Optimization of the Fuzzy Integrators in Ensembles of ANFIS Model for Time Series Prediction: The case of Mackey-Glass

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Abstract

This paper describes the optimization of the fuzzy integrators in Ensembles of ANFIS models for time series prediction, this with emphasis on its application to the prediction of Mackey-Glass time series, so this benchmark time series is used to the test of performance of the proposed ensemble architecture. We used fuzzy systems to integrate the outputs (forecasts) of each of the ANFIS models in the Ensemble. Genetic Algorithms (GAs) were used for the optimization of memberships function parameters of the fuzzy integrators. In the fuzzy integrators, we applied different noise levels. Simulation results show the effectiveness of the proposed approach.

Keywords: Interval Type-2 Fuzzy; ANFIS Model; Time Series; Genetic Algorithms.

1. Introduction

Time series predictions are very important because based on them past events can be analyzed to know the possible behavior of future events and thus take preventive or corrective decisions to help avoid unwanted circumstances. The choice and implementation of an appropriate method for prediction has always been a major issue for enterprises that seek to ensure the profitability and survival of business. Predictions give the company the ability to make decisions in the medium and long term, and due to the accuracy or inaccuracy of data this could mean predicted growth or profits and financial losses [1]. It is very important for companies to know the behavior that will be the future development of their business, and thus be able to make decisions that improve the company's activities, and avoid unwanted situations, which in some cases can lead to the company's failure.

ANFIS (adaptive neuro-fuzzy system) [2] put forward by Jang in 1993 integrate the advantages of both neural network and fuzzy systems, which not only have good learning capability, but can be interpreted easily also. ANFIS has been used in many applications in many areas, such as function approximation, intelligent control and time series prediction.

As to sample selection, many papers on time series prediction have not given good methods. On the one hand, they just partition the training data and testing data randomly. So, the training data sometimes do not always reflect the real distribution of the prediction model and the effectiveness of the prediction algorithm can't be assured. On the other hand, when there are too many training data, the training time is long. So how to choose a set of training data to reflect the real distribution of the prediction model and decrease the training time in the huge training data is a very important problem in time series prediction.

Genetic algorithms are adaptive methods which may be used to solve search and optimization problems. They are based on the genetic process of living organisms. Over generations, the populations evolve in line with the principles of natural selection and survival of the fittest, postulated by Darwin, in imitation of this process; genetic algorithms are capable of creating solutions to real world problems. The evolution of these solutions to optimal values of the problem depends largely on the proper coding of them. The basic principles of genetic algorithms were established by Holland [3,4] and are well described in texts Goldberg [5,6] and Davis [7].

This paper reports the results of the simulations, in which the Mackey-Glass time series [8-13] prediction using genetic optimization for the fuzzy integrators in ensembles of ANFIS models. The results for each ANFIS are evaluated by the metric of the root mean square error (RMSE). For the integration of the results of each modular in the ensemble of ANFIS we use the following integration methods: type-1 fuzzy system, interval type-2 fuzzy systems

2. Basic concepts

2.1. ANFIS model

ANFIS is a neural network implementation of a TSK (Takagi-Sugeno-Kang) fuzzy inference system. ANFIS applies a hybrid algorithm, which integrates BP (Backpropagation) and LSE (least square estimation) algorithm [2], so it has rapid learning speed (Fig. 1) and its architecture represented ANFIS model in the following.

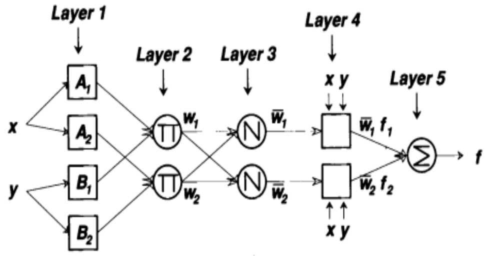

Fig. 1: ANFIS architecture.

2.2. Ensemble learning

Ensemble is a learning paradigm, where multiple component learners are trained for a same task, and the predictions of the component learners are combined for dealing with future instances [14]. Since an ensemble is often more accurate than its component learners, such a paradigm has become a hot topic in recent years and has already been successfully applied to optical character recognition, face recognition, scientific image analysis, medical diagnosis, etc. [15].

