Improving Beam Shaping Effect with an Optimum Background Correction Method

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***Abstract*—An optimum background correction method based on the Gerchberg-Saxton (GS) algorithm is proposed to improve the beam shaping effect of a computer-generated hologram. The zero-value pixels of a black-white image which is used as the target optical field in a beam shaping process is replaced with an optimum background correction value obtained from a fitting function, which has been deduced from simulations in advance and can be generally applied for the calculation of the optimum value for the target optical field. The simulation results show that the root-mean square error of the reconstructed field obtained by the proposed method can be much lower than that obtained directly by the GS algorithm.**

Keywords-optimum background correction method; GS algorithm; beam shaping; fitting function; root-mean-square error.

I. INTRODUCTION

Laser beam shaping technique, which uses a computer-generated hologram (CGH) [1] to shape an input beam into a desired optical field, is extensively applied in the areas such as laser machining, optical tweezers, optical communications, optical imaging, optical storage, optical encryption and decryption, and so on [2-7]. With the technique researchers have realized many novel beams, such as doughnuts, curves, triangles, and so on [8-11]. Novel optical beams have been used for many applications. For example, a vortex beam in holographic optical tweezers can be used to manipulate microparticles, and a flat-top beam is utilized to improve drilling effect in laser machining and expand disk space in data storage. Many algorithms have been developed to design CGHs for beam shaping. Among them, the Gerchberg-Saxton (GS) algorithm [12] is the most extensively adopted as it is fast to achieve a result and easy to use. With the known intensity distributions of both the input and output optical fields the GS algorithm generates a phase-only hologram, which can be used to reconstruct the target field. However, the GS algorithm is easily stuck in a local minimum and as a result the diffraction efficiency of the generated CGH cannot be increased further even with more iteration. Many modified algorithms [13-16] were proposed to improve the beam shaping effect. For example, in [13] the target field was modulated with different numerical values for different regions and then used as the new target field in the GS algorithm, but the improvement varied with the modulating values and how to choose the modulating values was unknown. In [14] a dummy area was added around the target image to increase the freedom degree in the iteration, but the computation load was also increased remarkably due to the expanded data size. [15] presented an amplitude adjustment method which set the zero–value pixels of the amplitude of a target field as small but non-zero values. The method can enhance the beam shaping effect significantly, but the non-zero values are obtained manually by a trial and error manner. Obviously, an optimum value is not always found for an arbitrary target field. Thus, the improvement of the method is also limited.

Based on [15] we find that in the GS algorithm a target field whose intensity distribution can be represented with a black-white image always has an optimum background correction value that can help improve the beam shaping effect with the lowest root-mean square (RMS) error. To obtain the optimum background correction value for an arbitrary target field, we work out a fitting function based on the simulation results. With the fitting function we can find the optimum value automatically for an arbitrary target optical field. As the value is uniformly added to the zero-value pixels of the target image, the pattern of the target image will not be affected except for the slight decreasing of the contrast. It is found that the CGH obtained by the proposed method reconstructs with an RMS error 80% lower than that with the normal GS algorithm. The proposed algorithm, which is named as the optimum background correction method, costs almost the same computation time as the GS algorithm and is easy to implement.

II. THE METHOD

In our method the intensity distribution of a target optical field is represented with a black-white image, which has binary grey levels 0 and 255. The zero-value area in the image is called as the background of the target field. We use $\bar{\mathrm{I}}$ to represent the average intensity of the target field and $\bar{\mathrm{I}}$ is expressed by Eq. (1),

$$\bar{\mathrm{I}} = \sum_{m,n}^{M,N} A(m,n) \Big/ (M \cdot N), \qquad [\ A(m,n) = 0 \ \ or \ \ 255] \qquad (1)$$

where $\mathrm{A}(m,n)$ is the normalized intensity of a pixel element of the target field, and $M \times N$ is the total number of pixels of the target field. The intensity distribution of the input beam is set as a Gaussian profile. With the constraints on the amplitudes of the input and output beams we can run the GS algorithm and obtain a phase distribution for the CGH which will reconstruct the desired pattern on the output plane. We use RMS to evaluate the beam shaping effect of the algorithm. The RMS is defined by Eq. (2),

$$RMS = \sqrt{\sum_{m,n}\left[\frac{|\mathrm{A}(m,n)|}{\max(\mathrm{A})} - \frac{|\mathrm{A}'(m,n)|}{\max(\mathrm{A}')}\right]^2 \Big/ (M \times N)} \quad (2)$$

where $\mathrm{A}'(m,n)$ is the intensity of a pixel of the reconstructed field on the output plane. $\max(\mathrm{A})$ and $\max(\mathrm{A}')$ represent the maximum intensities of the target field and the reconstructed field, respectively.

