Numerical investigation of thermomagnetic convection of electrically conducting fluids under an inclined magnetic field using lattice Boltzmann method

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Abstract: In the present work, natural convection in a square cavity filled with an electrically conducting fluid has been numerically investigated in the presence of an inclined magnetic field using lattice Boltzmann method. The vertical walls of this two-dimensional cavity are heated differentially while the horizontal walls are assumed to be adiabatic. The flow and temperature field distributions are obtained and the average Nusselt numbers on the left hot wall is calculated. Available results reveal that both the Hartmann number and the inclination angles of magnetic field have significant influence on the heat transfer.

Introduction

Convective heat transfer in the presence of magnetic field comes to be a research focus nowadays because of its wide reach in engineering practices, and applying magnetic field to strengthen or control heat transfer properties in the cavity gains its popularity among a large quantity of researchers.Investigators have already proved that the Rayleigh number has a positive effect on the heat transfer while the Hartmann number has a negative one[1-4]. Published literature [1-10] are based on the traditional Computational Fluid Dynamics methods such as the finite volume method, the finite difference method and so on, while the lattice Boltzmann method has already become an available and brand new numerical method after the theoretical breakthrough and research progress for more than 20 years. LBM is a programming easily numerical method with simple algorithms which also has capabilities of simulation for microflows, nanopaticles, crystal growth, porous media and many other complex convective heat transfer problems which traditional methods cannot achieve. So it is of great necessity to investigate the problem in an inclined magnetic field utilizing lattice Boltzmann method. In this study, standard two-dimensional model with nine velocities and temperature-density double distribution equations of LBM are employed to numerically investigate the thermomagnetic convection of an electrically conducting fluid in square cavity under an inclined magnetic field.

Problem statement

The geometry of present problem are clearly described in Fig.1, which consists of a two-dimensional square cavity with a height of L, and the cavity is placed horizontally. Vertical walls of this cavity are heated differentially, the temperature of left wall is maintained at Th while the temperature of right wall is fixed at Tc, and Th is hotter than Tc. The horizontal walls are assumed to be insulated. The inclination angle of inclined uniform magnetic field is θ. Cartesian coordinates are utilized and the origin of coordinates is fixed at the left bottom corner. The enclosure is filled with an electrically conducting fluids with a Prandtl number of 0.025.

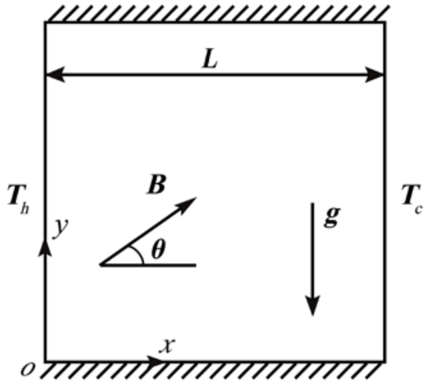

Fig.1 Physical model and Cartesian coordinates.

Governing equations

The electrically conducting fluid is assumed to be Newtonian and incompressible. The flow in this enclosure is steady and there is also no phase change, and all the effects of Joule heating, induced magnetic field and viscous dissipation on natural convection are neglected. Accordingly, the dimensionless governing equations can be described as[8]:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}-Ha^2U\sin^2\theta+Ha^2V\sin\theta\cos\theta) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+RaPr\Theta+Pr(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}-Ha^2V\cos^2\theta+Ha^2U\sin\theta\cos\theta) \tag{3}$$

$$U\frac{\partial \Theta}{\partial X}+V\frac{\partial \Theta}{\partial Y}=\frac{\partial^2 \Theta}{\partial X^2}+\frac{\partial^2 \Theta}{\partial Y^2} \tag{4}$$

Rayleigh number, Prandtl number and Hartmann number are defined as:

$$Ra=\frac{g\beta(T_h-T_c)L^3}{\nu\alpha},\quad Pr=\frac{\nu}{\alpha},\quad Ha=\sqrt{\frac{\sigma B^2L^2}{\nu\rho}} \tag{5}$$

