Elastoplastic Analysis of Casing Stress in Thermal Recovery

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Abstract. On the basis of the main thermal recovery wells casing damage characteristics, the temperature rise of the casing-cement is clearly defined by the simulation analysis of thermal recovery wells wellbore temperature field at the end of steam injection. Based on the theory of thick wall cylinder in the uniform inside and outside pressure, the paper establishes the elastic mechanical model of the combination composed of casing-cement ring and the surrounding rock, and discusses casing stress distribution in different injection-production cycle, combining with the thermal recovery routine parameters of a certain oilfield. Elastic-plastic analysis model of casing-cement under the condition of rigid formation is established while the pressure of surrounding rock is in the Elastic-plastic zone by using the Tresca yield criterion. The contact stress of casing when only the casing is into plastic state is also deduced, and discusses the degree of plastic yield of casing in different injection-production cycle in perspective of the elastic-plastic analysis.

Introduction

In recent years, with the wide application of technology of heavy oil thermal recovery, the number of casing damage increase sharply, the domestic and foreign scholars have conducted a lot of research. Garside R, etc. considering the influence of temperature on casing material properties, the curvature of wellbore and casing pre-stressed in steam injection stage, established the mathematical model of thermal stress of casing in thermal recovery [1]; On the combination of numerical simulation and experimental study, a numerical simulation model of thermal recovery wells casing was established by the finite element method, Joao C.R et al analyzed the effects of the thermal cycle load and casing steel grade, yield strength [2]; The domestic scholars began to adopt three axis stress calculation model and finite element analysis to study of casing damage mechanism in-depth [3-5]. Tan Chengjin, etc. research shows that the maximum stress intensity value of casing was obtained on the inner wall under three axial stress condition. Gao Deli, Wang Zhaohui, etc. on the basis of experimental related data and industry standards, used regression theory to obtain the relationship between the yield strength, elastic modulus and linear expansion coefficient with the change of temperature of three kinds of casing steel, and established the calculation model of casing under three axial stress in thermal recovery wells. In this paper, based on the above theoretical research, study on the effect of steam temperature on thermal recovery well casing strength, and the relationship between steam stimulation injection production cycle and the degree of casing damage. For the first time, the plastic yield level of casing in different injection production period was discussed in the elastic-plastic analysis.

The Finite Element Model of Heat Transfer of Thermal Recovery Wells

The difference between thermal recovery wells and oil wells is the steam injection in the wellbore. Under normal circumstances, the annulus between the tubing and casing production will inject nitrogen or other low pressure gas, which plays good insulation properties in the process of steam injection in high temperature steam. While the depth of insulated tubing is above the position of oil layer casing perforating interval, as a result, the casing under the packer that near the reservoir will be fully exposed to the high temperature steam, which was said “the dangerous section” in literature [6].Make following assumptions to simplify the calculation:

1) The pressure, injection rate and dry degree of the high temperature steam always stay the same;

2) Use the packer under the wells ,for which the steam will not enter the annular space between casing and tubing;

3) Ignoring the frictional loss caused by the fluid flow;

4) Fluid Characteristics of high temperature steam which injected into the wells does not change with temperature and the depth of well.

In this paper, a certain depth of the casing-cement-formation system that from the packer of thermal recovery wells to reservoir is the object of study.

Table.1 The material properties of the unit

Material	Density (kg/m^2)	Thermal conductivity (W/m・℃)	Specific heat (J/kg・K)	Modulus of elasticity (GPa)	Poisson's ratio	Coefficient of expansion (1×10^{-6}/℃)
Casing	7850	43.27	468.92	178	0.26	14.4
Cement	1830	0.81	879.23	15~28	0.12~0.17	10.3
Formation	2720	0.65	866.67	14~20	0.18~0.22	10.3

Consideration of heat loss of high temperature steam which transfer along the shaft, assuming that the steam temperature of study was 283℃ at the highest [7]. The temperature load of casing inner wall is 283℃.

