Advanced Wavelet-Based Multilevel Discrete-Continual Finite Element Method for Three-Dimensional Local Structural Analysis

P.A. Akimov
Department of Applied Mathematics and Computer Science
Moscow State University of Civil Engineering
Russia

M. Aslami
Department of Applied Mathematics and Computer Science
Moscow State University of Civil Engineering
Russia

M.L. Mozgaleva
Department of Applied Mathematics and Computer Science
Moscow State University of Civil Engineering
Russia

O.A. Negrozov
Department of Applied Mathematics and Computer Science
Moscow State University of Civil Engineering
Russia

***Abstract*-This paper is devoted to advanced wavelet-based discrete-continual finite element method of local structural analysis. Structures with regular (in particular, constant or piecewise constant) physical and geometrical parameters along so-called "basic" direction are under consideration. High-accuracy solution of the corresponding problems at all points of the model is not required normally, it is necessary to find only the most accurate solution in some pre-known local domains. Wavelet analysis is a powerful and effective tool for corresponding researches. Initial continual and discrete-continual formulations of multipoint boundary problem of three-dimensional structural analysis are presented. The last formulation is transformed to corresponding localized one by using the discrete Haar wavelet basis and finally, with the use of averaging and reduction algorithms, the localized and reduced governing equations are obtained. Special algorithm of localization with respect to each degree of freedom is presented.**

Keywords-advanced wavelet-based discrete-continual finite element method; local structural analysis; multipoint boundary problem; three-dimensional problem; operational formulation; discrete-continual formulation; averaging, reduction; Haar basis; localization with respect to each degree of freedom

I. INTRODUCTION

Research and development of correct mathematical models and methods of structural mechanics are the most important aspects of ensuring safety of buildings and complexes. The analysis and design of structures normally require accurate computer-intensive calculations using numerical (discrete) methods. The field of application of discrete-continual finite element method (DCFEM) [1], that is now becoming available for computer realization, comprises structures with regular (in particular, constant or piecewise constant) physical and geometrical parameters in some dimension (so-called "basic" direction (dimension)). Considering problems remain continual along "basic" direction while along other directions discrete-continual methods presuppose finite element approximation. Solution of corresponding resultant multipoint boundary problems for systems of ordinary differential equations with piecewise constant coefficients and immense number of unknowns is the most time-consuming stage of the computing, especially if we take into account the limitation in performance of personal computers, contemporary software and necessity to obtain correct semi analytical solution in a reasonable time. However, high-accuracy solution at all points of the model is not required normally, it is necessary to find only the most accurate solution in some pre-known domains. Generally the choice of these domains is a priori data with respect to the structure being modelled. Designers usually choose domains with the so-called edge effect (with the risk of significant stresses that could potentially lead to the destruction of structures, etc.) and regions which are subject to specific operational requirements. It is obvious that the stress-strain state in such domains is of paramount importance. Specified factors along with the obvious needs of the designer or researcher to reduce computational costs by application of DCFEM cause considerable urgency of constructing of special algorithms for obtaining local solutions (in some domains known in advance) of boundary problems. Wavelet analysis provides effective and popular tool for such researches [5]. Solution of the considering problem within multilevel wavelet analysis is represented as a composition of local and global components [2, 3, 7].

II. OPERATIONAL FORMULATION

Let x_3 be direction along which physical and geometrical parameters of three-dimensional structure are piecewise constant ("basic" direction). In is necessary to note that these parameters can be changed arbitrarily along x_1 and x_2 . Operational formulation of corresponding resultant multipoint boundary problem of three-dimensional theory of elasticity at extended domain [7], embordering considering structure, within DCFEM has the form:

$$\begin{cases} \bar{U}_k' = \tilde{L}_k \bar{U}_k + \bar{S}_k, \quad x_3 \in (x_{3,k}^b, x_{3,k+1}^b), \quad k = 1,\dots,n_k - 1 \\ \tilde{B}_k^- \bar{U}_{k-1}(x_{3,k}^b - 0) + \tilde{B}_k^+ \bar{U}_k(x_{3,k}^b + 0) = \tilde{\bar{g}}_k^- + \tilde{\bar{g}}_k^+, \quad k = 2,\dots,n_k - 1 \\ \tilde{B}_1^+ \bar{U}_1(x_{3,1}^b + 0) + \tilde{B}_{n_k}^- \bar{U}_{n_k-1}(x_{3,n_k}^b - 0) = \tilde{\bar{g}}_1^+ + \tilde{\bar{g}}_{n_k}^-, \end{cases} \quad (1)$$

$$\tilde{L}_k = \begin{bmatrix} 0 & E \\ L_{k,vv}^{-1}(L_{k,uu} + C_k) & L_{k,vv}^{-1}\tilde{L}_{k,uv} \end{bmatrix}; \quad \bar{S}_k = -\begin{bmatrix} 0 \\ L_{k,vv}^{-1}\tilde{\bar{F}}_k \end{bmatrix}; \quad (2)$$

$$L_{k,vv} = \begin{bmatrix} \bar{\mu}_k & 0 & 0 \\ 0 & \bar{\mu}_k & 0 \\ 0 & 0 & \bar{\lambda}_k + 2\bar{\mu}_k \end{bmatrix}; \quad L_{k,uv} = \begin{bmatrix} 0 & 0 & \partial_1^*\bar{\lambda}_k \\ 0 & 0 & \partial_2^*\bar{\lambda}_k \\ \partial_1^*\bar{\mu}_k & \partial_2^*\bar{\mu}_k & 0 \end{bmatrix}\partial_3; \quad (3)$$

$$L_{k,uu} = \sum_{j=1}^{2} \partial_j^* \bar{\mu}_k \partial_j \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \partial_1^*\bar{\mu}_k\partial_1 & \partial_2^*\bar{\mu}_k\partial_1 & 0 \\ \partial_1^*\bar{\mu}_k\partial_2 & \partial_2^*\bar{\mu}_k\partial_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \partial_1^*\bar{\lambda}_k\partial_1 & \partial_1^*\bar{\lambda}_k\partial_2 & 0 \\ \partial_2^*\bar{\lambda}_k\partial_1 & \partial_2^*\bar{\lambda}_k\partial_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4)$$

$$C_k = (\theta_k + \delta_{\Gamma,k}) \begin{bmatrix} c_{k,1} & 0 & 0 \\ 0 & c_{k,2} & 0 \\ 0 & 0 & c_{k,3} \end{bmatrix}; \quad \bar{v}_k = \partial_3 \bar{u}_k; \quad \bar{U}_k' = \partial_3 \bar{U}_k; \quad (5)$$

$$\tilde{L}_{k,uv} = L_{k,uv} - L_{k,vu}; \quad L_{k,vu} = L_{k,uv}^*; \quad \tilde{\bar{F}}_k = \theta_k \bar{F}_k + \delta_{\Gamma,k} \bar{f}_k \quad (6)$$

$$\theta_k(x) = \begin{cases} 1, & x \in \Omega_k \\ 0, & x \notin \Omega_k \end{cases}; \quad \delta_{\Gamma,k}(x) = \frac{\partial \theta_k}{\partial \bar{n}_k}; \quad \delta_{\Gamma,k}'(x) = \frac{\partial \delta_{\Gamma,k}}{\partial \bar{n}_k} \quad (7)$$

