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A reflection on fuzzy "complements"

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Abstract

A fuzzy set can be seen as a measure of the meaning of a predicate in a given universe. Generally, a predicate covers some elements of the universe and not others. In this paper the problem of determining the properties a fuzzy set should check to cover elements not covered by a predicate represented by a fuzzy set is reviewed. Thus the problems of determining the negation and antonym of a predicate and the possibility of finding a good measure to represent them are studied, and new concepts as the kernel of an antonymy are introduced.

Keywords: Negation, complement, antonym, Kernel of negation, Kernel of antonym

1. Introduction

1.1. When fuzzy sets were introduced in the mid sixties and first seventies of the XX Century, the only back referents for building its theory were the (naïve) theory of classical sets, classical logic and multiple valued logics. Instead, and fifty years after Zadeh's first seminal paper [1], it seems that we should look for referents to those aspects of language and common sense reasoning not well fitted by crisp sets and formal deduction. New points of view are necessary to face the future of fuzzy logic and, to foresee which kind of research should be "tomorrow" conducted, today cannot be hidden in teaching to university students [2]. For this reason this paper is presented in a Special Session devoted to Didactics, and for tentatively trying to reflect on what can be understood by "not being covered by P" in a universe X of discourse under a predicate or linguistic label P. A question that can be thought in several forms for understanding what such "coverage" can mean, and that in fact has a long history [3].

What "is P" is represented by fuzzy sets $p : X \longrightarrow [0,1]$ being "measures", and coming from the linguistic and empirically perceived binary relation "$x \leq_P y \Leftrightarrow x$ is less P as y", without which is impossible to assert that P is imprecise in X, and assuring P is not metaphysical [4]. If P is precise in X, the relation $\leq_P$ collapses in the symmetric relation $=_P \equiv \leq_P \cap \leq_P^{-1}$, that shows $x =_P y \Leftrightarrow x$ is equally P to y.

1.2. A measure in the graph $(X, \leq_P)$ is a mapping [4] $p : X \longrightarrow [0,1]$, such that: 1) $x \leq_P y \Rightarrow p(x) \leq p(y)$; 2) If z is maximal under $\leq_P$, then it is $p(z) = 1$; 3) If z is minimal, then $p(z) = 0$. Property (1) is essential, and properties (2) and (3) not only depend on the existence of maximals and minimals, but do not impede the existence of non-maximals or non-minimals whose measure is equal, respectively, to 1 or to 0. Fuzzy sets are just measures p of meaning that, only if P is precise, they belong to $\{0,1\}^X$.

For what relates to the idea of "not being captured by P", the problem is finding a measure, or fuzzy set, q: $X \longrightarrow [0,1]$ being a measure for a graph $(X, \leq_Q)$, where Q is a predicate naming a suitable and predicative interpretation of that idea. Some possible cases, corresponding to different interpretations of Q, either in fuzzy, or in crisp terms, will be taken into account.

2. "Not being covered by P" as negation

2.1. If P is a predicate, and coming from traditional logic, fuzzy logicians usually interpret "not being covered by P" as the negation $Q = notP = P'$, always existing and unique in each case, although the antonym aP, if existing, not always is unique [5]. Of course, the fuzzy representation of P, aP, and P' is not unique, but context dependent and purpose driven. The former laws defining a measure are not sufficient to specify a single measure, but more information is needed for it [4].

The first problem is to know the relation $\leq_{P'}$, once $\leq_P$ is known, and at this respect it only can be stated $\leq_P^{-1} \subseteq \leq_{P'}$, that is, "if y is less P than x" then "x is less P' than y", but without the guarantee that the reciprocal can hold. This inclusion only states the measurability of P' in stating $\leq_{P'} \neq \emptyset$, but not the existence of a measure p' obtained from p; for instance, without fully knowing $\leq_{P'}$, it is not possible to capture the relationship between the maximals and the minimals of both binary relations.

