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Developing Membership Functions and Fuzzy Rules from Numerical Data for Decision Making

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Abstract

Nowadays, decision making using fuzzy logic is a major research area for scientists, researchers and project managers. Construction of membership functions and fuzzy rules from numerical data is very important in various applications of the fuzzy set theory. Therefore, in this paper a model is proposed for development of membership functions and fuzzy rules from numerical data for decision making. The main advantage of the proposed model is its simplicity. The proposed model is applied on Fisher's Iris data for decision making. The validation result shows that proposed model has a higher accuracy than existing models.

Keywords**:** Membership function, Decision tree, Fuzzy rule, Histogram analysis**,** Entropy

1. Introduction

Zadeh had provided a new way of the thinking about uncertainty, vagueness and imprecision [1]. Membership functions and fuzzy rule base play a very vital role in fuzzy decision making system building. Membership function and fuzzy rule can be generated either with the help of domain expert or real data. However, most of the research articles in journals dealing with fuzzy logic either use domain expert knowledge or appear without using membership function [2]. Constructing fuzzy profile is one of the basic step in the design of a problem which is to be solved by fuzzy set theory.

The problem that makes membership function construction a vital task is the lack of consensus on the definition and interpretation of membership functions. The personal interpretations of a meaning of the concept vary from one person to another person. It is likely that different fuzzy profile may be developed to define the same concept [3]. The problem of development of fuzzy profile and the fuzzy rule base is very important because the success of a process depends on the membership functions and fuzzy rule base used. Therefore, it is needful to explain that how the membership function, and fuzzy rule base is derived.

In literature, many methods have been found of membership function generation based on heuristics, histograms, neural networks, clustering, genetic algorithm [4, 5, 6]. The heuristic method uses predefined shape of membership functions [7]. Histograms of attribute provide information about the distribution of input attribute values. Generating membership function using histogram analysis is simple and convenient [5].

Many research articles have been proposed for generation of fuzzy if- then rule from numeric data [7-17]. The drawback of majority of the models is that the membership functions still need to be predefined. Hong and Lee [11] proposed a method for membership function construction which needs predefine membership functions of input variable. Ping and Chen [16] proposed a new method for fuzzy profile generation based on α-cut of equivalence relations. Mitra et al. [17] developed a method for automatic linguistic discretization of continuous attributes using quantiles. Recently, Makrehchi et al. [13] proposed a method for generating optimal fuzzy membership function through genetic algorithm. These fuzzy profiles affect the performance and predictability of decision tree. Graphically, fuzzy profiles are represented in the form of membership functions. Actually, different types of membership functions exist in literature. Triangular and trapezoidal shapes provide a convenient representation of domain expert knowledge and it also simplifies the process of computation [18-22]. Therefore, in the proposed model, only triangular membership functions are considered in order to reduce programming and mathematical complexity.

Numerous classification models were developed in the decision making by several authors [24-26]. However, compared to the traditional decision making models, decision tree models are robust and involve only fewer amounts of computational efforts. Reasons for the incorporation of decision tree induction in the proposed model are listed below.

- Classification trees are found to be one of the most important, easy to understand and robust method of decision making.
- The method is straightforward and easy to handle huge amount of data. Therefore, it is more convenient to assimilate with human perception.
- It requires less amount of computational time and thus yields the result as faster.

Based on literature survey, in this paper, we proposed a method for the development of membership functions, decision tree and fuzzy rules from numerical data for decision making. The rest of this paper is organized as follows: The proposed methodology is explained in section 2. In section 3, a case study is explained to demonstrate the proposed method. In section 4, the conclusion is presented.

2. Proposed Method

The proposed model consists of four main steps such as,

(i) Development of membership functions.
(ii) Development of fuzzy decision tree.
(iii) Extracting fuzzy rules from the fuzzy decision tree
(iv) Decision making

Membership functions are evolved from numerical data using histogram analysis. Histogram of training data is used for the determination of membership functions. Development of fuzzy decision tree involves the determination of a root node and test nodes based on information gain values. This process results in obtaining a complete set of fuzzy classification rules. With respect to the training data set, the set of fuzzy rules that are obtained through the development of fuzzy decision tree model may be useful in the decision making. The steps involved in the proposed model are shown in Fig. 1.