In this proposed architecture, noise levels are applied to each fuzzy integrator to measure performance between these models used in each ensemble. This measurement is performed in order to see that this model is more effective to predict the time series.

We applied different experiments and the best result are when we used a set of 3 ANFIS models for ensemble learning, it is noteworthy that the type of membership functions is assigned differently to each ANFIS and the desired goal error is assigned to each Ensemble of 0.01 to 0.000001 and the prediction error of each fuzzy integrator is calculated with the root mean squared error RMSE (3).

2.3. Fuzzy integrators

Type-2 fuzzy sets are used to model uncertainty and imprecision; originally they were proposed by Zadeh [16-18] and they are essentially ''fuzzy–fuzzy'' sets in which the membership degrees are type-1 fuzzy sets.

The uncertainty is represented by a region called footprint of uncertainty (FOU). When; we have an interval type-2 membership function.

The uniform shading for the FOU represents the entire interval type-2 fuzzy set [19-23] and it can be described in terms of an upper and a lower membership function.

2.4. Genetic algorithms

Genetic Algorithms (GAs) are adaptive heuristic search algorithms based on the evolutionary ideas of natural selection and the genetic process [24]. The basic principles of GAs were first proposed by John Holland in 1975, inspired by the mechanism of natural selection, where stronger individuals are likely to be the winners in a competing environment [25-27]. GAs assume that the potential solution of any problem is an individual and can be represented by a set of parameters [28].

2.5. Mackey-Glass time series

One of the chaotic time series data used in many works is defined by the Mackey-Glass [8-13] time series, whose in (1) is given by:

$$x(t) = \frac{0.2x(t-\tau)}{1+x^{10}(t-\tau)} - 0.1x(t-\tau) \quad (1)$$

For obtaining the values of the time series at each point, we can apply the Runge-Kutta method for the solution of in (1). The integration step was set at 0.1, with initial condition $x(0) = 1.2$, $\tau = 17$, $x(t)$ is then obtained for $0 \leq t \leq 1200$, which is illustrated in (Fig. 2).

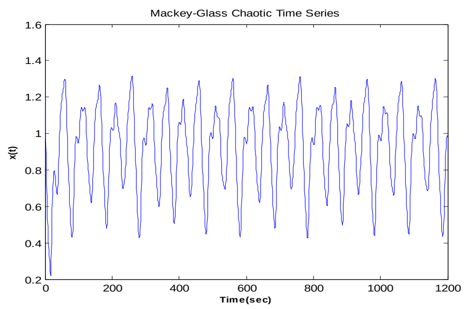

Fig. 2: Mackey-Glass time series.

3. Proposed architecture model

The proposed method combines the ensemble of ANFIS models and the use of genetic algorithms for the optimization of the interval type-2 and type-1 fuzzy systems as response integrators (Fig. 3).

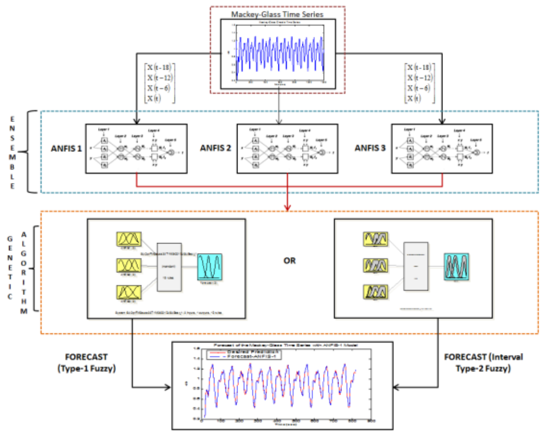

Fig. 3: The general architecture of the proposed method.

This architecture is divided into 4 sections, where the first phase represents the data base to simulate in the Ensemble of ANFIS, which in our case is the historical data of the Mackey-Glass [29, 30] time series. From the Mackey-Glass time series we used 800 pairs of data points (Fig. 2), similar to [8-10, 13].