2.2. Under the "case" $\leq_P^{-1} = \leq_{P'}$, z is a maximal of P' if and only if z is a minimal of P, and it is a minimal of P if and only if it is a maximal of P'. In this hypothesis, it suffices to count with a family [6] $\{N_x : x \in X\}$, of functions $N_x : [0,1] \longrightarrow [0,1]$, such that:

a) $z \leq y \Rightarrow N_x(y) \leq N_x(z)$;

b) $N_x(0) = 1$;

c) $N_x(1) = 0$,

called negation functions, and defining $p'(x) = N_x(p(x))$, for all x in X. The function p' verifies:

- $p \le q \Leftrightarrow p(x) \le q(x) \Rightarrow N_x(q(x)) \le N_x(p(x)) \Leftrightarrow q' \le p'$
- $p_0(x) = 0$, for all x in X, implies $p_0'(x) = N_x(0) = 1$, that is, $p_0' = p_1$, with $p_1(x) = 1$ for all x in X.
- Analogously, $p_1' = p_0$,
- No problem arises to prove that p' is a measure. If z is a maximal for P', then it is a minimal for P, and $p'(z) = N_x(p(z)) = N_x(0) = 1$, analogously for a minimal, and, also: $x \le_{P'} y \Leftrightarrow y \le_P x \Rightarrow p(y) \le p(x) \Rightarrow N_x(p(x)) \le N_x(p(y)) \Leftrightarrow p'(x) \le p'(y)$.

Hence, p' is a measure for P', a fuzzy set representing P'. Notice that if p represents a crisp subset of X, that is, $p \in \{0,1\}^X$ with $p^{-1}(1) = P$, then $p'(x) = N_x(p(x)) = N_x(1) = 0$ if $x \in P$, and $p'(x) = N_x(p(x)) = N_x(0) = 1$ if $x \in P^c$; that is p' is the membership or characteristic function of the crisp subset P^c, the classical complement of P.

2.3. Notice that these fuzzy sets p' are not functionally expressible from p, and there are not known criteria to assert when p' should be functionally expressible from p, but they allow to use different negations in different parts of the universe of discourse X. Nevertheless, fuzzy logic almost always works in the hypothesis of functional expressibility reached with all the functions N_x equal to a single negation function N, that is, with $p' = N \circ p$. Anyway, the advancement of Computing with Words will make necessary to represent large statements where the negation could be only unique at its component parts.

One of the important features in using fuzzy sets is the continuity of the membership functions p when X is a part of $\mathbb{R}^n$, as it is usual in most applications, that reflects the flexibility of P. Since it is not convenient that p' can show more discontinuities than those eventually shown by p, this is the reason for which only continuous negation functions as for instance, $N(x) = 1-x$ and $N(x) = 1-x^2$, are used for counting with either a strict order-reversing negation, or with its continuity. Also and inherited from classical logic, it is usually supposed that functions N_x verify $N_x \circ N_x = N_x^2 = id$, under which these functions are continuous (and, hence, strictly decreasing), and are called "strong negations". Such negations benefit from a characterization [7] through "order- automorphims f " of the unit interval:

$$N_x(x) = f_x^{-1}(1 - f(x)), \text{ for all } x \in X,$$

with $f_x : [0,1] \longrightarrow [0,1]$, bijective and verifying: $z < y \Rightarrow f_x(z) < f_x(y), f_x(0) = 0$, and $f_x(1) = 1$, for all x in X. Order auto-morphisms f_x are not unique for each N_x, but several of them can define the same strong negation.

It should be pointed out that P not always coincide with its double negation $(P')' = P''$, and that, in general, from the inclusion $\le_P^{-1} \subseteq \le_{P'}$ it just follows the two inclusions $\le_{P'}^{-1} \subseteq \le_P$, and $\le_{P'}^{-1} \subseteq \le_{P''}$. The semantic analysis of "negation" is still incomplete in fuzzy logic.

2.4. Two fuzzy sets p and q are (logically) "contradictory", provided it is $p \le q'$, equivalent to $p(x) \le q'(x)$, for all x in X, and mimicking the conditional "if x is P, then x is not Q", for all x in X. Then, p is "self-contradictory" if $p \le p'$, or $p(x) \le N_x(p(x)) = f_x^{-1}(1 - f_x(p(x)) \Leftrightarrow p(x) \le f_x^{-1}(1/2)$, that is, p is below the curve $c(x) = f_x^{-1}(1/2)$, of the fixed points of the family $N_x : N_x(f_x^{-1}(1/2)) = f_x^{-1}(1 - f_x(f_x^{-1}(1/2)) = f_x^{-1}(1/2) \in (0,1)$, since 0 and 1 are not fixed points [8]. When the negation is functionally expressible, $p' = N_f \circ p$, that is, the family of negations is reduced to a single one, there is just the single fixed point $f^{-1}(1/2) \in (0,1)$.

