

## Reading Correction of Two-dimensional Electric Rotating-mirror Based on Optical Transmission Matrix

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**Abstract.** Aiming at correcting two-dimensional rotating-mirror's reading in optical scanning or rotation, in this paper, a reading correction method based on optical transmission matrix is proposed. A mathematical model based on the measurement principle of two-dimensional turntable in the system is applied to get pitching angle and azimuth angle of the emergent ray. By using the high-precision theodolite to proofread the direction of the adjusted emergent ray, the correctness of the model is verified. The experimental comparison and analysis indicate that this method without increasing the accuracy of processing and installing can revise the reading of two-dimensional rotating-mirror. It is proved that the precision can be 4".

### Introduction

Two-dimensional electric turntable which describes the location of the object using pitching angle and azimuth angle is an optical-mechanical-electrical integrative modern equipment. It is widely used in the fields such as tracking, measuring and simulating, which is an integral part of precision instrument[1].

In this paper, the two-dimensional rotating-mirror is an application of two-dimensional electric turntable on the optical measurement. In optical instruments, two-dimensional turntable is applied to the optical measurement and adjustment. However, due to the characteristics of the two-dimensional rotating-mirror adjustment, the emergent ray in pitch and azimuth direction is not consistent with the pitching angle and azimuth angle of two-dimensional turntable[2,3]. By analyzing this problem using optical transmission matrix, this paper presents a software correction method.

### Principle of Two-dimensional Rotating-mirror Reading

In the sight line alteration measurement system aiming at IR aiming sight[4,5], the reflecting collimator which is used to simulate an infinitely far target includes black body, target, plane mirror and parabolic mirror. IR aiming sight observes infinity target by plane mirror, as shown in Figure 1.

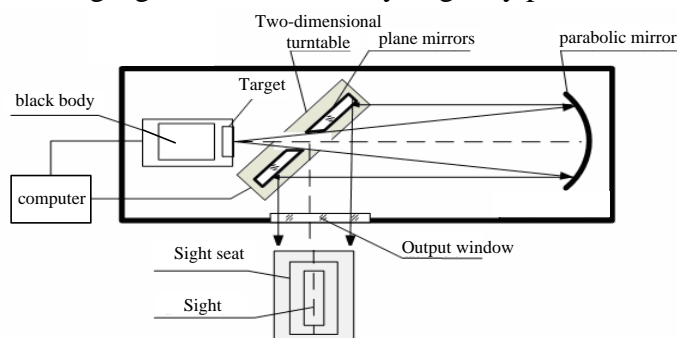


Fig.1 System of reflection type collimator

Plane mirror is installed on a two-dimension precision adjustment stage which is electric control, as shown in Figure 2. Two spindles are equipped with an absolute angular encoder respectively.

When they work, the absolute angular encoders will give turn angle of the Two-dimensional turntable. Owing to revolve of the Plane mirror in space, emergence angle and incidence angle is not 2 times but function relationship because of mechanical coupling. Therefore, the system error must be researched.

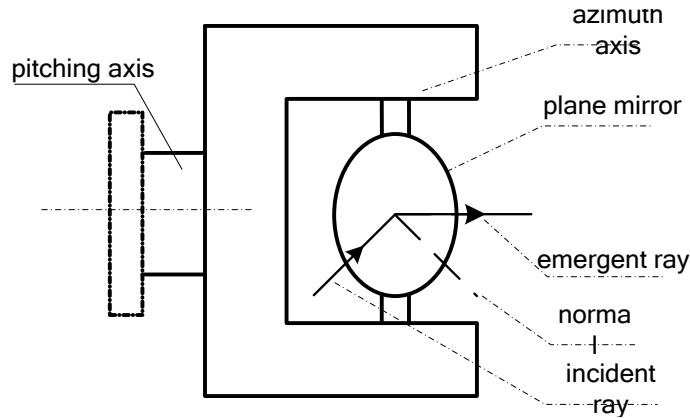


Fig.2 Structure diagram of turntable

### Calculation of Emergence Angle

For system of reflection type collimator, let the coordinate of black body be  $(x, y, z)$ , relative to the coordinate of parabolic mirror. The direction vector of incident ray reaching two-dimensional turntable is:

$$A_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (1)$$

If the original position of the plane mirror is perpendicular to the incident ray, the normal vector of the plane mirror is  $N_1 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$ . In three-dimensional space, if the optical axis coincides with z-axis,

the coordinate transformation of figure after rotating  $\theta$  degrees around the x-axis is formula (2) and the coordinate transformation of figure after rotating  $\theta$  degrees around the z-axis is formula (3).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \cos \theta - z \sin \theta \\ y \sin \theta + z \cos \theta \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \cos \theta - y \sin \theta \\ x \sin \theta + y \cos \theta \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad (3)$$

Let the clockwise direction be my positive direction, plane mirror rotates  $-\alpha$  degrees around the z-axis(pitching angle). Then, it rotates  $-\beta$  degrees around the x-axis(azimuth angle). Normal unit vector of plane mirror after two rotate is:

$$\begin{aligned}
N_2 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & -\sin \beta \\ 0 & \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} N_1 \\
&= \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \cos \beta \sin \alpha & \cos \beta \cos \alpha & -\sin \beta \\ \sin \beta \sin \alpha & \sin \beta \cos \alpha & \cos \beta \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin \alpha \\ -\cos \alpha \cos \beta \\ -\cos \alpha \sin \beta \end{pmatrix} \quad (4)
\end{aligned}$$

For the plane mirror, the relationship between incident ray and emergent ray is shown in figure 3.