We predict $x(t)$ from three past (delays) values of the time series, that is, $x(t-18)$, $x(t-12)$, and $x(t-6)$. Therefore the format of the training and checking data is:

$$[x(t-18), x(t-12), x(t-6); x(t)] \quad (2)$$

Where t = 19 to 818 and $x(t)$ is the desired prediction of the time series.

In the second phase, training (the first 400 pairs of data are used to train the ANFIS) and validation (the second 400 pairs of data are used to validate the ANFIS models) is performed sequentially in each of the ANFIS model, where the number of ANFIS to be used can be from 1 to n depending on what the user wants to test, but in our case we are dealing with a set of 3 ANFIS in the Ensemble. Therefore each ANFIS model has three input variables ($x(t-18), x(t-12), x(t-6)$) and one output variable ($x(t)$), which is the desired prediction.

In the fourth phase we integrate the overall results of each Ensemble of ANFIS (ANFIS 1, ANFIS 2 and ANFIS 3) models, and such integration is performed by type-1 and interval type-2 fuzzy integrators of Mamdani type, but each fuzzy integrators will optimized (GAs) of the MFs parameters. Finally the forecast output determined by the proposed architecture is obtained and it is compared with desired prediction.

4. Simulation results

This section presents the results obtained through experiments on the architecture of optimization interval type-2 and type-1 fuzzy integrators in ensembles of ANFIS models with genetic algorithms, which show the performance that was obtained from each experiment to simulate the Mackey-Glass time series.

We performed different experiments and the best result is when we used a set of 3 ANFIS models for ensemble learning, it is noteworthy that the type of membership functions was assigned differently to each ANFIS and the desired goal error was assigned to each Ensemble of 0.01 to 0.000001 and the prediction error of each fuzzy integrator is calculated using (3).

GAs are used to optimize the parameters values of the MFs in each type-1 and interval type-2 fuzzy integrators. The representation of GAs is of Real-Values and the chromosome size will depend of the MFs that are used in each design of the type-1 and interval type-2 fuzzy inference systems integrators.

The objective function is defined to minimize the prediction error (Root Mean Square Error) as follows:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (a_t - p_t)^2}{n}} \quad (3)$$

Where $\boldsymbol{a}$, corresponds to the real data of the time series, $\boldsymbol{p}$ corresponds to the output of each fuzzy integrator, $\boldsymbol{t}$ is de sequence time series, and $\boldsymbol{n}$ is the numbers of data points of time series.

In (Fig. 3) is illustrated the general representation of the chromosome and represent the utilized interval type-2 fuzzy membership functions. In those figures, the first phase represented each input/output variables of the fuzzy systems, the second phase represents the MFs containing each input (MFs1 “Small”, MFs2 “Middle” and MFs3 “Large”) and output (MFs1 “OutANFIS1”, MFs2 “OutANFIS2” and MFs3 “OutANFIS1”) variables of the fuzzy systems, the third phase represents the MFs parameter “PL = Lower Parameter” where PL_1 ... PL_N (0.15…1.2) are the size parameter of the MFs, the fourth phase represent the MFs parameter “PU = Upper Parameter” PU_1 ... PU_N (0.35…1.4) are the size parameter of the MFs that corresponds to each input and output. The number of parameters varies according to the kind of MFs of the type-1 fuzzy system (e.g. two parameter are needed to represent a Gaussian MF’s are "sigma and mean ") and interval type-2 fuzzy system (e.g. three parameter are needed to represent “igaussmtype2” MF’s are "sigma, mean1 and mean2") illustrated in (Fig. 4).

Therefore the number of parameters that each fuzzy inference system integrator depends of the MFs type assigned to each input and output variables.

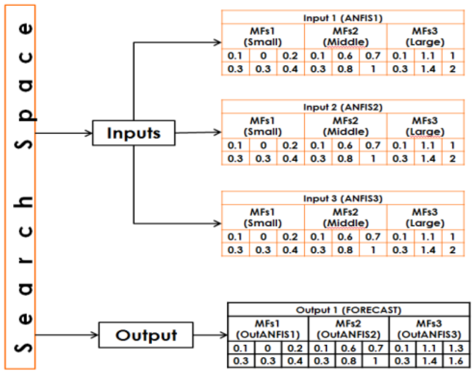

Fig. 4: Representation of the Chromosome for the Optimization of the Interval Type-2 Fuzzy Gaussian MFs.