With precise predicates, that is, specified by crisp subsets of X, the only self-contradictories are those specified by the empty set: $A \subseteq A^c \Rightarrow A = A \cap A^c = \emptyset$. This does not hold in the imprecise case; for instance, with $N(x) = 1 - x$, it is $p(x) \le p'(x) = 1 - p(x) \Leftrightarrow p(x) \le \frac{1}{2}$, for all x in X, that is, p is below the constant fuzzy set $p_{\frac{1}{2}}(x) = \frac{1}{2}$.

3. "Not being covered by P" as an opposite

3.1. When learning a natural language people often play with either the negation, or the antonyms of P, for well empirically capturing the meaning of P in X. It is difficult (if not impossible) to recognize, for instance, that "John is tall" without also being able to recognize that "Peter is short", or that "Peter is not tall". In some sense, and for an as good as possible linguistic management of words, natural language employs both a kind of sharp and soft ways to "separate" what is covered from what is not covered by P. Hence, it should be studied the measurability of the antonyms [5] aP of P, and try to find possible measures, or fuzzy sets, representing them. In most applications, for instance, problems can be posed thanks to linguistic variables interpreting the available information, and for which antonyms are crucial.

3.2. The opposites aP show an "opposition to P in the universe of discourse X". This implies, by one side, $\le_{aP} = \le_P^{-1}$, and by the other that it should exist a symmetry $s_P : X \longrightarrow X (s_P \circ s_P = id_X)$, such that "$x$ is P" and "$s_P(x)$ is aP" can be simultaneously stated, as well as that s_P maps each minimal and maximal of aP into those of P, and reverses the relation $\le_P$. Under these conditions, aP is measurable [4]: It is fully known $\le_{aP}$ $(\subseteq \le_{P'})$, and the mapping $p_a(x) = p(s_P(x))$, for all x in X, and all symmetry s_P, is a measure for aP, since:

1) $x \leq_{aP} y \Leftrightarrow y \leq_P x \Rightarrow s_P(x) \leq_P s_P(y) \Rightarrow p(s_P(x)) \leq p(s_P(y)) \Leftrightarrow p_a(x) \leq p_a(y)$.

2) If z is a maximal of aP, then $p_a(z) = p(s_P(z)) = 1$, since $s_P(z)$ is a maximal of P.

3) Analogously, if z is a minimal of aP, $p_a(z) = p(s_P(z)) = 0$.

Even if it is not always $P'' = P$, it is always $a(aP) = P$; for instance, $a(a(\text{tall})) = a(\text{short}) = \text{tall}$, or $a(a(\text{rich})) = a(\text{poor}) = \text{rich}$. Thus, $p_a = p \circ s_P$ does reproduce this property in, at least, the form $p_{a(aP)} = p$: Since, it is $\leq_{a(aP)} = \leq_{aP}^{-1} = (\leq_P^{-1})^{-1} = \leq_P$, and $p_{a(aP)} = p_{aP} \circ s_{aP} = (p \circ s_P) \circ s_{aP} = p \circ (s_P \circ s_{aP})$, it suffices $s_{aP} = s_P$, that the symmetry associated to aP is the same than that associated to P, to satisfy $p_{a(aP)} = p$.

Notice that a "restricted" concept of (linguistic) self-contradiction could be defined by $p \leq p_a (\leq p')$. These self-contradictory fuzzy sets (a subset of the logically self-contradictory), are characterized by the functional equation $p = p \circ s_P$, to which reduces the former inequality.

It should be noticed that P can have no antonyms, a single one, or several, and that a is just a mapping between linguistic terms and, hence, aP is a single linguistic term, but not a set of them. Provided P has more than one antonym, a_iP, for $1 \leq i \leq n$, it is always $a_i(a_iP) = P$. For instance, with $P = \text{sweet}$, $a_1P = \text{bitter}$, $a_2P = \text{sour}$, $a_3P = \text{salty}$, it is $a_1(a_1P) = a_2(a_2P) = a_3(a_3P) = \text{sweet}$, but no antonym aP is $aP = \{\text{bitter}, \text{sour}, \text{salty}\}$. What is still unknown, and deserves to be studied, is what happens in cases like $a_1(a_3P) = a_1(\text{salty})$, although $a_1(\text{salty})$ could seem to be a synonym of $P = \text{sweet}$, like it seems to be $a_2(a_1P) = a_2(\text{bitter})$, etc. This opens a curious problem that, as far as the authors know, is not considered in Linguistics, and can show the interest of using fuzzy methodologies.