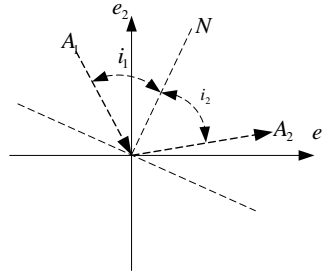


Fig.3 Refracted ray at the interface

Its expression is:

$$A_2 = HA_1$$

In the formula, H is reflective matrix:

$$H = 1 - 2NN^T$$

N is normal unit vector of plane mirror. The expression of ray  $A_2$  which ray  $A_1$  reflects by  $N_2$  is shown below.

$$A_2 = (1 - 2N_2N_2^T)A_1 \quad (5)$$

Putting the formula (1) and the formula (4) into equation (5) can obtain formula (6), as follows:

$$A_2 = (1 - 2N_2N_2^T)A_1 = \begin{pmatrix} \sin 2\alpha \cos \beta \\ 1 - 2\cos^2 \alpha \cos^2 \beta \\ -\cos \alpha \sin 2\beta \end{pmatrix} = \begin{pmatrix} A_{21} \\ A_{22} \\ A_{23} \end{pmatrix} \quad (6)$$

This emergent ray will go through two layers of germanium infrared window glass. Because the infrared window consists of two parallel flat mirrors, the ray which passes through the flat mirror would lead horizontal shifting (or longitudinal shift), without changing the direction of the ray. The final ray still is  $A_2$ . At this point, the angle of the ray is the pitching angle  $-\alpha'$  and azimuth angle  $-\beta'$  of the turntable.

$$\alpha' = \arcsin\left(\frac{A_{21}}{1}\right) = \arcsin\left(\frac{\sin 2\alpha \cos \beta}{1}\right)$$

$$\beta' = 180^\circ - \arccos\left(\frac{A_{22}}{1}\right) = 180^\circ - \arccos\left(\frac{1 - 2 \cos^2 \alpha \cos^2 \beta}{1}\right)$$

## Experiments and Data Analysis

### Data Processing Using MATLAB

Based on the derived formula, MATLAB is used to process data. The following list only when the  $0^\circ \sim 5^\circ$ ; when changes within the range of  $45^\circ \sim 50^\circ$ , corresponding to the corrected pitch angle and azimuth. The following tables only list the modified pitching angle  $\alpha'$  and modified azimuth angle  $\beta'$  when  $\alpha$  is in  $0^\circ \sim 5^\circ$  and  $\beta$  is in  $45^\circ \sim 50^\circ$ , as shown in table 1 and table 2. Images after fitting are shown in figure 4 and figure 5. Vector coordinates of the emergent ray is shown in table 3.

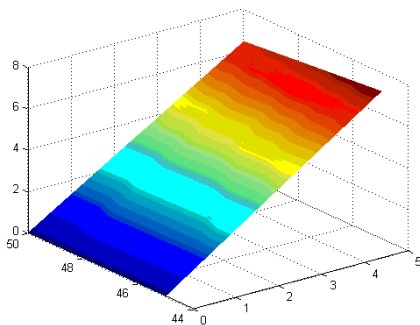


Fig.4(a)

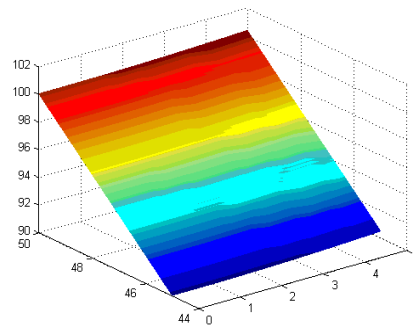


Fig.4(b)

Image of pitching angle  $\alpha'$  after fitting

Image of azimuth angle  $\beta'$  after fitting

Tab. 1 Modified pitching angle  $\alpha'$

	$45^\circ$	$46^\circ$	$47^\circ$	$48^\circ$	$49^\circ$	$50^\circ$
$0^\circ$	$0^\circ$	$0^\circ$	$0^\circ$	$0^\circ$	$0^\circ$	$0^\circ$
$1^\circ$	$1.414^\circ$	$1.389^\circ$	$1.363^\circ$	$1.338^\circ$	$1.311^\circ$	$1.28^\circ$
$2^\circ$	$2.827^\circ$	$2.777^\circ$	$2.726^\circ$	$2.675^\circ$	$2.623^\circ$	$2.570^\circ$
$3^\circ$	$4.238^\circ$	$4.164^\circ$	$4.087^\circ$	$4.011^\circ$	$3.932^\circ$	$3.853^\circ$
$4^\circ$	$5.648^\circ$	$5.548^\circ$	$5.446^\circ$	$5.343^\circ$	$5.239^\circ$	$5.132^\circ$
$5^\circ$	$7.053^\circ$	$6.928^\circ$	$6.801^\circ$	$6.672^\circ$	$6.542^\circ$	$6.409^\circ$