The GAs used the following parameters for the experiments: 100 individuals or genes, 100 generations and 30 iterations (running the GAs), the selection method are the stochastic universal sampling, the percentage of crossover or recombine is 0.8 and the mutation is 0.1. There are fundamentals parameters for test the performances of the GAs.

4.1. Optimization of type-1 fuzzy integration using gaussian MFs

In the design of the type-1 fuzzy integrator we consider three input variables and one output variable, so the input/output variables have three MFs. Therefore the number of parameters that are used in the representation of the chromosome is 24, because Gaussian MFs used two parameters (sigma and Mean) for their representation in the type-1 fuzzy systems integrator. The results obtained for the optimization of the Gaussian MFs with GAs are the following: the parameters obtained with the GAs for the type-1 fuzzy Gaussian MFs (Fig. 5). The forecast data (Fig. 6) is generated by optimization of the type-1 fuzzy integrators. Therefore the obtained evolution error with the GAs for this integration is of 0.013616.

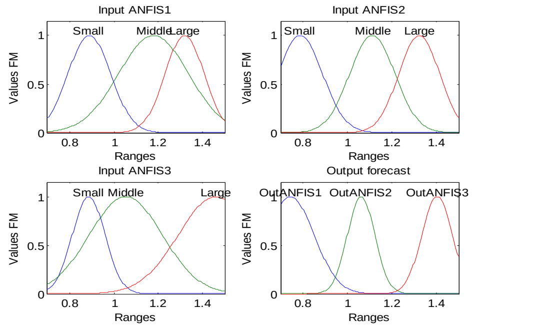

Fig. 5: Optimization of the Gauss MFs (input and output) parameters.

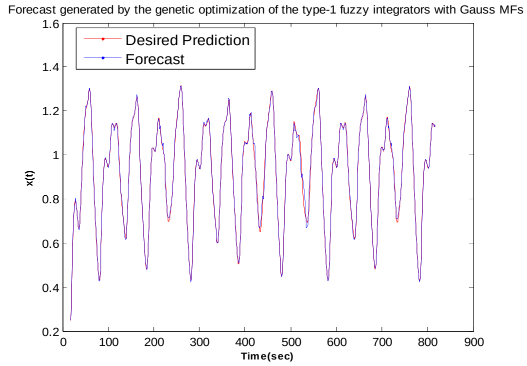

Fig. 6: Forecast generated by the genetic optimization of the type-1 fuzzy integrators with Gauss MFs.

4.2. Optimization of the interval yype-2 fuzzy integration using triangular "itritype2" MFs

In the design of the interval type-2 fuzzy integrator we consider three input variables and one output variable, so each input/output variable have three MFs. Therefore the number of parameters that are used in the representation of the chromosome is 72, because "itritype2" MFs used six's parameters (a1, b1, c1, a2, b2 and c2) for their representation in the interval type-2 fuzzy systems. The results obtained to the optimization of the "itritype2" MFs with GAs are the following: the parameters obtained with the GAs for the interval type-2 fuzzy "itritype2" MFs (Fig. 7). The forecast data (Fig. 8) is generated by optimization of the interval type-2 fuzzy integrators. Therefore the obtained evolution error with the GAs for this integration is of 0.017381.

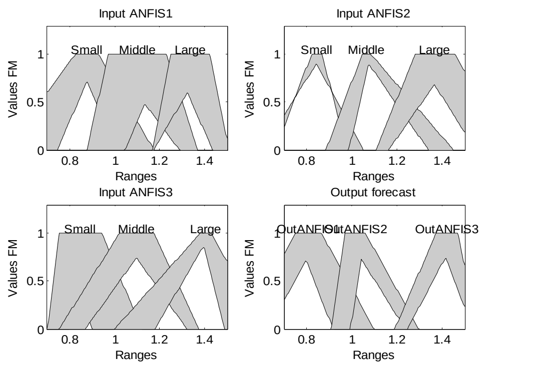

Fig. 7: Optimization of the three "itritype2" MFs (input and output) parameters.