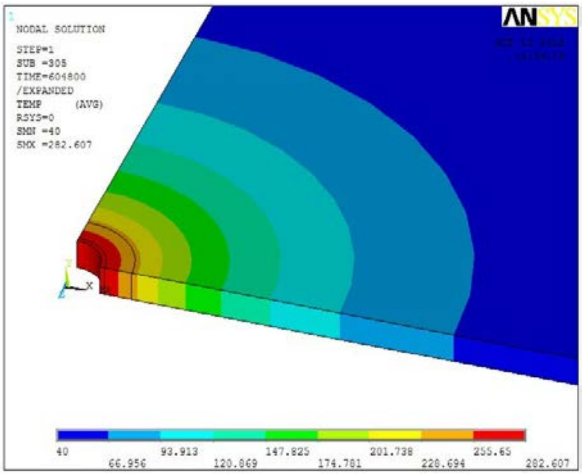

Fig.1 casing-cement-formation temperature distribution of steam injection later stage

Through the analysis on the temperature distribution, the casing temperature shortly after the start of the steam injection has been synonymous with the steam temperature, and cement ring temperature has been close to steam temperature. Calculation analysis of casing strength should be started at this period that is the later steam injection stage.

Casing elasticity analysis of the dangerous section in thermal recovery wells

Under ideal conditions, after the end of cementing casing-cement-formation will maintain close contact, casing-cement-formation system dynamics model as shown in figure 2. According to the theory of elastic mechanics, the casing-cement-formation system mechanics problem is simplified as a plane strain problem, assumptions as the following:

1） In the whole process of，casing-cement-formation is always in a consolidation state, full contact between the three;

2） Casing and cement ring is isotropic elastic material, while the formation is rigid formation;

3） Without considering the wear of casing deformation, the casing and cement are ideal cylinder, and casing, cement ring and the hole are concentric.

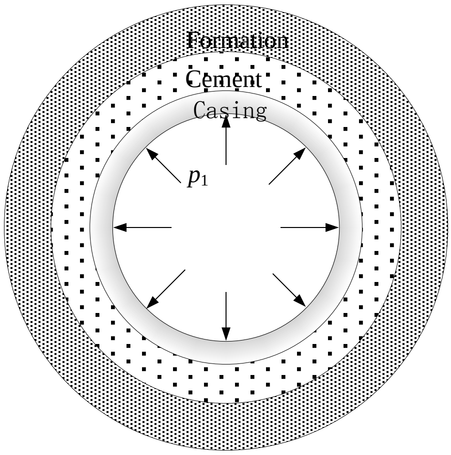

Fig.2 Casing-cement-formation system mechanics model

Three axial stress of thermal recovery casing in the late of steam injection stage is written as

$$\begin{cases} \sigma_{pfr} = \dfrac{p_1 r_{pi}^2 - p_2 r_{po}^2}{r_{po}^2 - r_{pi}^2} + \dfrac{r_{pi}^2 r_{po}^2 \left(p_2 - p_1\right)}{r_{po}^2 - r_{pi}^2} \dfrac{1}{r^2} \\ \sigma_{pf\theta} = \dfrac{p_1 r_{pi}^2 - p_2 r_{po}^2}{r_{po}^2 - r_{pi}^2} - \dfrac{r_{pi}^2 r_{po}^2 \left(p_2 - p_1\right)}{r_{po}^2 - r_{pi}^2} \dfrac{1}{r^2} \\ \sigma_{pz} = \dfrac{F_z}{\pi\left(r_{po}^2 - r_{pi}^2\right)} + 2v_p \dfrac{p_1 r_{pi}^2 - p_2 r_{po}^2}{r_{po}^2 - r_{pi}^2} \end{cases} \tag{1}$$

Where σ_{Rfr} (Pa) is the casing radial stress induced by the internal and external pressure in later steam injection; $\sigma_{Rf\theta}$(Pa) is the casing a stress induced by the inside and outside pressure in later steam injection; P_1(Pa) is the inner wall pressure of the casing caused by steam injection; P_2 (Pa) is the contact pressure of cement and casing; r_{pi}(m) is the inner radius of casing; r_{po}(m) is the outer radius of casing; υ_p is the Poisson's ratio of the casing.