Where the X and Y are the dimensionless coordinates. U and V are the dimensionless velocities. Θ is the dimensionless temperature. P is the dimensionless pressure. T0, Th and Tc are the reference temperature and steady temperature on the vertical walls. L is the length of square cavity. B is the magnitude of uniform magnetic field. μ is the dynamic viscosity. ν is the kinematic viscosity. θ is the angle of inclined magnetic field with respect to the horizontal plane. g is the gravity acceleration. β is the thermal expansion coefficient. σ is the conductivity of fluid. ρ is the fluid density. α is the thermal diffusivity. T0 is the reference temperature.

Lattice Boltzmann equations

In present research, standard D2Q9 model of LBM is employed for flow and temperature in this two-dimensional cavity. The discrete velocity set for the D2Q9 model is:

$$e_k=\begin{cases}(0,0), & k=0\\ c(\cos[(k-1)\frac{\pi}{2}],\sin[(k-1)\frac{\pi}{2}]), & k=1,2,3,4\\ \sqrt{2}c(\cos[(2k-1)\frac{\pi}{4}],\sin[(2k-1)\frac{\pi}{4}]), & k=5,6,7,8\end{cases} \tag{6}$$

Where e_k are the discrete velocities of lattice. The weighted factors of lattice ω_k are assigned as: ω_0=0, ω_{1-4} =1/9, ω_{5-8} = 1/36. The lattice Boltzmann equations of the flow and temperature field with external forces can be written as follows:

$$f_k(x+\Delta x,t+\Delta t)=f_k(x,t)[1-\omega_m]+\omega_m f_k^{eq}(x,t)+F_k \tag{7}$$

$$g_k(x+\Delta x,t+\Delta t)=g_k(x,t)[1-\omega_s]+\omega_s g_k^{eq}(x,t) \tag{8}$$

Here *f* is the density distribution functions, and *g* is the internal energy distribution functions. The external force term F_k, factors ω_m and ω_s are defined as:

$$F_k = 3\cdot\omega_k\cdot(\frac{Ha^2\mu}{L^2}(v\cos\theta\sin\theta - u\sin^2\theta) + g\beta\rho(T-T_0) + \frac{Ha^2\mu}{L^2}(u\cos\theta\sin\theta - v\cos^2\theta))$$

$$\omega_m = \frac{1}{3\cdot\nu+0.5},\quad \omega_s = \frac{1}{3\cdot\alpha+0.5} \tag{9}$$

And the equilibrium density distribution functions f^{eq} and the equilibrium internal energy distribution functions g^{eq} can be defined as:

$$f_k^{eq} = \omega_k\rho[1+3\frac{(e_k\cdot u)}{c^2}+4.5\frac{(e_k\cdot u)^2}{c^4}-1.5\frac{u^2}{c^2}] \tag{10}$$

$$g_k^{eq} = \omega_k T[1+3\frac{(e_k\cdot u)}{c^2}] \tag{11}$$

At last, the macroscopic variables can be calculated by the following formula:

$$\rho(x,t) = \sum_k f_k(x,t),\quad \rho u(x,t) = \sum_k f_k(x,t)e_k + F_k,\quad T = \sum_k g_k(x,t) \tag{12}$$

Description of numerical results

The average Nusselt number on hot wall is used in this work to describe the heat transfer properties:

$$Nu_{\mathrm{m}} = \int_0^1 -\frac{\partial\Theta}{\partial X}\bigg|_{X=0} dY \tag{13}$$

Results and discussion

Fig.2 depicts the effect of Hartmann number on the streamlines at Ra=104 and 105. It describes the suppression on the convective heat transfer by the applied magnetic field. The stream function on the central streamline decreases significantly with the increasing Hartmann number which indicates that both the intensity of flow and the heat transfer rate are weakening. The central region of streamlines diminishes gradually with the growing Hartmann number and breaks into two sections of extreme value on the upper and lower end of vertical center line eventually. Contours that nearly parallel to the vertical walls are spotted when the magnetic field is strong enough which declares the dominant role of heat conduction in the heat transfer .