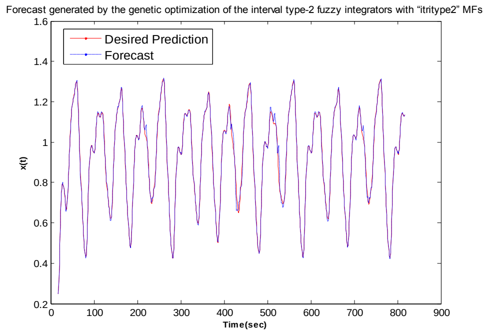

Fig. 8: Forecast generated by the genetic optimization of the interval type-2 fuzzy integrators with "itritype2" MFs.

4.3. Comparisons of results for optimization of the fuzzy integrators

This phase shows the results of the experiments that are obtained with the proposed method (Fig. 3).

Table 1 shows the results of 30 experiments that were performed with the optimization of the fuzzy integrators (using 2 MFs) in ensembles of ANFIS models for the time series prediction. The Best and Averages results for the type-1 fuzzy integrator are using Gaussian MFs, which obtained a prediction error of 0.018745 for the best and average prediction error of 0.019013. The best and average results for the interval type-2 fuzzy integrators are using Generalized Bell (igbelltype2) MFs, which obtained a prediction error is of 0.018154 for the best and average prediction error is of 0.018386.

Prediction Error (RMSE)	Type-1 Fuzzy Integrator			Interval Type-2 Fuzzy Integrator		
	Gauss	GBell	Triangular	igaussmtype2	igbelltype2	itritype2
Best	0.018745	0.02187	0.123708	0.01843	0.018154	0.123708
Average	0.019013	0.021928	0.124352	0.018552	0.018386	0.124352
Time (HH:MM:SS)	07:15:32	10:26:13	16:12:31	25:37:19	52:42:07	65:33:03

Table 1: Best and Average results of the prediction error of Mackey-Glass using two MFs.

Table 2 shows the results of 30 experiments that were performed with the optimization of the fuzzy integrators (using 3 MFs) in ensembles of ANFIS models for the time series prediction. The best and average results for the type-1 fuzzy integrator are using Generalized Bell MFs, which obtained a best prediction error is of 0.013434 and average prediction error of 0.013838. The best and average results for the interval Type-2 fuzzy integrator are using Gaussian (igaussmtype2) MFs, which obtained a best prediction error of 0.011248 and average prediction error is of 0.011888.

Prediction Error (RMSE)	Type-1 Fuzzy Integrator			Interval Type-2 Fuzzy Integrator		
	Gauss	GBell	Triangular	igaussmtype2	igbelltype2	itritype2
Best	0.013616	0.013434	0.03386	0.011248	0.01654	0.017381
Average	0.013888	0.013838	0.033931	0.011595	0.027234	0.017494
Time (HH:MM:SS)	13:48:09	16:12:18	21:54:07	31:13:03	60:32:19	81:09:15

Table 2: Best and Average results of the prediction error of Mackey-Glass using three MFs.

4.4. Applied levels noise in the fuzzy integrators

After fitting a time series model, one can evaluate it with forecast fit measures. When more than one forecasting technique seems reasonable for a particular application, then the forecast accuracy measures can also be used to discriminate between competing models. Therefore we consider the follows metrics for the comparison the performance of the fuzzy integrators. root mean square error (RMSE), mean error (ME), mean square error (MSE), mean absolute error (MAE), mean percentage error (MPE) and mean absolute percentage error (MAPE).

Tables 3, 4, 5, 6, 7 and 8 illustrate the results of 10 experiments in which Gaussian noise to the 3%, 5%, 7%, 9%, 10% and 30% was applied, to measure the performance of the proposed architecture. These results were obtained from the optimization of the fuzzy integrators (using 2 and 3 MFs).