As was mentioned earlier, the beam shaping effect can be improved with an optimum background correction value added on a target field in the GS algorithm. To find the empirical expression of the optimum value we will run the GS algorithm with different target fields. The procedures are described in the following.

Step 1, we choose one black-white image as the target field and the image sequence number is set as $t = 1$. The pixel number of the image is $M \times N = 256 \times 256$ and the grey values of the image are normalized to 0 or 255. The average intensity $\bar{\mathrm{I}}$ of the image is calculated by Eq. (1). Then we set values for the following parameters, the initial background correction value $\delta_0 = 0$, the search step $\Delta = 1$, the outer iteration counter $k = 1$, the inner iteration counter $L = 0$, and the new background correction value $\delta = \delta_0 + L \cdot \Delta$.

Step 2, replace the black area of $\mathrm{A}(m,n) = 0$ of the target field with δ, so the new target field is written as $\mathrm{A}_c = \mathrm{A} + \delta \cdot [1 - \mathrm{A}/\max(\mathrm{A})]$. Then we run the GS algorithm with the given intensity distributions of the input and output beams. The initial phase distribution of the input beam is randomly generated. We run the GS algorithm for 2000 iteration and obtain a reconstructed field with the corresponding RMS.

Step 3, increase L with a step of 1 and replace the background with $\delta = \delta_0 + L \cdot \Delta$ correspondingly. Then we repeat step 2 until $L = 20$, and for each δ we will get a reconstructed field and the corresponding RMS.

Step 4, find δ_{tk} that corresponds to the lowest RMS obtained in the L iteration. We can infer that the optimum δ is in the range of $\delta_0 = \delta_{tk} - \Delta$. Then we start to search from $\delta_0 = \delta_{tk} - \Delta$. And at the same time the search step is decreased as $\Delta = 0.1 \cdot \Delta$, and let $k = k + 1$.

Step 5, run steps 2-4 till $k = 3$.

Step 6, let $\bar{\mathrm{I}}_t = \bar{\mathrm{I}}$ and $\delta_t = \delta_{t3}$. Then, $\delta_{t=1}$ is the optimum background correction value for the first target field.

Step 7, input a new image as the target field and let $t = t + 1$.

Step 8, repeat steps 1-7 till $t = 50$. It is worth mentioning that there are 50 images used in our simulations. The images are chosen with the same size of 256×256 and different sizes of the bright areas with a rectangular shape ranging from 3×3, 8×8, 13×13, $\cdots$, to 253×253.

Step 9, obtain a fitting function from the curve.

We find that the optimum background correction value δ is determined by the average intensity $\bar{\mathrm{I}}$ other than other features of a target field, i.e., if target fields of different sizes and different patterns have the same average intensities, the optimum background correction values will also be the same. Hence, it is important to find the relationship between δ and $\bar{\mathrm{I}}$. The fitting function is deduced and expressed in Eq. (3). Therefore, with the fitting function we can conveniently obtain the optimum background correction value for an arbitrary target field. A flow-chart for the application of the fitting function for an arbitrary target field is shown in Figure 1.

Note that the proposed method is applicable for both the Fresnel transform (FNT) [17] and the Fourier transform (FT) in the beam propagation of the GS algorithm. However, the method is found to be suitable only for black-white images. Although the optimum background correction value of any target field can be calculated by Eq. (3), the improvement of the proposed method becomes unremarkable when the target field has a high average intensity, and at the same time the beam shaping effect will fluctuate significantly with a slight change of δ. Hence, the optimum background correction value for a target field with high average intensity should be expressed with a more precise fitting function.

$$\delta = \begin{cases} 0.22 \cdot \bar{\mathrm{I}} - 7.58 \cdot 10^{-4} \cdot \bar{\mathrm{I}}^2 + 0.45 & (0 < \bar{\mathrm{I}} < 133.17) \\ 25 - 0.0654 \cdot \bar{\mathrm{I}} & (133.17 \le \bar{\mathrm{I}} < 255) \end{cases} \quad (3)$$

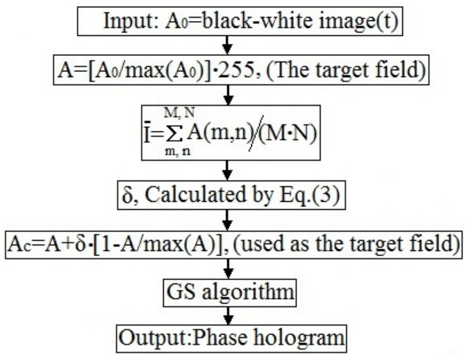

Figure 1. The flow chart of the application of the fitting function for an arbitrary target field.