3.3. There is a link between antonyms and negation [9] coming from examples like "if the bottle is empty then it is not full", where $a(\text{full}) = \text{empty}$. Such link is the rule: <If x is aP, then x is P'>, that only in rare cases reverses to <If x is P', then x is aP>, makes aP equivalent to P' and it is said that $aP = P'$ is a non-regular antonym for which it is $\leq_{aP} = \leq_P^{-1} = \leq_{P'}$. There is no independence [4] between aP and P': The two relationships given by $\leq_{aP} \subseteq \leq_{P'}$, and $p_{aP} \leq p'$ hold, and the second can be expressed by the inequality of "coherence", $p \circ s_P \leq N_P \circ p$, linking the symmetry and the negation that, consequently, cannot be independently chosen. For instance, if $X = [0, 10]$, and $P = \text{big}$ with $p(x) = \frac{x}{10}$, it should be $s(x) \leq 10N(x/10)$ that, provided it should be taken $N(x) = \frac{1-x}{1+x}$, implies $s(x) \leq \frac{10(10-x)}{10+x}$, and if it were $s(x) = 10 - x$, then $1 - x \leq N(x)$. All this reinforces the necessity of well designing in fuzzy terms any linguistic variable describing a system.

4. Kernels of negation and antonymy

4.1. The coherence between negation and antonym, $p_{aP} \leq p'$, should hold for all antonyms aP of P and thus, if existing, the function, $\sup\{p_{aP} : \text{for all } aP\} = K_P \leq p' = N_P \circ p$, can be seen as a "fuzzy kernel of negation" that can or cannot coincide with p'. For instance, if $X = [0, 1]$ and $P = \text{big}$ with $p(x) = x$, since now the symmetries s_{big} can be identified with the strong negations $N_f = f^{-1} \circ (1 - f)$, whose supremum is the discontinuous negation function $N_{\min}(x) = 1$, if $0 \leq x < 1$, and 0 if $x = 1$, It is $K_{\text{big}}(x) = \sup\{p_{aP}\} = \sup\{p(N_f(x))\} = \sup\{N_f(x)\} = N_{\min}(x)$. That is, $K_{\text{big}}(x) = 1$, for x in $[0, 1)$, and $K_{\text{big}}(1) = 0$, specifies the set $[0, 1)$, and, hence, the only element in the unit interval that always can be qualified as big is 1. Of course, the idea of a fuzzy kernel of negation is a new one still deserving a deep study.

4.2. A "crisp kernel of antonymy" of P can be reached, once an antonym aP is known, through the set $A_P = \{x \in X : p(x) \leq p_a(x)\}$, containing those elements of X showing "P less than aP" [10], those that are more covered by aP than by P. For instance, provided X is an interval $[a, b]$ of the real line, it will exist the number $\sup A_P$ separating what is P from what is aP. For instance, with $P = \text{big}$ $(aP = \text{small}) \in X = [0, 10]$ with $p(x) = \frac{x}{10}$, and the symmetry $s(x) = 10 - x$, it is $\frac{x}{10} \leq \frac{(10-x)}{10} \Leftrightarrow x \leq 5$, and $\sup A_P = 5$: the crisp kernel of antonymy is $[0, 5]$ and contains those elements that are less big than small, or more small than big. Nevertheless, with the symmetry $s(x) = 10\frac{10-x}{10+x}$, from $\frac{x}{10} \leq \frac{10-x}{10+x} \Leftrightarrow x^2 + 20x - 100 \leq 0$, follows $\sup A_P = 10\sqrt{2} - 10 \approx 4.1421$, and now the crisp kernel of antonymy is $[0, 10\sqrt{2} - 10]$, showing that the numbers greater than 4.1421 are less aP than P. The true elements that can actually be seen as big are those in the semi-open interval $(10\sqrt{2} - 1, 1]$.

