization (DFE) [1], vector perturbation [2], Tom Lin Senyuan island (TH) precoding [3] and ideal dirty paper coding [4] these four methods. Vector perturbation has put forward multi-user MIMO channel model which can achieve rate close to full capacity. It is a linear precoding technique with superior performance, but this approach requires joint selection which will be sent to all receivers. It is a multidimensional integer least-squares problem of grid signal vector perturbation, the complexity of its existence is much higher than TH precoding. It is a compromise which provide a good choice between performance and complexity, and received much attention in recent.

Like many precoding scheme, the main problems of TH precoding system are availability of channel state information (CSI) in the transmitter. In frequency division duplex (FDD) system, the transmitter can't estimate the convey information and CSI from receiver to transmitter through feedback channel. This article will focus on the implementation of the FDD system TH precoding scheme, using the results of the study in the quantification of the CSI at the transmitter [5] widely used in the multi-input multi-output (MIMO) system. This paper designed quantitative CSI ZF precoding based on a multi-user space TH. Assuming that quantitative CDI is only in the transmitter, the calculation of feedforward filter and feedback filter is based on available only quantitative CDI on the transmitter side. In addition, the proposed scheme system settings in the system the number of user K can be less than or equal to transmitting antenna number $\tilde{N}_T$. It suggested to quantification the function of the number of feedback bits based on the CSI - TH precoding scheme to realize the analysis of the average rate of combined quantitative characterization of the sum of the rate of average loss rate of CSI for each user. The derived upper limit of TH precoding closely tracked the real rate of loss and convergence speed than the upper limit for the number of feedback bits in the ZF precoding.

II. THE TRANSMISSION SYSTEM WITH QUANTITATIVE CSI

In the actual cases, there are not perfect CSI presenting in the transmission system. For example, in the FDD system transmitting end through each receiver downward limit of feedback B get CSI. According to the literature [6] for quantitative research of CSI, channel in each vector can be quantitative, its corresponding index feedback to the transmitter at the receiving end through a wrong channel without delay. A transmitter and the receiver can read the code $W = \{w_1, \cdots w_n\}(w_i \in C^{1\times n_T})$, direction vector can be written as after the quantitative of the receiving end first k selecting.

$$\hat{h}_k = \arg\max_{w_i \in W} \left\{ \left| \bar{h}_k w_i \right|^2 \right\} \tag{1}$$

Here $\bar{h}_k = \frac{h_k}{\|h_k\|}$ is the channel direction vector of user k. Here we use the RVQ code. The code in the measurement vector n is independent of shaft and the n_T inside spherical space is fragmented. Although RVQ this is not the most suitable for limited space system code, but it is suitable for the most close to the optimal quantitative analysis and its performance. For the user k, we can get

$$h_k = h_k \cos\theta_k + h_k \sin\theta_k \tag{2}$$

Here $\cos^2\theta_k = \left|\bar{h}_k \hat{h}_k^H\right|^2$, $\tilde{h}_k = C^{1\times n_r}$ is a baseline vector distribution of same axial units in the orthogonal complement of subspace $\hat{h}_k$ and $\sin\theta_k$ independent of each other. H can be written as:

$$H = \Gamma(\Phi H + \Omega H) \quad (3)$$

Here

$\Gamma = diag(\rho_1,\cdots,\rho_k), \rho_k = \|h_k\|, \Phi = diag(\cos\theta_1,\cdots,\cos\theta_k),$

$\Omega = diag(\sin\theta_1,\cdots,\sin\theta_k), \hat{H} = [\hat{h}_1^T,\cdots,\hat{h}_k^T]^T, \tilde{H} = [\tilde{h}_1^T,\cdots,\tilde{h}_k^T]^T$

As a simple analysis, in this article we reference for the quantitative element approximation method in [7]. Here quantitative unit is envisioned as a spherical theissen polygon area, whose the surface area is $\frac{1}{n}$ of the total n_T dimensions of the spherical surface. For a given code W, the actual unit vector quantization w_i, $R_i = \{\bar{h} : |\bar{h}w_i|^2 \geq |\bar{h}w_j|^2, \forall i \neq j\}$ can be approximate as $\tilde{R}_i \approx \{\bar{h} : |\bar{h}w_i| \geq 1-\delta\}, \delta = 2^{-\frac{\pi}{\pi r-1}}$. In quantitative CDI launch, the transmitting terminal get feedback precoding matrix F and feedback matrix B through its channel matrix QR decomposition using the same method of QR decomposition of matrix $\hat{H}$ which uses the same decomposition methods with the same QR decomposition matrix H. And here this matrix $\hat{R}$ and $\hat{Q}$ are respectively with the same structure like R and Q. we can get $F = \hat{Q}^H$ and $B = (diag\{\hat{R}\})^{-1}\hat{R} - I$. In addition, the scaling matrix at the receiving end becomes:

$$G = \sqrt{\frac{k}{p}}(\Gamma\Phi diag\{\hat{R}\})^{-1} \quad (4)$$

At the receiving end we use the same method under the condition of the optimal CSI to test and detect signals, and the detected signal vector $\hat{y}$ can be further written as:

$$\begin{aligned}\hat{y} &= G(\sqrt{\frac{p}{k}}HFx + n) = G\sqrt{\frac{p}{k}}\Gamma(\Phi\hat{H} + \Omega\tilde{H})Fx + Gn \quad (5)\\ &= v + (\Phi diag\{\hat{R}\})^{-1}\Omega\tilde{H}\hat{Q}^H + \sqrt{\frac{p}{k}}(\Gamma\Phi diag\{\hat{R}\})^{-1}n\end{aligned}$$