Ω is the domain, occupied by structure; $x = (x_1, x_2, x_3)$; x_1, x_2, x_3 are coordinates (x_3 corresponds to basic dimension); $x_{3,k}^b,\ k = 1,\dots,n_k$ are coordinates of boundary cross-sections of structure (in particular, coordinates of cross-sections with discontinuities of the first kind of physical and geometrical parameters of structure; l_3 is the length of structure along basic dimension, $x_3 \in [0, l_3]$; $\Omega_k,\ k = 1,\dots,n_k - 1$ are corresponding fragments of domain Ω with boundaries Γ_k, obtained by separation from domain Ω by cross-sections $x_3 = x_{3,k}^b$ and $x_3 = x_{3,k+1}^b$; $\omega_k,\ k = 1,\dots,n_k - 1$ are extended domains, embordering fragments $\Omega_k,\ k = 1,\dots,n_k - 1$; $\theta_k = \theta_k(x_1, x_2, x_3)$ is the characteristic function of domain Ω_k; $\delta_{\Gamma k} = \delta_{\Gamma k}(x_1, x_2, x_3)$ is the delta-function of border $\Gamma_k = \partial\Omega_k$; $\bar{n}_k = [\, n_{k,1} \ \ n_{k,2} \ \ n_{k,3} \,]^T$ is unit normal vector of domain boundary $\Gamma_k = \partial\Omega_k$; $\bar{u}_k,\ k = 1,\dots,n_k - 1$ is the unknown vector of displacements in domain Ω_k; $\tilde{B}_k^-, \tilde{B}_k^+,\ k = 2,\dots,n_k - 1$, $\tilde{B}_1^+$, $\tilde{B}_{n_k}^-$ are matrices (operators) of boundary conditions of the sixth order (x_3-independent); $\tilde{\bar{g}}_k^-, \tilde{\bar{g}}_k^+,\ k = 2,\dots,n_k - 1$, $\tilde{\bar{g}}_1^+$, $\tilde{\bar{g}}_{n_k}^-$ are right-side vectors of boundary conditions of the sixth order (x_3-independent); $\tilde{\bar{F}}_k$ is the right-side vector in domain Ω_k; $\bar{F}_k$ is the vector of body forces in domain Ω_k; $\bar{f}_k$ is the boundary traction vector in domain Ω_k; $\bar{\lambda}_k$, $\bar{\mu}_k$ are Lame coefficients of material in domain Ω_k; C_k is the matrix of elastic parameters of the supports (if any); $c_{k,i}$ is the coefficient of resistance in the direction of the axis Ox_i; $\partial_k = \partial / \partial x_k,\ \partial_k^* = -\partial / \partial x_k,\ k = 1,2$; $\bar{v}_k = \partial_3 \bar{u}_k = \bar{u}_k'$; $\bar{v}_k' = \partial_3 \bar{v}_k$.

III. DISCRETE-CONTINUAL FORMULATION

DCFEM presupposes finite element approximation of extended domain along directions of structure perpendicular to the basic direction, while along basic direction problem remain continual (thus extended domain is divided into discrete-continual finite elements). Resultant multipoint boundary problem for the first-order system of ordinary differential equations with piecewise-constant coefficients within DCFEM [1, 7] has the form:

$$\begin{cases} \bar{y}_k^{(1)} - A_k \bar{y}_k = \bar{f}_k, \quad x_3 \in (x_{3,k}^b, x_{3,k+1}^b), \quad k = 1, 2, \dots, n_k - 1 \\ B_k^- \bar{y}_k(x_{3,k}^b - 0) + B_k^+ \bar{y}_k(x_{3,k}^b + 0) = \bar{g}_k^- + \bar{g}_k^+, \quad k = 2, \dots, n_k - 1 \\ B_1^+ \bar{y}_k(x_{3,1}^b + 0) + B_{n_k}^- \bar{y}_k(x_{3,n_k}^b - 0) = \bar{g}_1^+ + \bar{g}_{n_k}^-, \end{cases} \quad (8)$$

where $N_1 - 1$ is the number of elements along x_1; $N_2 - 1$ is the number of elements along x_2 (mesh is topologically equivalent to rectangular (corresponding key features includes regular numeration of nodes and therefore convenient mathematical formulas, effective computational schemes and algorithms, simple data processing and so on)); $A_k,\ k = 1, 2, \dots, n_k - 1$ are matrices of constant coefficients of order $n = 6N_1N_2$ (discrete analogs of operators $\tilde{L}_k,\ k = 1, 2, \dots, n_k - 1$); $\bar{f}_k,\ k = 1, 2, \dots, n_k - 1$ are vectors of size $n = 6N_1N_2$ (discrete analogs of vector functions $\bar{S}_k,\ k = 1, 2, \dots, n_k - 1$);