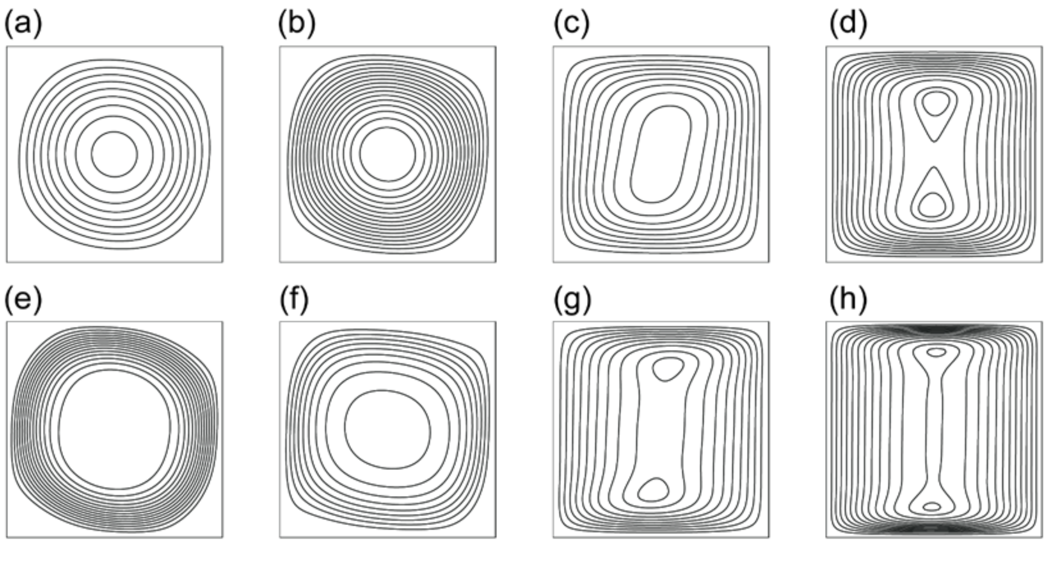

Fig.2 Streamlines under horizontal magnetic field: (a)Ra=10^4, Ha=0, (b)Ra=10^4, Ha=10, (c)Ra=10^4, Ha=50, (d)Ra=10^4, Ha=100, (e)Ra=10^5, Ha=0, (f)Ra=10^5, Ha=30, (g)Ra=10^5, Ha=160, (h)Ra=10^5, Ha=320.

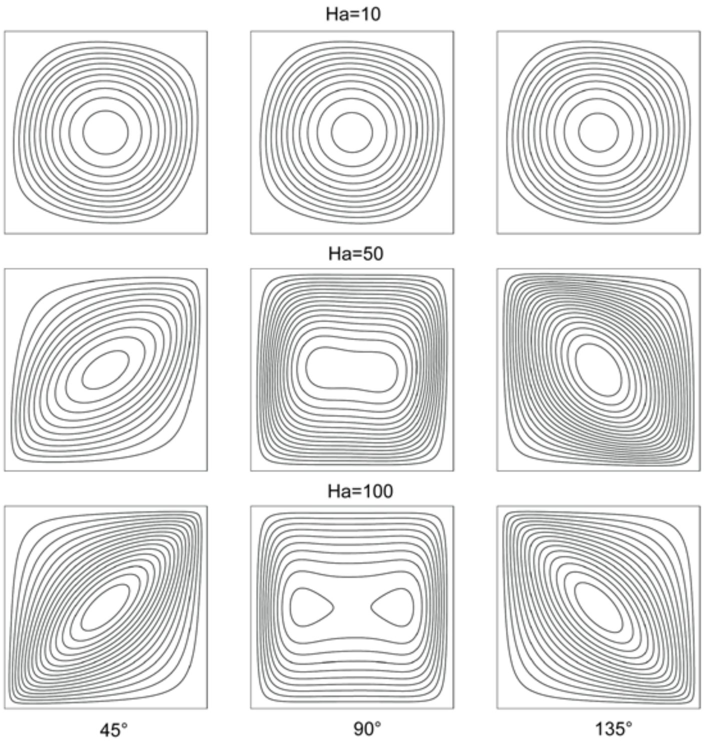

Fig.3 Streamlines of the fluid with a Prandtl number of 0.025 under inclined magnetic field at $Ra=10^4$