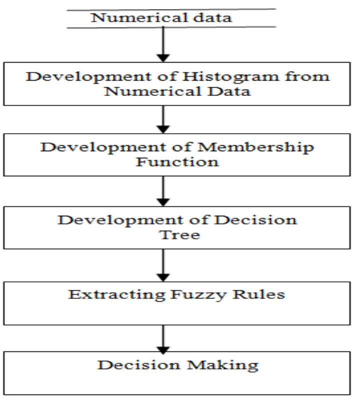

Fig. 1: Proposed model

Construction of the histogram is the first part that needs to be carried for the fuzzy profile development. The next part involves the development of a decision tree for the extraction of fuzzy rules. Decision making is done with the help of fuzzy rules.

2.1 Development of membership functions using histogram analysis

In order to construct membership functions, histograms are developed for all the training data. Further, in order to remove shallow local minima and maxima to the global maximum the histogram is smoothened using moving point average method. A fuzzy profile corresponding to each maximum point of the histogram is developed such that the center point of the triangular fuzzy membership function corresponds to the maximum point and its nearest local minimum points may be located on both sides. Fig. 2 illustrates the histogram developed using the actual raw data.

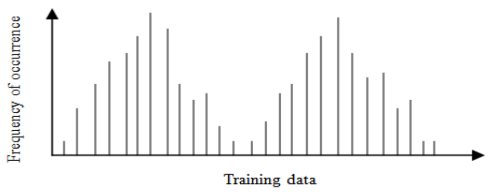

Fig. 2: Histogram of training data before smoothing

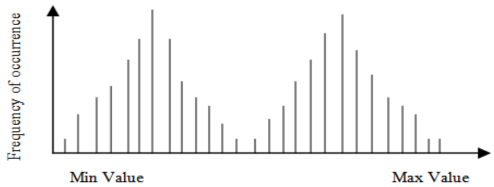

Fig. 3: Histogram of training data after smoothing

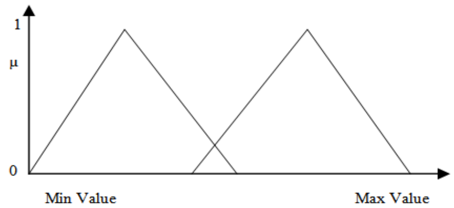

Fig. 4: Generated membership function using histogram

The result of histogram after smoothing the data using a moving point average method is illustrated in Fig. 3. It is found that piecewise linear functions are less sensitive to variations present in the training data. Even though different membership functions may be used with respect to the nature of input parameters, both triangular and trapezoidal membership functions are linear in nature. Further, triangular membership functions are more suitable when precise membership functions of a fuzzy set are not known [20].

Therefore, in most of the cases triangular membership functions are used for fuzzification of numerical data. Fig. 4 illustrates the generated membership function using histogram analysis. With the help of generated membership function, transform the quantitative values of data set into fuzzy set.

2.2 Development of a decision tree

A decision tree is a flowchart like tree structure, where, each internal node or non-leaf node denotes a test on the attribute, each branch represents an outcome of the test, and each leaf node or terminal node holds a class label. The topmost node in a tree is the root node. The first step involved in the development of fuzzy decision tree is the determination of root node. Information gain corresponding to an attribute is calculated and then the attribute corresponding to the maximum information gain is selected as root node. Further test nodes that are relevant to the rest of the input data set are also determined using information gain which is based on Shanon information theory. This iteration continues until all the classes in the dataset that is considered in the dataset become equal. The algorithm to generate a decision tree is as follows:

Step 1. Calculate information gain

Sub step 1.1 Form a knowledge representation system S= {A ∪ C}.

Assuming that there exist, a training data set (TDS) having 'n' condition attributes A= {A_1, A_2, …., A_n}, in which, each attribute has 'v' distinct values {a_1, a_2, …, a_v}. Let C={c} is set of decision label attribute and it contains only a single element. Suppose class label attribute has 'm' distinct values {c_1, c_2, …, c_m} defining 'm' distinct classes c_i where i=1, 2, …, m.

Sub step 1.2 Compute the entropy of each attribute

The expected information need to classify a given training data set is calculated by equation 1.

$$I(s_1,...,s_m) = -\sum_{i=1}^{m} p_i \log p_i \qquad (1)$$

Where,

P_i is the probability that a training sample belongs to class S_i.

S_i be the number of training samples of TDS in class c_i.