The results of the fuzzy integrators that use 2 MFs are observed for the following abbreviations (t1gauss3213, t1gbell3213, t1triang3213, it2gauss3213, it2gbell3213 and it2triang3213). Therefore the results of the fuzzy integrators that used 3 MFs are observed for the following abbreviations (t1gauss3313, t1gbell3313, t1triang3313, it2gauss3313, it2gbell3313 and it2triang3313).

The obtained results for the experiments with a degree Gaussian noise (3%) are shown on Table 3 for the fuzzy integrators used 2 and 3 MFs. We are also presenting results with different fuzzy integrators systems. Therefore de best prediction error is when we use the triangular MFs (it2triang3213), because de prediction errors are (RMSE is 0.0219, ME is -0.0143, MSE is 0.0005, MAE is 0.0164, MPE is -1.9012 and MAPE is 1.8971).

MFs	RMSE	ME	MSE	MAE	MPE	MAPE
t1gauss3213	0.0241	-0.0844	0.0007	0.0190	-9.4691	2.1550
t1gbell3213	0.0265	-0.0394	0.0010	0.0221	-4.6148	2.5034
t1triang3213	0.0815	-0.0103	0.0069	0.0658	-1.2236	7.4554
it2gauss3213	0.0222	-0.0457	0.0005	0.0166	-4.8102	1.9273
it2gbell3213	0.0223	-0.0275	0.0005	0.0166	-3.2190	1.9320
it2triang3213	0.0219	-0.0143	0.0005	0.0164	-1.9012	1.8971
t1gauss3313	0.0329	-0.0649	0.0015	0.0279	-7.9742	3.0895
t1gbell3313	0.0232	-0.0479	0.0007	0.0196	-5.7423	2.3539
t1triang3313	0.0384	-0.0193	0.0018	0.0303	-2.7180	1.9379
it2gauss3313	0.0220	-0.0286	0.0005	0.0170	-3.4708	1.9528
it2gbell3313	0.0228	-0.0107	0.0012	0.0218	-2.1058	2.5928
it2triang3313	0.0257	-0.0088	0.0008	0.0215	-1.4523	2.5060

Table 3: Results for optimization of the fuzzy integrators which applied Gaussian noise of 3%.

The obtained results for the experiments with a degree Gaussian noise (5%) are shown on Table 4 for the fuzzy integrators used 2 and 3 MFs. We are also presenting results with different fuzzy integrators systems. Therefore de best prediction error is when we use de generalized bell MFS (it2gbell3213), because de prediction errors are (RMSE is 0.0284, ME is -0.0315, MSE is 0.0009, MAE is 0.0220, MPE is -3.6786 and MAPE is 2.6005).

MFs	RMSE	ME	MSE	MAE	MPE	MAPE
t1gauss3213	0.0310	-0.0591	0.0012	0.0264	-7.0691	3.0993
t1gbell3213	0.0327	-0.0416	0.0015	0.0289	-4.7247	3.3507
t1triang3213	0.0961	-0.0189	0.0101	0.0809	-2.1523	9.2653
it2gauss3213	0.0287	-0.0281	0.0009	0.0220	-3.3488	2.6043
it2gbell3213	0.0284	-0.0315	0.0009	0.0220	-3.6786	2.6005
it2triang3213	0.0287	-0.0119	0.0009	0.0220	-1.6924	2.5543
t1gauss3313	0.0375	-0.0555	0.0026	0.0360	-7.2124	4.1079
t1gbell3313	0.0291	-0.0464	0.0010	0.0241	-5.6508	2.8149
t1triang3313	0.0433	-0.0198	0.0030	0.0406	-2.9416	2.9869
it2gauss3313	0.0285	-0.0257	0.0009	0.0228	-3.0142	2.6383
it2gbell3313	0.0297	-0.0063	0.0032	0.0382	-2.3091	4.6618
it2triang3313	0.0309	-0.0078	0.0016	0.0303	-1.5474	3.5934

Table 4: Results for optimization of the fuzzy integrators which applied Gaussian noise of 5%.