$$\bar{y}_k = \bar{y}_k(x_3) = [\, \bar{u}_k^T(x_3) \quad \bar{v}_k^T(x_3) \,]^T; \quad (9)$$

$$\bar{u}_k = \bar{u}_k(x_3) = [\, (\bar{u}_n^{(k,1,1)})^T \ (\bar{u}_n^{(k,2,1)})^T \dots (\bar{u}_n^{(k,N_1,1)})^T \ \dots \ \dots (\bar{u}_n^{(k,1,2)})^T \ (\bar{u}_n^{(k,2,2)})^T \dots (\bar{u}_n^{(k,N_1,2)})^T \dots (\bar{u}_n^{(k,1,N_1)})^T \ (\bar{u}_n^{(k,2,N_1)})^T \dots (\bar{u}_n^{(k,N_1,N_2)})^T \,]^T; \quad (10)$$

$$\bar{v}_k = \bar{v}_k(x_3) = [\, (\bar{v}_n^{(k,1,1)})^T \ (\bar{v}_n^{(k,2,1)})^T \dots (\bar{v}_n^{(k,N_1,1)})^T \ \dots \ \dots (\bar{v}_n^{(k,1,2)})^T \ (\bar{v}_n^{(k,2,2)})^T \dots (\bar{v}_n^{(k,N_1,2)})^T \dots (\bar{v}_n^{(k,1,N_1)})^T \ (\bar{v}_n^{(k,2,N_1)})^T \dots (\bar{v}_n^{(k,N_1,N_2)})^T \,]^T; \quad (11)$$

$$\bar{u}_n^{(k,p,q)} = \bar{u}_n^{(k,p,q)}(x_3) = [\, u_1^{(k,p,q)} \ u_2^{(k,p,q)} \ u_3^{(k,p,q)} \,]^T, \ p = 1, 2, \dots, N_1, \ q = 1, 2, \dots, N_2 \quad (12)$$

$$\bar{v}_n^{(k,p,q)} = \bar{v}_n^{(k,p,q)}(x_3) = [\, v_1^{(k,p,q)} \ v_2^{(k,p,q)} \ v_3^{(k,p,q)} \,]^T, \ p = 1, 2, \dots, N_1, \ q = 1, 2, \dots, N_2 \quad (13)$$

$u_i^{(k,p,q)} = u_i^{(k,p,q)}(x_3),\ p = 1, 2, \dots, N_1,\ q = 1, 2, \dots, N_2,\ k = 1, 2, \dots, n_k - 1$ are components of displacement u_i in the node with coordinate (x_1^p, x_2^q, x_3) on the interval $x_3 \in (x_{3,k}^b, x_{3,k+1}^b)$.

IV. REDUCED WAVELET-BASED DISCRETE-CONTINUAL FORMULATION

Let Q be transition matrix consisting from Haar basis vectors, located in columns; Q_b be block matrix; P_{12} be permutation matrix,

$$Q_b = \begin{bmatrix} Q & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & Q \end{bmatrix}; \quad P_{12}\bar{y}(x_2) = [\,(\bar{u}_1(x_3))^T \quad (\bar{u}_2(x_3))^T \quad (\bar{u}_3(x_3))^T\,]^T ; \quad (14)$$

$$\bar{u}_i(x_3) = [\,(u_i^{(1,1)})^T \quad (u_i^{(2,1)})^T \quad \ldots \quad (u_i^{(N,1)})^T \quad \ldots \quad (u_i^{(1,2)})^T \quad (u_i^{(2,2)})^T \ \ldots (u_i^{(N,2)})^T \ldots (u_i^{(1,N)})^T \quad (u_i^{(2,N)})^T \ \ldots (u_i^{(N,N)})^T\,]^T, \quad i = 1,2,3. \quad (15)$$