Let the attribute A_i is used to partition TDS into v subsets {S_1, S_2, …., S_v} where S_{ij} (j=1, 2, …., v) be the number of samples of class c_i in subset S_j, therefore, the entropy of attribute A_i is calculated by equation 2.

$$E(A_i) = \sum_{1}^{v} \frac{s_{1j} + ...s_{mj}}{s} I(s_{1j},...,s_{mj}) \qquad (2)$$

Therefore, the information gain of particular attribute A_i is calculated by equation 3.

$$Gain(A_i) = I(s_{1j},...,s_{mj}) - E(A_i) \qquad (3)$$

Step 2. Highest information gain attribute is selected as the attribute to split the tree.

Step 3. Repeat (1)-(2) recursively until,

(i) There are no more remaining attribute on which the data may be further partitioned.
(ii) There are no data available for the selected node.
(iii) All data points for a selected node belong to the same class

2.3 Extracting fuzzy rules from the fuzzy decision tree

Fuzzy classification rules are extracted from the decision tree by tracing a path from the root node to a decision level node. Each path starting from the root traversing down to a decision level node is converted into a rule [23]. The fuzzy rule directly extracted from the fuzzy decision tree, enhance an overall performance of the classifier and improve the efficiency. These fuzzy rules can be used for decision making in many areas for various applications.

3. Case study

In order to validate the proposed approach, we classify the Fisher's Iris data [27]. It contains three types of flower such as: Iris Setosa, Iris Versicolor, and Iris Verginica. Each flower can be recognized by four kinds of attribute such as: sepal length, sepal width, petal length, and petal width. We use 50% of the Iris data as the training data set and the other 50% as the testing data set.

3.1 Development of membership functions using histogram analysis

The histogram of all attributes is shown in Fig 5. Generated membership function using histogram analysis is shown in Fig 6, 7, 8, and 9.

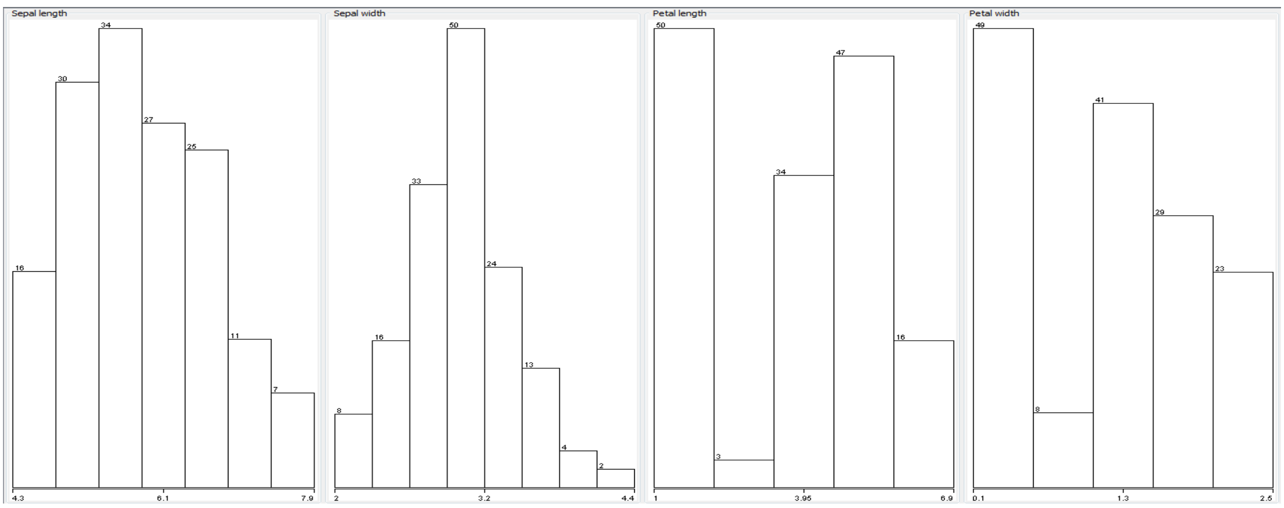

Fig. 5: Histogram of training data before smoothing (sepal length, sepal width, petal length, and petal width)