The total displacement of casing is a superposition of the radial displacement which was caused by the internal and external pressure, temperature effect and the axial force. By solving the radial displacement of the three kinds of loads, the total displacement of casing can be obtained.

$$u_{pz} = \frac{1+v_p}{E_p\left(r_{po}^2 - r_{pi}^2\right)}\left[\left(1-2v_p\right)\left(p_1 r_{pi}^2 - p_2 r_{po}^2\right) r - \frac{r_{pi}^2 r_{po}^2\left(p_2 - p_1\right)}{r}\right] + \alpha_p \Delta T r - v_p \frac{\sigma_{pz}}{E_p} r \tag{2}$$

Where E_p(Pa) is the elastic modulus of the casing.

Similarly, the total displacement of cement is a superposition of the radial displacement which were caused by the internal and external pressure and temperature effect.

$$u_{cz} = \frac{1+v_c}{E_c\left(r_{co}^2 - r_{ci}^2\right)}\left[\left(1-2v_c\right)\left(p_2 r_{ci}^2 - p_3 r_{co}^2\right) r - \frac{r_{ci}^2 r_{co}^2\left(p_3 - p_2\right)}{r}\right] + \alpha_c \Delta T r \tag{3}$$

According to the continuity condition, that the displacement of casing outer wall is equal to the one of cement inner wall and the displacement of cement outer wall is zero, the stress distribution of casing can be obtained by simultaneous solution. Based on the fourth strength theory of material mechanics, the ultimate strength of casing of steam injection thermal recovery shall meet the requirements of Mises yield condition, as shown in (2-11).

$$\sigma_i = \sqrt{\frac{1}{2}\left[\left(\sigma_r - \sigma_\theta\right)^2 + \left(\sigma_\theta - \sigma_z\right)^2 + \left(\sigma_z - \sigma_r\right)^2\right]} \le \sigma_s \tag{4}$$

Where σ_s is the yield strength of the casing

Casing elastic-plastic analysis of the dangerous sect ion in thermal recovery wells

As shown in figure 5, the deformation of casing section can be divided into two parts, that a part of the inner wall of casing is plastic zone, while the outer wall is elastic area.

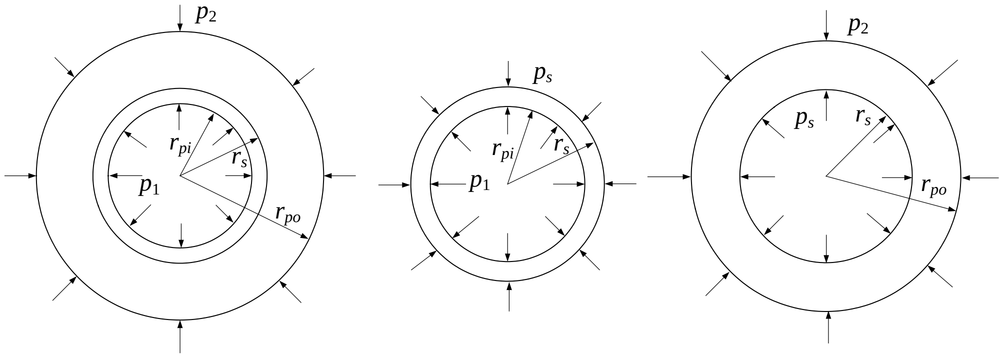

Fig.3 Casing mechanical model of elastic-plastic conditions

According to the theory of elastic-plastic mechanics, equilibrium equations and geometric equations used in elasticity will continue to apply in plastic mechanics, only in the physical relationship between the two is different. The balance equation of plastic zone is written as

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}+\frac{\sigma_r-\sigma_\theta}{r}=0 \tag{5}$$

According to the Tresca yield condition and boundary condition , the stress component of casing plastic zone is obtained

$$\begin{cases}\sigma_r=\sigma_s\ln\dfrac{r}{r_{pi}}-p_1\\ \sigma_\theta=\sigma_s\left(1+\ln\dfrac{r}{r_{pi}}\right)-p_1\end{cases} \tag{6}$$

In the elastic zone where $r = r_s$, the material has entered the plastic yield state. Stress components must satisfy the yield condition, there are

$$\sigma_r-\sigma_\theta=-\frac{2r_s^2r_{po}^2\left(p_2-p_s\right)}{r_{po}^2-r_s^2}\frac{1}{r_s^2}=\sigma_s \tag{7}$$