Here we use the relationship of $v = (diag\{\hat{R}\})^{-1}\hat{R}x$ .In the formula (5), the first item is beneficial to all user signal vector, the second item is jamming signal caused by quantitative CSI. According to the formula (5), output signal of receiver k and interference plus noise ratio γ_k can be written as:

$$\begin{aligned}\gamma_k &= \frac{1}{\frac{\sin^2\theta_k}{|\hat{\gamma}_{k,k}|^2\cos^2\theta_k}\|\tilde{h}_k\hat{Q}^H\| + \frac{k}{p}\frac{1}{\rho_k^2|\hat{r}_{k,k}|^2\cos^2\theta_k}} \quad (6)\\ &= \frac{\frac{p}{k}\rho_k^2|\hat{r}_{k,k}|^2\cos^2\theta_k}{\frac{p}{k}\rho_k^2\|\tilde{h}_k\hat{Q}^H\|^2\sin^2\theta_k + 1}\end{aligned}$$

III. ANALYSIS OF AVERAGE TOTAL RATE UNDER THE QUANTITATIVE CSI FEEDBACK

It is very difficult to obtain accurate implicit function expression about SINR output γ_k distribution law, not to mention the accurate rate of average total closed expression. Therefore, in order to simplify the analysis, it is suggested that the average total rate of some boundaries and the average total rate loss instead of accurate result. In order to be easier, in this section we assumed that each user channel was Rayleigh fading. In the next section, we will study statistical distribution of the interference signal power for each user due to quantify CSI [8].

A. The Interfere Part

In this section, we assumed that the attenuation of Rayleigh channel and the RVQ feedback of quantify CSI, we will get the statistical distribution of interference parts in formula $\frac{P}{k}\rho_k^2\|\tilde{h}_k\hat{Q}^H\|^2\sin^2\theta_k$. As known to all, ρ_k^2 has a $x_{2n_T}^2$ distribution and $\sin^2\theta_k$ distribution has been given in [10]. Because we can use $\hat{h}_k(k=1,\cdots,K)$ to determine $\tilde{h}_k \perp \tilde{h}_k(k=1,\cdots,K)$ and $\hat{Q}$, so $\hat{h}_k(k=1,\cdots,K)$ is associated with $\hat{Q}$. But the distribution of $\|\tilde{h}_k\hat{Q}^H\|^2$ is not known yet, and the accuracy cannot be ignored. The theory next can reveal precision of the interference of distribution.

For $1 < K < n_T$, and the random variable $\varepsilon_k = \|\tilde{h}_k\hat{Q}^H\|^2, k = 1,\cdots K$. follow the same beta distribution until the $(K-1),(n_T-K)$, then ε_k can be noted as $Beta(K-1, n_T-K)$. In addition, the probability density function (p,d,f) of ε_k can be given

$$f_{\varepsilon,k}(x) = \frac{1}{\beta(K-1, n_T-k)}x^{K-2}(1-x)^{n_T-k-1} \quad (7)$$

Here $\beta(a,b) = \int_0^1 t^{a-1}t^{b-1}dt$ is a beta function. When $k=1$, there is no interference. When $K = n_T, \varepsilon_k = \|\tilde{h}_k\hat{Q}^H\|^2$ is the constant of 1.

The logistic expectations of interfere ε_k are given as follows

$$E_{H,W}[-\log_2(\|\tilde{h}_k\hat{Q}^H\|^2)] = \log_2 e \times \sum_{m=K-1}^{n_T-2}\sum_{l=0}^{n_T-m-2}\frac{(n_T-2)}{m!n!(n_T-m-2-l)!} \quad (8)$$
$$\times(-1)^l\frac{1}{m+l}$$

B. Above the Average rate of Total loss And Total Rate

User k under optimal CSI and quantitative CSI instantaneous gain rate can be respectively written as:

$$R_{p,k} = \log_2(1+\xi_k) \tag{9}$$

$$R_{Q,\ k} = \log_2(1+\gamma_k) \tag{10}$$

Based on this, under the condition of each user feedback section B, quantitative CSI feedback user average total loss rate of k can reach maximum by^2.

$$\Delta R_k = E_{H,W}\{R_{P,k} - R_{Q,k}\} \le \Delta R = \log_2(1+cP2^{\frac{B}{n_T-1}}) + \frac{\log_2(e)^{n_T-1}}{n_T-1}\sum_{i=1}^{n_T-1}\beta(n,\frac{i}{n_T-1}) \tag{11}$$

Here $c = \frac{(K-1)n_T}{k(n_T-1)}$ and the size of the code is $n = 2^B$.