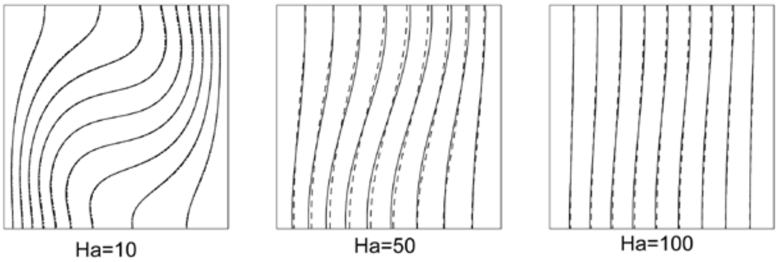

Fig.4 Isotherms in the inclined magnetic field: 45°(solid), 90°(dashed), 135°(dotted).

Streamlines in the enclosure under inclined magnetic field are shown in Fig.3. The square cavity is placed horizontally with inclination angles varying from 45°, 90° to 135° at Ra=104, and the set value of Ha are 10, 50 and 100. It is found that the increase of Hartmann number significantly augments the effect of inclination angles on the streamlines and symmetrical contours are obtained when tilting the magnetic field about 45° and 135°. This vertical symmetry is caused by the combined influence of gravity, buoyancy lift and the inclined magnetic field etc. When the uniform magnetic field is tilted about 90°, the center streamline stretches horizontally and divides into two sections when the magnetic field is powerful. Fig.4 describes the effect of inclination angles on the isotherms and it can be observed that the effect is insignificant.

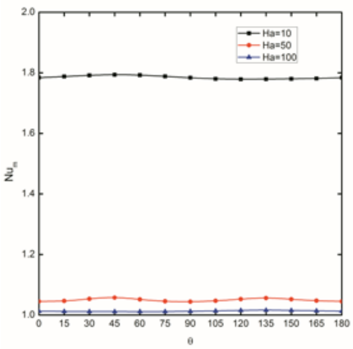

Fig.5 Effect of inclined angle on average Nusselt numbers for different Hartmann numbers.

Fig.5 depicts the variation of average Nusselt number with the changing inclination angles for various Hartmann numbers at Ra=104. Only one obvious peak value of average Nusselt number can be observed at Ha=10 while at Ha=50, there exist two peak values at 45° and 135° in the same period as a consequence of the increasing magnetic field. At Ha=100, there is no obvious change of the average Nusselt number which indicates the effect of inclination angles is insignificant when the magnetic field is strong.

Conclusions

In this paper, thermomagnetic convection in a square cavity with differentially heated vertical walls and adiabatic horizontal walls is numerically investigated utilizing lattice Boltzmann method under an inclined magnetic field. The double distribution equations and D2Q9 model of LBM are employed to analyze the heat transfer characteristics. Relevant parameters are: the Rayleigh number 104, Hartmann number from 0 to 100 with intervals of 10, inclination angles of the magnetic field varying from 0° to 180° with intervals of 15°. Comparisons of the flow, temperature distributions and heat transfer properties are illustrated clearly and conclusions are summarized as follows:

a) Numerical investigation in present research proves the validity and correctness of the lattice Boltzmann method in handling the thermomagnetic convection of electrically conducting fluids in a square cavity.

b) The average Nusselt number decreases obviously with the increasing Hartmann number. Therefore, augment of the magnetic field strength will tremendously suppresses the flow intensity and the heat transfer rate.

c) The inclination angles obviously effects the average Nusselt number at low Hartmann number. In the weak magnetic field, its effect on the flow structure and heat transfer is remarkable.

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