

A fuzzy edge-based image segmentation approach

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Abstract

In this paper, we present a way to define the concept of fuzzy image segmentation, which has not been clearly defined in the literature. In this work the term *Fuzzy image segmentation* is characterized by means of a fuzzy set over the set of edges, which can be understood as the fuzzy boundary of the image. Also we discuss a visualization of an image segmentation in terms of edge detection. But first, we define two concepts of crisp image segmentation on an image network, one based on nodes and the other one focusing on edges.

Keywords: Fuzzy set, Image segmentation, Crisp image segmentation, Graph-based fuzzy image segmentation

1. Introduction

Segmentation is a technique used to partition the pixels of an image in regions [1, 2], where each detected region or object is delimited by their boundaries. However, sometimes these boundaries are not sharp and clear (not crisply defined). Generally, let us remark that even when using fuzzy-based techniques on noisy image, the resulting segmentation uses to be crisp. On the contrary, in this paper, we extend the concept of crisp image segmentation [1] into a fuzzy framework. In this way, we deal with the noise or imperfections that could have an image. Also, as we will see, the output of a fuzzy image segmentation is more similar to a human segmentation.

Then, a first issue is how to translate the relevant concepts into a fuzzy framework. The concept of partition of an image network is easily translated but it is not obvious how to do the same with connectivity. In [3, 4] some classical concepts (area, perimeter or connectivity) in image analysis has been extended into a fuzzy framework. Nevertheless, these concepts are drawn for crisp objects or regions. Based in those definitions, in [4, 5, 6] a fuzzy connectedness is modelled into an image segmentation framework by means of a fuzzy relation, that is, connectivity is expressed by the degree up to which two elements are connected in a given subset. In such a way it is possible to build a membership function either to represent the connectivity degree

of a specific region or to measure some fuzzy concepts [6]. Even so, based on these concepts it is not possible to know if a set of fuzzy classes R_1, \dots, R_c is suitable to be a fuzzy image segmentation, or even what should be a suitable fuzzy output of an image segmentation.

Actually, a fuzzy image segmentation should be a set of fuzzy regions R_1, \dots, R_k where the membership function $\mu_{R_1}, \dots, \mu_{R_k}$ of each region represents the degree up to which each pixel of the image belongs to a region.

In [7, 8], some covering, connectivity and redundancy properties has been studied in relation with a set of fuzzy regions R_1, \dots, R_k with the purpose of analyzing and generalizing standard fuzzy classification.

Notice that, a crisp image segmentation can be characterized in terms of the set of edges that separates adjacent regions, that is, there is a bijection between the usual definition of crisp image segmentation and the set of boundary edges that connects nodes of different regions as is mentioned in [9]. The authors of [9] established a procedure to build a fuzzy image segmentation from a hierarchical image segmentation and they proposed a notion of how to visualize a fuzzy boundary. More details can be found in [10]. Following this idea, we intend to formally introduce an alternative way to define the concept of fuzzy image segmentation by means of the fuzzy boundary of the image. Furthermore, we define alternatives approaches to image segmentation which were introduced in general way in [9].

The concept of fuzzy boundary was introduced in the framework of fuzzy edge detection problems [11, 12, 13]. But it should be emphasized that the output of a fuzzy edge detection problem does not produce a suitable fuzzy image segmentation output. They are related but distinct problems. Similarly, an edge detection output does not produce a suitable image segmentation, and conversely. In the framework of image segmentation, the authors [14, 15, 16, 17, 18] have proposed to construct the fuzzy boundary based on an adaptation of a hierarchical segmentation algorithm [17, 18].

2. Crisp image segmentation graph theory based

First at all, we model a digital image from a graph theory point of view. After that, in this section we present two definitions of crisp image segmentation based on this model: one of them is based on the nodes of the graph and while the other is specified in terms of the edges of the image.

First, we begin by establishing how we treat a digital image.

Definition 2.1. Given a digital image having $I = r \times s$ pixels, we model it as a connected graph $G = (V, E)$. Let $V = \{p_{ij} = (i, j) | 1 \leq i \leq r; 1 \leq j \leq s\}$ be the set of pixels. Let $E = \{e = \{p_{ij}; p_{i'j'}\} | p_{ij}, p_{i'j'} \in V\}$ be the set of non-ordered pairs of neighbor pixels, such that an edge $e = \{p; p'\} \in E$ exists only if two pixels p and p' are neighbors; otherwise $\{p; p'\} \notin E$.

Based on the characteristics of each pixel, it is possible to build a dissimilarity measure between adjacent pixels. For simplicity, in this paper we have chosen the standard 4-connectivity topology.