The obtained results for the experiments with a degree Gaussian noise (7%) are shown on Table 5 for the fuzzy integrators used 2 and 3 MFs. We are also presenting results with different fuzzy integrators systems. Therefore de best prediction error is when we use de triangular MFS (it2triang3213), because de prediction errors are (RMSE is 0.0356, ME is -0.0138, MSE is 0.0014, MAE is 0.0272, MPE is -1.9278 and MAPE is 3.1327).

MFs	RMSE	ME	MSE	MAE	MPE	MAPE
t1gauss3213	0.0385	-0.0472	0.0018	0.0319	-5.6793	3.6986
t1gbell3213	0.0391	-0.0378	0.0019	0.0326	-4.4823	3.7075
t1triang3213	0.0979	-0.0184	0.0103	0.0816	-2.1929	9.3545
it2gauss3213	0.0358	-0.0378	0.0014	0.0270	-4.3173	3.1285
it2gbell3213	0.0357	-0.0291	0.0013	0.0269	-3.4264	3.1377
it2triang3213	0.0356	-0.0138	0.0014	0.0272	-1.9278	3.1327
t1gauss3313	0.0436	-0.0602	0.0029	0.0405	-7.6893	4.5582
t1gbell3313	0.0362	-0.0399	0.0016	0.0291	-5.1850	3.3852
t1triang3313	0.0493	-0.0153	0.0034	0.0434	-2.4562	3.1166
it2gauss3313	0.0356	-0.0293	0.0014	0.0274	-3.7053	3.1395
it2gbell3313	0.0361	-0.0096	0.0031	0.0394	-2.6355	4.8795
it2triang3313	0.0387	-0.0068	0.0021	0.0347	-1.5276	4.0896

Table 5: Results for optimization of the fuzzy integrators which applied Gaussian noise of 7%.

The obtained results for the experiments with a degree Gaussian noise (9%) are shown on Table 6 for the fuzzy integrators used 2 and 3 MFs. We are also presenting results with different fuzzy integrators systems. Therefore de best prediction error is when we use de gaussian MFS (it2gauss3313), because de prediction errors are (RMSE is 0.0422, ME is -0.0267, MSE is 0.0019, MAE is 0.0326, MPE is -3.3131 and MAPE is 3.8219).

MFs	RMSE	ME	MSE	MAE	MPE	MAPE
t1gauss3213	0.0438	-0.0419	0.0022	0.0352	-5.1293	4.1604
t1gbell3213	0.0453	-0.0327	0.0024	0.0366	-4.1603	4.1780
t1triang3213	0.0851	-0.0144	0.0089	0.0758	-1.6998	8.7227
it2gauss3213	0.0429	-0.0365	0.0019	0.0321	-4.4353	3.7608
it2gbell3213	0.0423	-0.0284	0.0019	0.0315	-3.5629	3.7041
it2triang3213	0.0425	-0.0122	0.0019	0.0318	-1.8274	3.7087
t1gauss3313	0.0477	-0.0535	0.0033	0.0428	-7.0724	4.9408
t1gbell3313	0.0429	-0.0463	0.0020	0.0335	-5.6012	3.9161
t1triang3313	0.0584	-0.0132	0.0044	0.0495	-2.2123	3.6948
it2gauss3313	0.0422	-0.0267	0.0019	0.0326	-3.3131	3.8219
it2gbell3313	0.0428	-0.0063	0.0038	0.0440	-2.2881	5.3395
it2triang3313	0.0452	-0.0092	0.0027	0.0399	-1.8014	4.7482

Table 6: Results for optimization of the fuzzy integrators which applied Gaussian noise of 9%.

The obtained results for the experiments with a degree Gaussian noise (10%) are shown on Table 7 for the fuzzy integrators used 2 and 3 MFs. We are also presenting results with different fuzzy integrators systems. Therefore de best prediction error is when we use the Gaussian MFS (it2gauss3213), because de prediction errors are (RMSE is 0.0451, ME is -0.0396, MSE is 0.0021, MAE is 0.0334, MPE is -4.3041 and MAPE is 3.8817).