Tab. 2 Modified pitching angle  $\beta'$

	$45^\circ$	$46^\circ$	$47^\circ$	$48^\circ$	$49^\circ$	$50^\circ$
$0^\circ$	$90^\circ$	$92^\circ$	$94^\circ$	$96^\circ$	$98^\circ$	$100^\circ$
$1^\circ$	$90.017^\circ$	$92.017^\circ$	$94.016^\circ$	$96.016^\circ$	$98.016^\circ$	$100.015^\circ$
$2^\circ$	$90.070^\circ$	$92.067^\circ$	$94.065^\circ$	$96.063^\circ$	$98.061^\circ$	$100.059^\circ$
$3^\circ$	$90.157^\circ$	$92.152^\circ$	$94.146^\circ$	$96.141^\circ$	$98.136^\circ$	$100.132^\circ$
$4^\circ$	$90.279^\circ$	$92.269^\circ$	$94.26^\circ$	$96.251^\circ$	$98.242^\circ$	$100.234^\circ$
$5^\circ$	$90.435^\circ$	$92.420^\circ$	$94.406^\circ$	$96.392^\circ$	$98.379^\circ$	$100.365^\circ$

Tab. 3 Vector coordinates of the emergent ray(A,B,C)

	45 °	46 °	47 °	48 °	49 °	50 °
0 °	(0,0,-1)	(0,0.034,-0.999)	(0,0.070,-0.998)	(0,0.104,-0.995)	(0,0.139,-0.990)	(0,0.173,-0.985)
1 °	(0.024,0,-0.999)	(0.024,0.035,-0.999)	(0.020,0.070,-0.997)	(0.023,0.104,-0.994)	(0.022,0.139,-0.989)	(0.022,0.173,-0.984)
2 °	(0.049,0.001,-0.998)	(0.048,0.036,-0.998)	(0.048,0.071,-0.996)	(0.047,0.106,-0.993)	(0.046,0.141,-0.989)	(0.045,0.176,-0.984)
3 °	(0.074,0.003,-0.997)	(0.073,0.038,-0.997)	(0.071,0.073,-0.995)	(0.070,0.108,-0.992)	(0.069,0.143,-0.988)	(0.067,0.178,-0.982)
4 °	(0.098,0.005,-0.995)	(0.097,0.040,-0.995)	(0.094,0.075,-0.993)	(0.093,0.110,-0.990)	(0.091,0.145,-0.985)	(0.089,0.180,-0.980)
5 °	(0.123,0.008,-0.992)	(0.121,0.042,-0.992)	(0.118,0.077,-0.990)	(0.116,0.112,-0.987)	(0.114,0.147,-0.983)	(0.112,0.182,-0.977)

**Verification Experiment**

In order to verify the correctness of the above model, the verification experiment using high precision theodolite measures emergent ray, as shown in Figure 6. In this experiment, the emergent ray in pitch and azimuth direction measured by high precision theodolite is compared with the pitching angle and azimuth angle calculated by the established model,as shown in table 4.Then, the accuracy of the established functional can be tested.

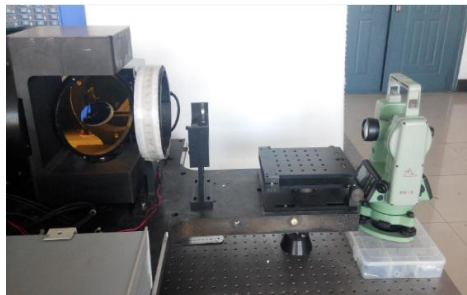


Fig. 6 Experimental setup

Tab.4 Results contrast

pitching angle of theodolite	azimuth angle of theodolite	pitching angle of established model	azimuth angle of established model
0	216"	0	218"
-216"	0	-216"	0
216"	216"	218"	216"
432"	432"	429"	433"
648"	216"	649"	218"
864"	432"	862"	430"
-216"	432"	-214"	432"
1080"	-432"	1082"	-434"
1296"	-864"	1296"	-862"
-1080"	-1080"	-1076"	-1080"

**Conclusion**

Through the above experiment, the reading of two-dimensional electric rotating-mirror is modified by optical transmission matrix. The precision of this method can be 4" in pitch and azimuth direction without changing structure of the turntable, which is in keeping with the accuracy

requirement 5" in the sight line alteration measurement system. And this method has guiding significance for future research and application.

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