Provided P is precise, it is $A_P = \{x \in X : 1 = p_a(x)\}$, that, if p_a is not coincidental with p' (P is regular), cannot be coincidental with the crisp complement of the crisp set specified by P in X. For instance, if $p_a = p' = 1 - p$, it follows $A_P = \{x \in X : p(x) = 0\}$, the crisp complement of the set $\{x \in X : p(x) = 1\}$ specified by P, but, in general, with $p_a = p \circ s \neq p'$, the result will depend on s. For instance, provided $X = [0, 1]$, $p(x) = 1$, if $0 \leq x \leq 0.5$, and $p(x) = 0$ otherwise, with $s(x) = \frac{1-x}{1+x}$, it is $1 = p(\frac{1-x}{1+x}) \Leftrightarrow 0 \leq 1 - \frac{x}{1+x} \leq 0.5 \Leftrightarrow \frac{1}{3} \leq x$, and $A_P = [\frac{1}{3}, 1]$, showing that in this case there is no coincidence between A_P and the complement $(0.5, 1]$ of the crisp set specified by P in $[0, 1]$.

4.3. A way of having a "crisp kernel of negation" comes, also, from the inequality, $p \leq p'$: It is the crisp set $N_P = \{x \in X : p \leq p'\} = \{x \in X : p(x) \leq f^{-1}(1/2)\}$, with the fixed point of the corresponding negation, and containing those points in X showing P less than they show not P, those that are more covered by P' than by P. If X is an interval $[a, b]$ of the real line, the number $\sup N_P$ in $[a, b]$, is a "separation" point between P and not P and allows to see the interval $[a, \sup N_P]$ as the crisp kernel of negation. For instance, if $X = [0, 10], P =$ big with $p(x) = x/10$, and the negation $N(x) = \frac{1-x}{1+x}$ with fixed point $\sqrt{2} - 1$, it is $N_P = \{x \in [0, 10] : \frac{x}{10} \leq \sqrt{2} - 1\}$, with which it follows $\sup N_P = 10(\sqrt{2} - 1) \approx 4.1421$, and the crisp kernel of negation is the interval $[0, 4.1421]$. The numbers that can be considered big are those greater than $\sqrt{2} - 1$.

Notice that the crisp kernel of negation of a precise predicate P, always coincides with its crisp complement since being $p(x) \in \{0, 1\}$, and $p'(x) = 1 - p(x)$ with fixed point $\frac{1}{2}$, it is $N_P = \{x \in X : p(x) \leq \frac{1}{2}\} = \{x \in X : p(x) = 0\}$, the crisp complement of the subset $\{x \in X : p(x) = 1\}$ specified by P.

4.4. Provided antonym and negation are coherent, that is, $p_a \leq p'$, it is $A_P \subseteq N_P$. This happens, for instance, with $P =$ big in $X = [0, 10]$, with $p(x) = \frac{x}{10}, N(x) = \frac{1-x}{1+x}$, and $s(x) = 10(\frac{1-10x}{1+10x})$.

5. Non-regular precise predicates

There are doubts on the existence of regular antonyms of precise predicates, and it is not clear for which reasons a precise predicate P should be non-regular: $aP = P'$. What has been said can help to clarify such question: existing or not words in language designating them, some of such crisp antonyms can be found mathematically and, hence, there is the possibility of designating them by words. The problem lies in finding an adequate symmetry s such that $p_a = p \circ s \leq 1 - p$, whenever $p(x) = 1$ for x in A, and $p(x) = 0$ for x in A^c, with $A \subseteq X$. For those x such that $p(x) = 1$, it should be $p(s(x)) = 0$ since $1 - p(x) = 0$, and for those y such that $p(y) = 0$ it should be $p(s(y)) \leq 1$ implying either $p(s(y)) = 1$, or $p(s(y)) = 0$. Hence, provided these sets have the same cardinality, it suffices that s maps A into A^c and A^c into A; on the contrary, some elements in A should be "moved" to the same A.