After the transition from a unit basis to the basis of the Haar, we can write that

$$\bar{w}_i(x_3) = Q^T \bar{u}_i(x_3), \quad i = 1,2,3 \quad \text{ορ} \quad \bar{u}_i(x_3) = Q\bar{w}_i(x_3), \quad i = 1,2,3 \quad (16)$$

where $\bar{w}_i(x_3)$ are components of decomposition of $\bar{u}_i(x_3)$ in Haar basis. We have

$$\bar{y}(x_2) = P_{12}^T Q_b \bar{w}(x_3); \quad \bar{w}(x_3) = [\,(\bar{w}_1(x_3))^T \quad (\bar{w}_2(x_3))^T \quad (\bar{w}_3(x_3))^T\,]^T ; \quad P_{12}^T P_{12} = E, \quad (17)$$

where E is identity matrix of corresponding order. Without loss of generality let's consider the interval $x_3 \in (x_{3,k}^b, x_{3,k+1}^b)$ and introduce notation

$$\bar{w}_i^{(k)} = \bar{w}_i^{(k)}(x_3) = \bar{w}_i(x_3), \quad i = 1,2,3, \quad x_3 \in (x_{3,k}^b, x_{3,k+1}^b); \quad (18)$$

$$\bar{w}_k(x_3) = [\,(\bar{w}_1^{(k)}(x_3))^T \quad (\bar{w}_2^{(k)}(x_3))^T \quad (\bar{w}_3^{(k)}(x_3))^T\,]^T. \quad (19)$$

Let $\bar{w}_i^{(k),red}(x_3), \ i = 1,2,3$ be vectors of corresponding reduced components of vectors $\bar{u}_i^{(k)}(x_3), \ i = 1,2,3$ in Haar basis, $R_{k,i}$ be so-called reduction matrix of size $n \times n_{red,i}^{(k)}$, $n_{red,i}^{(k)}$ is the size of vector $\bar{w}_i^{(k),red}(x_3)$) on interval $x_3 \in (x_{3,k}^b, x_{3,k+1}^b)$. We have

$$R_{k,i}: \ \bar{w}_i^{(k),red} \to \bar{w}_i^{(k)}, \quad i = 1,2,3, \quad \text{ι.ε.} \quad \bar{w}_i^{(k)} = R_{k,i}\bar{w}_i^{(k),red}, \quad i = 1,2,3; \quad (20)$$

$$\bar{w}_k(x_3) = R_{b,k}\bar{w}_k^{red}(x_2); \quad R_{b,k} = \begin{bmatrix} R_{k,1} & 0 & 0 \\ 0 & R_{k,2} & 0 \\ 0 & 0 & R_{k,3} \end{bmatrix}; \quad \bar{w}_k^{red}(x_3) = \begin{bmatrix} \bar{w}_1^{(k),red}(x_3) \\ \bar{w}_2^{(k),red}(x_3) \\ \bar{w}_3^{(k),red}(x_3) \end{bmatrix}; \quad (21)$$

$$\bar{y}_k(x_3) = P_{12}^T Q_b R_{b,k}\bar{w}_k^{red}(x_3) \quad \text{ορ} \quad \bar{y}_k(x_3) = S_k \bar{w}_k^{red}(x_3), \quad S_k = P_{12}^T Q_b R_{b,k}. \quad (22)$$

Then, obviously, we can provide the appropriate expression for defining the functional as follows:

$$\Phi(\bar{y}) = \sum_{k=1}^{n_k - 1} \Phi_k(\bar{y}_k);$$

$$\Phi_k(\bar{y}_k) = 0.5 \cdot [(A_{k,w}\partial_3\bar{y}_k, \partial_3\bar{y}_k) + (\tilde{A}_{k,uv}\partial_3\bar{y}_k, \bar{y}_k) + (A_{k,uu}\bar{y}_k, \bar{y}_k)] - (\bar{b}_k, \bar{y}_k) \quad (23)$$