III. Simulation validation

In the simulations we use target fields of different sizes and patterns to verify the beam shaping effect of the proposed method. Figure 2 shows four images to be used as target fields in the proposed method. The image sizes for Figure 2(a-d) are 512×512, 256×256, 128×128, and 64×64 pixels, respectively. Then we follow the flow chart in Figure 3 to implement the method for the given images one by one. The RMS errors for the reconstructed fields obtained by the proposed method and the GS algorithm are calculated correspondingly. Furthermore, beam propagations with both Fourier and Fresnel transforms in the proposed method and the GS algorithm are implemented for comparison.

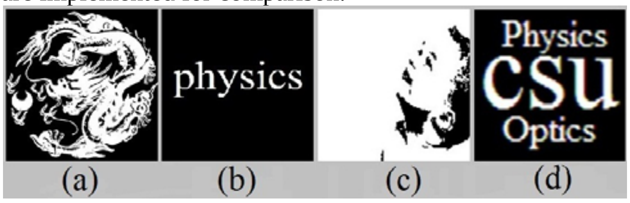

Figure 2. Black-white images used as the target fields in the beam shaping algorithms.
The image sizes are (a) 512×512, (b) 256×256, (c) 128×128, and (d) 64×64 pixels, respectively.

Table I lists the simulation results for the employed images shown in Figure 2. Tables I(a-d) present the results for the target fields in Figure 2(a-d), respectively. The average intensities for the images are calculated as 102.60, 10.33, 222.64, and 46.48, respectively. As a result, the optimum background correction values can be obtained based on Eq. (3). In Table I we use δ to represent the background correction value added in the target field, and the bold numbers are for the calculated optimum background correction values. $\delta=0$ means that the GS algorithm is employed as there is no background value added to the target field. RMS(FT) and RMS(FNT) represent the RMS errors obtained by the beam propagation in an algorithm which adopts the Fourier transform or the Fresnel transform.

It can been from Table I that the lowest RMS errors appear at the algorithms using the optimum background correction values whereas the highest RMS errors appear at the GS algorithms with $\delta=0$. The RMS errors in the algorithms with the optimum values are nearly 80% lower than those in the GS algorithms. It is worth mentioning that as we obtain the fitting function empirically, a calculated optimum value may not be exactly the one with the lowest RMS error. However, the variation caused by the deviation from the ideal optimum background correction value is found to be negligible, as less than 2%.

Table I. Simulation results for the different target fields. The bold numbers are for the calculated optimum correction values.

		(a)		
δ	0	13.04	**15.04**	17.04
RMS(FT)	0.2615	0.0596	0.0566	0.0591
RMS(FNT)	0.2982	0.0602	0.0581	0.0592
		(b)		
δ	0	1.64	**2.64**	3.64
RMS(FT)	0.0724	0.0235	0.0141	0.0149
RMS(FNT)	0.0609	0.0220	0.0137	0.0150
		(c)		
δ	0	9.04	**11.04**	13.04
RMS(FT)	0.0920	0.0281	0.0249	0.0271
RMS(FNT)	0.0945	0.0273	0.0240	0.0268
		(d)		
δ	0	7.04	**9.04**	11.04
RMS(FT)	0.1287	0.0369	0.0342	0.0426
RMS(FNT)	0.1290	0.0386	0.0353	0.0390

To further compare the beam shaping effects we present three-dimensional views of a target field, the reconstructed field by the proposed method, and the reconstructed field by the GS algorithm in Figure 3(a-c), respectively. The target field is the dragon image shown in Figure 2(a). Beam propagations in the proposed method and the GS algorithm adopt the Fresnel transform and the propagation distance is 50 cm.

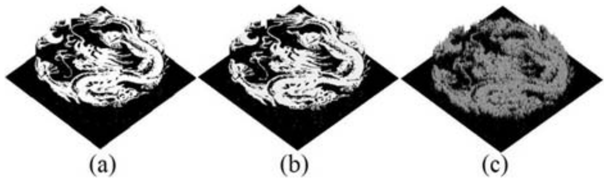

Figure 3. Three-dimensional views of the intensity distributions of (a) the target field, (b) the reconstructed field by the optimum background correction method, and (c) the reconstructed field by the GS algorithm, respectively.

From Figure 3 we can observe that the image reconstructed by the proposed method is very close to the target field whereas the image reconstructed by the GS algorithm has poorer quality.

IV. Conclusion

To summarize, we have proposed an optimum background correction method to improve the beam shaping effect of a CGH. A non-zero value can be added to the background of a black-white image which is used as the target optical field in a beam shaping process. We have worked out a fitting function to automatically find the optimum value. The simulation results have shown that the reconstructed field by the proposed method can have the

RMS error 80% lower than that reconstructed by the GS algorithm.

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