Let us consider a very simple example with $X = [0, 10]$, $P =$ smaller than five, and $P' =$ greater than five, where $p(x) = 1 \Leftrightarrow 0 \leq x < 5$, and $p(x) = 0$ otherwise, and $p'(x) = 1 \Leftrightarrow 5 \leq x \leq 10, p'(x) = 0$ otherwise. P specifies the crisp set $[0, 5)$, and $[5, 10]$ is the set specified by P'. Among the possible antonyms of P are those given by $p_a(x) = p(s(x))$, for all symmetry $s : [0, 10] \longrightarrow [0, 10]$, verifying the coherence inequality $p_a = p \circ s \leq p'$. Then:

$$s(x) \in [0, 5) \Rightarrow p_a(x) = 1; s(x) \in [5, 10] \Rightarrow p_a(x) = 0,$$

show that aP is actually a precise predicate. Since it should be $s(0) = 10$, and $s(10) = 0$, it suffices that $s(5) \neq 5$ for having $aP \neq P'$. For instance, there is such coincidence with the symmetry $s(x) = 10 - x$, but it is not with $s(x) = 10(\frac{10-x}{10+x})$ with which it is $s(5) = \frac{10}{3}$, and the antonym is given by:

- $p_a(x) = 1 \Leftrightarrow 0 \leq 10(\frac{10-x}{10+x}) < 5 \Leftrightarrow \frac{10}{3} < x \leq 10$,
- $p_a(x) = 0$, otherwise,

corresponding to the interval $(\frac{10}{3}, 10]$ linguistically named by the predicate $aP =$ greater than $\frac{10}{3}$, and different from $P' =$ greater than 5.

Hence, fuzzy logic opens a door to newly looking at opposites by even offering a methodology able to obtain antonyms without words for them in language.

6. Last comment.

There are logically-driven fuzzy theoreticians who derive the negation function N from the residuated implication functions $J_T(a, b) = \sup\{z \in [0, 1] : T(a, z) \leq b\}$, with T a continuous t-norm, and defining $N(a) = J_T(a, 0)$ (see, for example, [2]). These operators come from the identity $a' + b = \sup\{z : a \cdot z \leq b\}$, only valid in (complete) Boolean algebras, and as a consequence of defining "implication" by the binary operations $\rightarrow$ verifying the Modus-Ponens inequality $a.(a \rightarrow b) \leq b$, equivalent (only in Boolean algebras) to $a \rightarrow b \leq a' + b$. This election is out of the necessity to represent conditional linguistic statements in fuzzy terms and, of course, in Boolean algebras is $a' = a' + 0 = a \rightarrow 0$. For continuous t-norms not being an ordinal-sum of t-norms, it is:

- If either $T = \min$, or $T = f^{-1} \circ (\text{prod} \times \text{prod}) \circ f$ (T is in the family of the product), it is $J_T(a, 0) = N_{min}(a)$,
- If T is in the family of Lukasiewicz, $T = f^{-1} \circ (W \times W) \circ f$, it is $J_{Wf}(a, 0) = f^{-1}(1 - f(a)) = N_f(a)$.

Hence, under this "logical" interpretation of the negation coming from reading "if a, then b" as "not a or b", strong negations only appear linked to Lukasiewicz's t-norms, that is, to a very restrictive way of representing linguistic rules and as a typical reminiscence of classical logic. Notice that if the rules are represented, as it is done in fuzzy control engineering, by the conditional operators $J(a, b) = \min(a, b)$ (Mamdani), or by $J(a, b) = a \cdot b$ (Larsen), $J(a, 0)$ gives no negation whatsoever. The interests of logic are mainly placed in formal languages, but not in natural ones where there is not a universal representation of conditional statements, and where, in most cases, which is the negation of the antecedent is not known.

7. Conclusion

This paper only tries to reflect on a subject fuzzy logic praxis perhaps manages in a too simplistic form coming from a logic point of view, and not from that of ordinary reasoning that should be the true "worry" in Zadeh's Computing with Words. It refers to a general view on what is not properly under a linguistic label and for what, in language, there is not only the negation but also the opposites. It would like to open the eyes of theoreticians not only towards the true grounds on which fuzzy logic is anchored, natural language and common sense reasoning practices, but by taking into account that fuzzy logic neither can be an isolate mathematical subject, but a broader "physics" of imprecision and non-random linguistic uncertainty and closer to experimental science than to traditional logic [9].

What can be seen as "not being covered by P" still deserves more study; the subject is neither closed by what is currently known on strong negation functions for the negation, nor by the use of symmetries for the opposites, and there is particularly a lack of knowledge when the negation is not known to be either effectively measurable, or not strong. In the progress towards Computing with Words and Perceptions, the time for considering large phrases soon or later will arrive and call for a deep knowledge on the linguistic separation between what is under P and what is not under P, that is, calling for a new view on the idea of "complement". This paper is not, and currently cannot be, conclusive; it is but a tentative helping to offer a new focus for the question.

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