After transformation we obtain:

$$\Phi_k(\bar{w}_k^{red}) = 0.5 \cdot [(A_{k,2,s}\partial_3\bar{w}_k^{red}, \partial_3\bar{w}_k^{red}) + (\tilde{A}_{k,1,s}\partial_3\bar{w}_k^{red}, \bar{w}_k^{red}) + (A_{k,0,s}\bar{w}_k^{red}, \bar{w}_k^{red})] - (\bar{b}_{k,s}, \bar{w}_k^{red}); \quad (24)$$

$$A_{k,2,s} = S_k^T A_{k,w} S_k; \quad \tilde{A}_{k,1,s} = S_k^T \tilde{A}_{k,uv} S_k; \quad A_{k,0,s} = S_k^T A_{k,uu} S_k; \quad \bar{b}_{k,s} = S_k^T \bar{b}_k, \quad (25)$$

where $\bar{b}_s$ is vector of size $n_{red}^{(k)} = n_{red,1}^{(k)} + n_{red,2}^{(k)} + n_{red,3}^{(k)}$; $A_{k,2,s}$, $\tilde{A}_{k,1,s}$, $A_{k,0,s}$ are matrices of size $n_{red}^{(k)} \times n_{red}^{(k)}$. Thus, after reduction and transformation we get

$$\bar{V}_k' = A_k \bar{V}_k + \tilde{\bar{F}}_k; \quad (26)$$

$$\tilde{B}_k^- \bar{V}_{k-1}(x_{3,k}^b - 0) + \tilde{B}_k^+ \bar{V}_k(x_{3,k}^b + 0) = \tilde{\bar{g}}_k^- + \tilde{\bar{g}}_k^+, \quad k = 2, \ldots, n_k - 1 \quad (27)$$

$$\tilde{B}_1^+ \bar{V}_1(x_{2,1}^b + 0) + \tilde{B}_{n_k}^- \bar{V}_{n_k - 1}(x_{2,n_k}^b - 0) = \tilde{\bar{g}}_1^+ + \tilde{\bar{g}}_{n_k}^-, \quad (28)$$

where $\tilde{B}_k^-, \ k = 2, \ldots, n_k$ and $\tilde{B}_k^+, \ k = 1, \ldots, n_k - 1$ are matrices of size $6n \times 2n_{red}^{(k)}$ and $6n \times 2n_{red}^{(k)}$; $\tilde{\bar{g}}_k^-, \ k = 2, \ldots, n_k$ and $\tilde{\bar{g}}_k^+, \ k = 1, \ldots, n_k - 1$ are vectors of size $6n$,

$$\tilde{B}_k^- = B_k^- S_{b,k}, \quad \tilde{B}_k^+ = B_k^+ S_{b,k}, \quad k = 2, \ldots, n_k - 1;$$

$$\tilde{\bar{g}}_k^- = S_{b,k}^T \bar{g}_k^-, \quad \tilde{\bar{g}}_k^+ = S_{b,k}^T \bar{g}_k^+, \quad k = 2, \ldots, n_k - 1; \quad (29)$$

$$\tilde{B}_1^+ = S_{b,k}^T B_1^+ S_{b,k}; \quad \tilde{B}_{n_k}^- = S_{b,k}^T B_{n_k}^- S_{b,k}; \quad \tilde{\bar{g}}_1^+ = S_{b,k}^T \bar{g}_1^+; \quad \tilde{\bar{g}}_{n_k}^- = S_{b,k}^T \bar{g}_{n_k}^-; \quad (30)$$

$$A_k = \begin{bmatrix} 0 & E_k \\ A_{k,2,s}^{-1}\tilde{A}_{k,1,s} & A_{k,2,s}^{-1}A_{k,0,s} \end{bmatrix}; \quad \tilde{\bar{F}}_k = -\begin{bmatrix} 0 \\ A_{k,2,s}^{-1}\bar{b}_{k,s} \end{bmatrix}; \quad (31)$$

$$\bar{V}_k = \begin{bmatrix} \bar{z}_k \\ \bar{t}_k \end{bmatrix}; \quad \bar{V}_k' = \partial_3 \bar{V}_k = \begin{bmatrix} \partial_3 \bar{z}_k \\ \partial_3 \bar{t}_k \end{bmatrix} = \begin{bmatrix} \bar{z}_k' \\ \bar{t}_k' \end{bmatrix}; \quad (32)$$

$$\bar{z}_k = \bar{w}_k^{red}; \quad \bar{t}_k = \partial_3 \bar{z}_k = \bar{z}_k'; \quad \bar{t}_k' = \partial_3 \bar{t}_k; \quad (33)$$