Simultaneous casing-cement-formation displacement continuity conditions and stress continuity conditions, bend radius can be obtained

$$\begin{aligned}&\frac{2\left(1-v_p^2\right)r_s^2r_{po}^2}{E_p\left(r_{po}^2-r_s^2\right)}\left(p_1-\sigma_s\ln\frac{r_s}{r_{pi}}\right)-\frac{\left(1-v_p\right)\left[\left(1-2v_p\right)r_{po}^3+r_s^2r_{po}\right]}{E_p\left(r_{po}^2-r_s^2\right)}\left(p_1-\sigma_s\ln\frac{r_s}{r_{pi}}-\sigma_s\frac{r_{po}^2-r_s^2}{2r_{po}^2}\right)\\&=k_4\left(p_1-\sigma_s\ln\frac{r_s}{r_{pi}}-\sigma_s\frac{r_{po}^2-r_s^2}{2r_{po}^2}\right)+k_3k_5\left[k_6\left(p_1-\sigma_s\ln\frac{r_s}{r_{pi}}-\sigma_s\frac{r_{po}^2-r_s^2}{2r_{po}^2}\right)+\alpha_c\Delta Tr_{co}\right]+\alpha_c\Delta Tr_{ci}\\&\qquad-\alpha_p\Delta Tr_{po}\end{aligned} \tag{8}$$

Where, $k_3=\dfrac{E_c\left(r_{co}^2-r_{ci}^2\right)}{\left(1+v_c\right)\left[r_{ci}^2r_{co}+\left(1-2v_c\right)r_{co}^3\right]}$

$$k_4=\frac{\left(1+v_c\right)\left[r_{co}^2r_{ci}+\left(1-2v_c\right)r_{ci}^3\right]}{E_c\left(r_{co}^2-r_{ci}^2\right)}$$

$$k_5=\frac{2\left(v_c^2-1\right)r_{co}^2r_{ci}}{E_c\left(r_{co}^2-r_{ci}^2\right)}$$

$$k_6 = \frac{2\left(1-v_c^2\right) r_{ci}^2 r_{co}}{E_c\left(r_{co}^2 - r_{ci}^2\right)}$$

Eq. 6 has given the casing function expressions of border radius r_s, as long as given the value of r_s , the casing stress distribution of plastic zone and elastic zone can be obtained .

The Results of Calculation and Analysis

1000m deep formation in a certain Oilfield as an example, using three different kinds of casing N-80, P-105 and P-110 that diameter is 139.7mm. Calculate stress distribution of casing where well depth is 950m, and temperature rise is 240℃.

The same calculation parameters of the three casings are: the outer radius of casing is $r_{po} = 69.85$ mm; the inside radius of casing is $r_{pi} = 62.13$mm; the thickness of casing is 7.72mm; the thickness of cement is 50mm; the modulus of elasticity of casing is $E_c = 2 \times 10^4$ MPa; Poisson's ratio is$v_c = 0.23$.

Using Visual Basic software calculation, the equivalent stress distribution of casing P-110 after the first injection cycles was shown as figure 4.

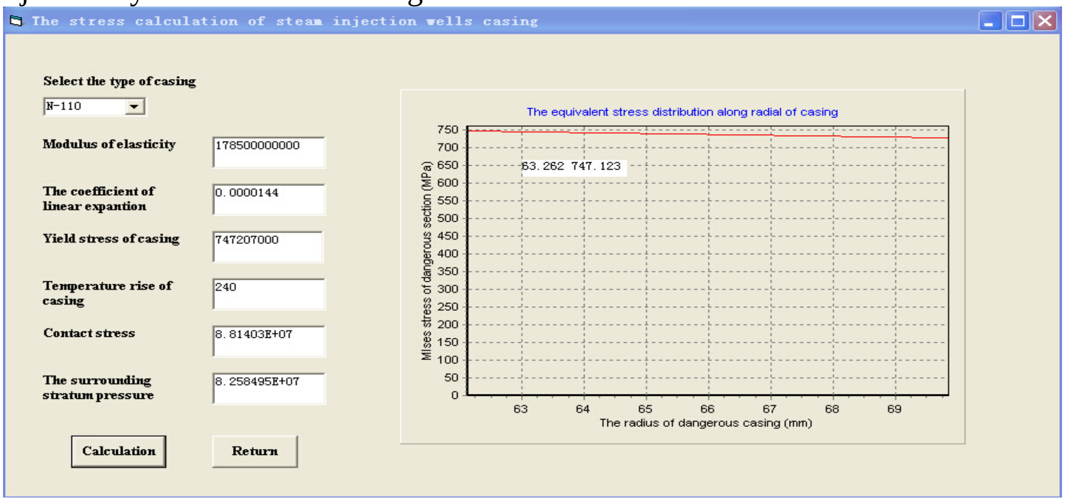

Fig 4.The equivalent stress distribution of P-110 after the first injection cycle