Therefore, when only quantitative CSI effect in transmitter, the second item in right formula (11) comparing with linear precoding can be considered as derivative of nonlinear precoding. In addition, the result of linear ZF beam, rate loss of the nonlinear prediction coding is also an increasing function of system signal to noise ratio (P). Therefore, with accurate feedback rate system under the condition of high SNR signal is limited, it can be got in the following theorem.

Under TH precoding quantitative CSI cases through the feedback section B, the average total rate range of user k are

$$R_{Q,K} \le \log_2 e(\sum_{m=K-1}^{n_T-2}\sum_{l=0}^{n_T-m-2}\frac{(n_T-2)!}{m!l!(n_T-m-2-l)!}\times(-1)^l\frac{1}{m+l}+\frac{1}{n_T-1}\sum_{l=1}^{n}\frac{1}{l}) \tag{12}$$

Here, the code range are $n = 2^B$.

If $n \to \infty$, the upper bound is 0.Therefore, when n with SNR linear expansion, for any given constant, we can find a number of a normal $N(\varepsilon)$, when $n > N(\varepsilon)$,

$$\Delta R \le \log_2(1+cP2^{-\frac{B}{n_T-1}}) + \varepsilon \tag{13}$$

In order to shape a zoom the sufficient condition for feedback rate, setting the right side of (13) the item is the in maximum allowable difference $\log_2 b$. Through some simple processing, we can get

$$B = (n_T-1)\log_2 P - \log_2(b-2^{\varepsilon}-1) + \log_2 c = (n_T-1)\frac{\log_2 10}{10}P_{dB} - \log_2(b-2^{\varepsilon}-1) + \log_2 c \tag{14}$$

Figure 1(a) showed a system average total rate curve under the case of $n_T = 4, K = 4$. Feedback rate is set according to the given relationship between the proportions in (14). Noted that when B is big enough, ε can be very small. In the simulation we set $\varepsilon = 0$ to get strong conditions than formula (14). In the optimal CSI TH precoding, limited feedback is respectively about 4 dB and 5.5 dB when b = 3 and b = 4.

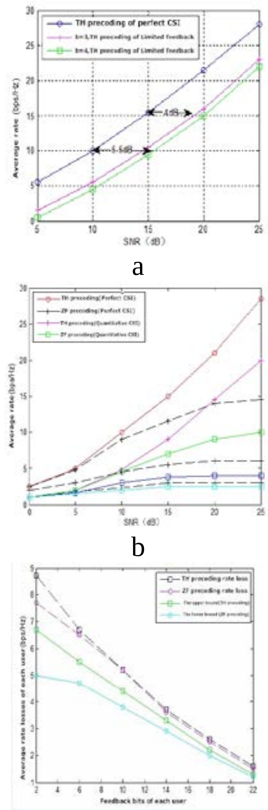

FIGURE I. A: THE INCREASING NUMBER OF FEEDBACK BITS OF 4*4 SYSTEM; B: THE AVERAGE RATE OF PERFECT CSI AND QUANTIZATION IN CSI TH AND ZF PRECODING. C: AVERAGE RATE LOSSES AND THE CORRESPONDING LIMIT NUMBER OF FEEDBACK BITS OF EACH USER. P = 25 DB.

IV. THE RESULTS OF SIMULATION

Setting $n_T = K = 4$, then the system SNR is classified as P. Figure 1(b) shows TH precoding and linear ZF precoding performance under the optimal CSI and quantitative CSI, and feedback section of each user B = 4,8,15. We can see that TH precoding performance is superior to linear precoding under optimal CSI and quantitative CSI circumstances.When SNR is small and slow, the average total rate obtained by TH precoding of quantitative CSI is superior to linear ZF precoding under optimal CSI.

Figure 1(c) showed that when the SNR of the system is 25 dB. Taking the average total loss rate of each user as a function obtained in the case of TH and ZF precoding the number of feedback section. We can also see from the picture that nonlinear precoding is more affected by the imperfect CSI than linear precoding. However, when the SNR is not very big or quantitative feedback processing is very thorough, nonlinear precoding performance is superior to linear precoding. In addition, we can notice that the upper limit of TH precoding is very close to the actual loss rate. With the increase of B at the same time, its convergence speed is faster than linear precoding in the literature.

V. CONCLUSION

This paper studies the multi-user MIMO systems to realize TH precoding with quantitative CSI in case in the downlink. Especially when the number of users K in the system transmitting antenna number is less than or equal to K system, this scheme can be extended to general system. In this paper, by derived average total rate and the average total rate expression of the maximum loss under the condition of quantitative CSI studied the average total rate of the scheme. With quantitative results showed in both the optimal CSI and CSI cases, nonlinear TH precoding get far superior performance of linear ZF precoding. In addition, with the increase of number of feedback section, derived from TH precoding ceiling linear ZF precoding had faster convergence than the real rate.

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