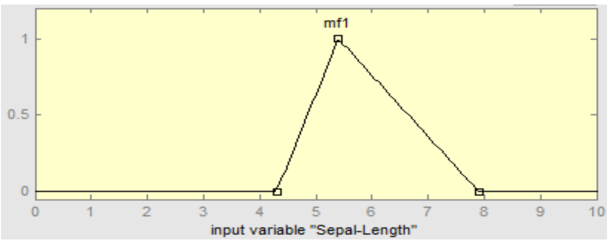

Fig. 6: Generated membership function of sepal length using histogram analysis

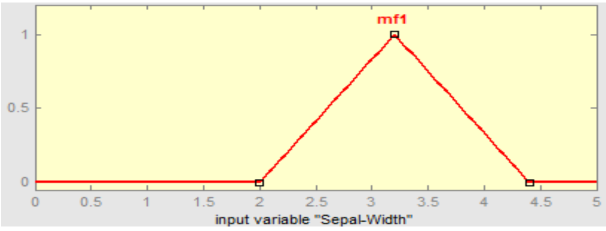

Fig. 7: Generated membership function of sepal width using histogram analysis

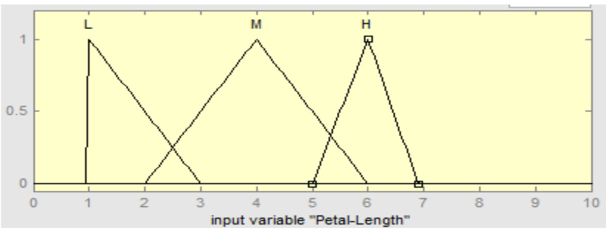

Fig. 8: Generated membership function of petal length using histogram analysis

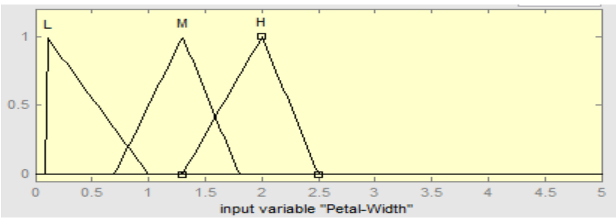

Fig. 9: Generated membership function of petal length using histogram analysis

3.2 Development of a decision tree

Fig. 10 illustrates the complete structure of decision tree obtained as a result of applying the decision tree algorithm for Fisher's Iris data [27].

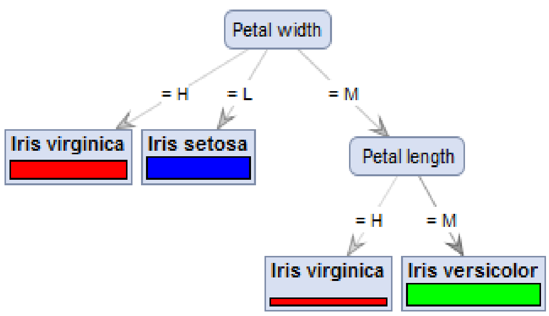

Fig. 10: Decision Tree of Fisher's Iris data

3.3 Extracting fuzzy rules from the fuzzy decision tree

The extracted fuzzy rules are as follows:

1. If Petal Width is H then the flower is Iris Virginica.
2. If Petal Width is L then the flower is Iris Setosa.
3. If Petal Width is M and Petal Length is M then the flower is Iris Versicolor.
4. If Petal Width is M and Petal Length is H then the flower is Iris Virginica.

3.4 Decision making and model validation

In order to classify the Fisher's Iris data [27] a program is written in C language considering fuzzy classification rules as conditions. The predicted result is shown in Table 1. The predicted result is compared with earlier work of Hong and Lee model [11], and Wu and Chen model [16] in terms of the number of the fuzzy rules and average accuracy. They used same Fisher's Iris data to classify. From Table 2 and Table 3 it is clear that the proposed model has higher accuracy in decision making than Hong and Lee model [11], and Wu and Chen model [16].

Setosa	Virginica	Versicolor	Average
100 %	97.83 %	98.83 %	98.88%

Table 1: Average accuracy rate of proposed model

	Hong and Lee model [11]	Wu and Chen model [16]	Proposed model
No of Fuzzy Rule	6	3	4

Table 2: Comparison of fuzzy rule of proposed model with others model

	Hong and Lee model [11]	Wu and Chen model [16]	Proposed model
Average Accuracy	95.57%	96.21 %	98.88%

Table 3: Comparison of average accuracy of proposed model with others model

Furthermore, the proposed approach does not need to predefine any fuzzy profile of the input variables and the output variables. The fuzzy profile and fuzzy rules are constructed by the proposed approach from the numerical data.

4. Conclusions

In this paper, a simple approach is proposed to develop the fuzzy profile and fuzzy rule from numerical data for decision making. The proposed method is better than Hall et al. algorithm [11], and Wu and Chen [16]. These satisfactory validating results give confidence in the membership function, and rule base development for decision making, but of course further validation using big datasets would provide even greater confidence in the suitability and applicability of the proposed model. The proposed model may be applied in decision making by project managers in various domain area where project metrics data are available.

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