After the end of the first injection cycle and the beginning of second injection cycle, part of casing material along radial inner wall has been completely surrender and can't restore the elastic state. In order to simplify the calculation, when second injection cycle begins, the bottom condition of casing is exactly same with the first cycle expect of thickness of casing. Through the analysis of equivalent stress of different casings in different steam injection cycle, the results of the three kinds of casing were shown in Table 2.

Table 2 Elastic analysis of casing

Type of casing	N-80		P-105		P-110		
Steam injection cycle	1	2	1	2	1	2	3
Thickness of casing(mm)	7.72	5.713	7.72	5.97	7.72	6.588	1.757
Thickness of yield(mm)	2.007	5.713	1.75	5.97	1.132	4.831	1.757

Using Visual Basic software calculation, take an example that the elastic-plasticity analysis of casing N-110 in the first injection cycles was shown as figure 5.

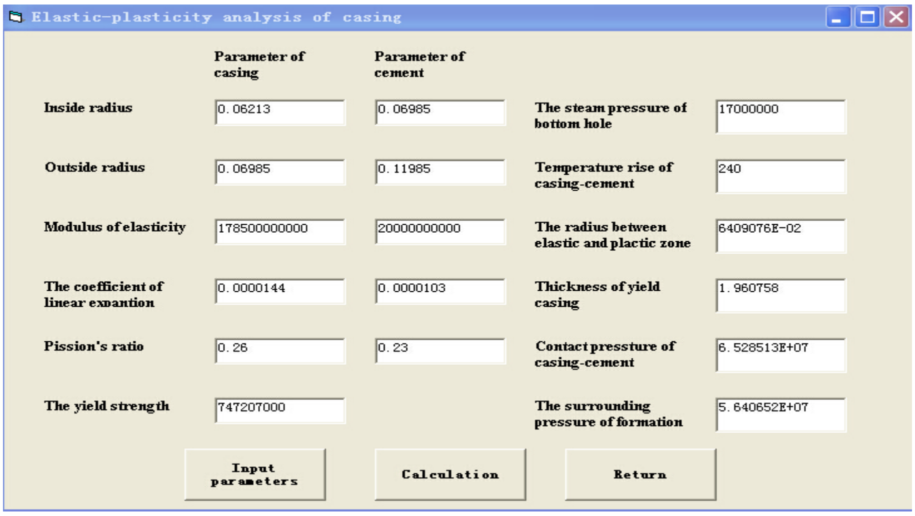

Fig 5.The elastic-plasticity analysis of P-110 after the first injection cycle

Through the analysis of different casings in different steam injection cycle, the results of the three kinds of casing were shown in Table 3.

Table 3 Elastic-plasticity analysis of casing

Type of casing	N-80	P-105		P-110	
Steam injection cycle	1	1	2	1	2
Inside radius of casing(mm)	62.13	62.13	66.48	62.13	64.09
Thickness of casing(mm)	7.72	7.72	3.37	7.72	5.76
Thickness of yield(mm)	7.71	4.35	3.36	1.96	4.82

Summary

In this paper, the temperature rise of casing-cement system was defined; Expression of contact stress between casing and cement and the surrounding pressure under elasticity and elastic-plasticity condition was derived. Combined with the thermal recovery in a certain Oilfield conventional parameters, the stress distribution of casing in different injection cycles were analyzed. The elastic analysis results show that, N-80 casing yield failure completely in the second injection cycle; P-105 casing yield failure completely in the third injection cycle, and P-110 casing yield failure completely in the third injection cycle, while the elastic-plasticity analysis results show that, N-80 casing yield failure completely in the first injection cycle; P-105 casing yield failure completely in the second injection cycle, and P-110 casing yield failure completely in the second injection cycle.

Acknowledgement

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