MFs	RMSE	ME	MSE	MAE	MPE	MAPE
t1gauss3213	0.0459	-0.0564	0.0024	0.0362	-6.6743	4.1853
t1gbell3213	0.0469	-0.0391	0.0025	0.0369	-4.6426	4.2248
t1triang3213	0.1005	-0.0140	0.0110	0.0852	-1.8085	9.7548
it2gauss3213	0.0451	-0.0396	0.0021	0.0334	-4.3041	3.8817
it2gbell3213	0.0451	-0.0304	0.0021	0.0336	-3.9031	3.9121
it2triang3213	0.0449	-0.0122	0.0021	0.0335	-1.8455	3.8840
t1gauss3313	0.0494	-0.0607	0.0035	0.0441	-8.0002	5.1133
t1gbell3313	0.0453	-0.0488	0.0021	0.0341	-5.7902	3.9786
t1triang3313	0.0555	-0.0166	0.0042	0.0484	-2.8256	2.9128
it2gauss3313	0.0452	-0.0276	0.0022	0.0346	-3.7248	4.0417
it2gbell3313	0.0453	-0.0107	0.0043	0.0474	-2.8603	5.7837
it2triang3313	0.0474	-0.0084	0.0028	0.0402	-1.6847	4.7200

Table 7: Results for optimization of the fuzzy integrators which applied Gaussian noise of 10%.

The obtained results for the experiments with a degree Gaussian noise (30%) are shown on Table 8 for the fuzzy integrators used 2 and 3 MFs. We are also presenting results with different fuzzy integrators systems. Therefore de best prediction error is when we use de generalized bell MFs (it2gbell3213), because de prediction errors are (RMSE is 0.1124, ME is -0.0229, MSE is 0.0127, MAE is 0.0801, MPE is -4.5501 and MAPE is 9.6284).

MFs	RMSE	ME	MSE	MAE	MPE	MAPE
t1gauss3213	0.1127	-0.0413	0.0131	0.0829	-6.8045	9.8710
t1gbell3213	0.1129	-0.0310	0.0130	0.0815	-5.2249	9.7520
t1triang3213	0.1451	-0.0030	0.0229	0.1164	-1.4017	13.8344
it2gauss3213	0.1126	-0.0267	0.0128	0.0810	-4.7428	9.7600
it2gbell3213	0.1124	-0.0229	0.0127	0.0801	-4.5501	9.6284
it2triang3213	0.1125	-0.0105	0.0128	0.0807	-3.1778	9.7415
t1gauss3313	0.1132	-0.0505	0.0134	0.0854	-8.1969	10.3174
t1gbell3313	0.1124	-0.0254	0.0128	0.0809	-5.0072	9.7362
t1triang3313	0.1179	-0.0149	0.0148	0.0911	-3.8629	8.1356
it2gauss3313	0.1128	-0.0199	0.0129	0.0819	-4.2288	9.8681
it2gbell3313	0.1129	-0.0104	0.0138	0.0876	-3.8357	10.8010
it2triang3313	0.1126	-0.0060	0.0133	0.0848	-2.7739	10.2825

Table 8: Results for optimization of the fuzzy integrators which applied Gaussian noise of 30%.

Therefore the best performance is achieved with the optimization of the fuzzy integrators (level of noise is 3%) with noise in the data test.

5. Conclusion

We have presented simulation results of the Mackey-Glass time series (forecasting) with different hybrid intelligent approaches. The interval type-2 fuzzy integrator is better than type-1 fuzzy integrator (using two and three MFs, show Table 1 and 2), because in most of the experiments that were performed with the proposed architecture of ensembles of ANFIS. Applying different levels of noise (3%, 5%, 7%, 9%, 10% and 30%) we observe that the optimization of the interval type-2 fuzzy integrator has better behavior and tolerance to noise than the type-1 fuzzy integrator, and this is shown in Tables 3 and 8. This conclusion was found by observing that the interval type-2 fuzzy integrator present lower prediction errors than the other methods.

We conclude that the results obtained with the architecture are good, since we achieved 98% of accuracy with the Mackey-Glass time series, therefore we can conclude that our proposal offers efficient results in the prediction of such time series, which can help us make decisions and so avoid unexpected events in the future.

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