E_k is identity matrix of size $n_{red}^{(k)} \times n_{red}^{(k)}$; $\bar{V}_k$ and $\tilde{\bar{F}}_k$ are vectors of size $2n_{red}^{(k)}$; $\bar{z}$, $\bar{t}$ are vectors of size $n_{red}^{(k)}$; A_k is matrix of size $2n_{red}^{(k)} \times 2n_{red}^{(k)}$.It can be shown that

$$\bar{U}_k(x_3) = S_{b,k}\bar{V}_k(x_3) = \begin{bmatrix} \bar{y}_k(x_3) \\ \partial_3 \bar{y}_k(x_3) \end{bmatrix}; \quad S_{b,k} = \begin{bmatrix} S_k & 0 \\ 0 & S_k \end{bmatrix};$$
$$S_{b,k}^T S_{b,k} = T_{b,k}^T T_{b,k}. \qquad (34)$$

Thus, considering problem (8) is transformed to a multilevel space by multilevel wavelet transform. Due to the high efficiency of the localization process and the simplicity of the computational algorithms and computer realization, discrete Haar wavelet basis has been used and corresponding direct and inverse algorithms of transformations has been performed [7]. We believe that this is one of the simplest and best-suited approach for local structural analysis. Due to special algorithms of averaging within multi grid approach, reduction of the problems is provided. This advanced wavelet-based DCFEM allows reducing the size of the problems and obtaining accurate results in selected domains simultaneously. This is rather efficient method for evaluation of local phenomenon (such as, for instance, stress concentration or concentrated force or even stress in special member) in buildings and structures. Furthermore, the proposed method allow qualitative and quantitative assessments of the degree of localization of various kinds of design factors and evaluation of the effect of each degree of freedom on behavior of the structure. Thus it often turns out to be possible to construct not only a high-quality design model, but also to make some reasonable design changes.

V. RESULTANT MULTIPOINT BOUNDARY PROBLEM

Solution of problem (26)-(28) is accentuated by numerous factors. They include boundary effects (stiff systems) and considerable number of differential equations (several thousands). Moreover, matrices of coefficients of a system normally have eigenvalues of opposite signs and corresponding Jordan matrices are not diagonal. Special method of solution of multipoint boundary problems for systems of ordinary differential equations with piecewise constant coefficients in structural analysis has been developed. Not only does it overcome all difficulties mentioned above but its major peculiarities also include universality, computer-oriented algorithm, computational stability, optimal conditionality of resultant systems and partial Jordan decomposition of matrix of coefficient, eliminating necessity of calculation of root (principal) vectors [4].

VI. VERIFICATION SAMPLES

For verification and illustrating the efficiency of the proposed method in multilevel localization and reduction of problem size, a lot of numerical samples have been considered [7]. The obtained results show an efficiency of the proposing method for localization and reducing the size of the problem. After comparison between conventional FEM (ANSYS Mechanical simulation software has been used for solution of problem in terms of FEM) [6, 7]and advanced wavelet-based DCFEM (for local structural analysis), it become clear that the localization of the problem, provide high-precision results for selected domains even in high level of reduction in wavelet coefficients. This localization can be imposed to any desired domain in the structure and, by choosing an optimum reduction matrix, high accuracy solution of the problem with an acceptable reduced size can be obtained. However, results of such local analysis may be unacceptable in the other (unselected) domains. Analysis of the behavior of the fundamental functions of boundary problems was under consideration as well in order to ensure the correct choice of reducing parameters. Generally it was confirmed that advanced wavelet-based DCFEM is more effective in the most critical, vital, potentially dangerous domains of structure in terms of fracture (areas of the so-called edge effects), where some components of solution are rapidly changing functions and their rate of change in many cases can't be adequately taken into account by the conventional FEM [6, 7].

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