Definition 2.2. Let $d_e \geq 0$ be the degree of dissimilarity between two joined pixels p and p' ($e = \{p; p'\} \in E$), the greater d_e is, the more dissimilar p and p' are. Then, let $D = \{d_e | e \in E\}$ be the set of all dissimilarities between adjacent nodes.

In this way, the information of a digital image I can be summarized by the image network

$$N(I) = \{ G; D \}. \quad (1)$$

Example 2.1. Figure 1 shows a 4×5 -image network with three different types of pixel: Red, Blue and White; where the distances between these types are 0 if they are equal and 0.3, 0.6 and 0.9 respectively between white and blue, blue and red, and red and white.

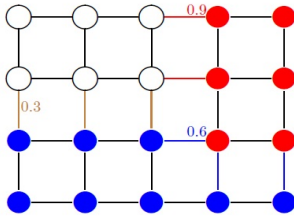


Figure 1: Image network of Example 2.1.

2.1. Node-based segmentation

A crisp image segmentation [1] can be defined as a partition of the set of pixels (nodes in our graph-based approach) into a set of connected subsets or regions. As a first step we modeled the image as a network, then we shall provide a formal graph-based

definition of image segmentation. Consequently image segmentation can be viewed as a partition of the set of pixels with some properties [7, 8].

Definition 2.3. Let $N(I) = \{ G = (V, E); D \}$ be a network image, the family $S = \{R_1, \dots, R_t\}$, with $R_j \subset V \forall j \in \{1, 2, \dots, t\}$, is a segmentation of the image $N(I)$ if and only if the following holds:

1. Non overlapping regions (i.e., for all $i \neq j$, $R_i \cap R_j = \emptyset$).
2. Covering (all the pixels are covered by regions): $\bigcup_{j=1}^t R_j = V$.
3. Connectivity of all regions: for all $j \in \{1, \dots, t\}$, the subgraph $(R_j, E|_{R_j})$ is a connected graph.

Thus pixels belonging to the same region are graph-connected. Obviously, two different and not adjacent regions can share the same characteristics (in terms of the dissimilarity distance d). These regions will be associated to the same class if a consistent classification procedure is applied on the segmented image.

In figure 2 we illustrate the node-based concept of segmentation on the image network of Example 2.1: three regions (white, blue and red) are obtained.

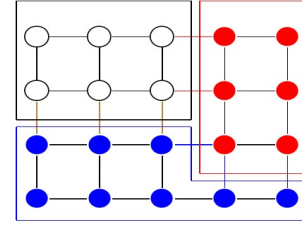


Figure 2: Segmentation of image network of Example 2.1 by nodes.

Remark 1. Various authors have defined image segmentation as stated above but without imposing a connectivity condition (3) over the set of feasible image segmentation solutions. Nevertheless, solutions which satisfy (1)-(2) can be easily transformed into a solution that also satisfies condition (3). From now on, and following [1] among others, we will associate image segmentation problems to connected regions, meanwhile non-connected regions will be associated to image clustering problems.

2.2. Edge-based segmentation

An alternative and equivalent definition of a segmented image is obtained through the minimal set of boundary edges $B \subset E$ which separate the regions of the segmentation of the image $N(I)$, see [19].

Definition 2.4. Given a network image $N(I) = \{ G = (V, E); D \}$, a subset $B \subset E$ characterizes an image segmentation if and only if the number of connected components of the partial graph generated

by the edges $E - B$, denoted as $G(E - B) = (V, E - B)$, decreases when any edge of B is deleted.

Remark 2. Given a boundary edge set B verifying Definition 2.4, then the family $S = \{R_1, \dots, R_t\}$ of connected components of the partial graph $G(E - B)$ is a segmentation.

An important aspect of these concepts is that there exists a bijection between a partition of the set of nodes into connected regions $S = \{R_1, \dots, R_t\}$ and the set of edges $B \subset E$ that produce image segmentations [9]. In such a way, we can say that to find a partition of the set of pixels V in the sense of Definition 2.3 is equivalent, to find a subset of edges in the sense of Definition 2.4.

Thereby, an image segmentation of $N(I)$ is characterized through a subset of edges $B \subset E$, and any of its edges links two different regions of the segmentation. Moreover, if an edge is deleted from B , the two adjacent regions of its endpoints are joined in one region.

Remark 3. Let us note that even if there is an equivalence between these two concepts, there exist a problem with the visualization of both of them. Since how to visualize the set of edges that breaks the image into regions is still an open issue. We will address this problematic in section 4.

3. Into the fuzzy image segmentation concept

The classical concept of partition of a set N , as a set of $P = \{P_1, \dots, P_k\}$ that satisfies

- $P_i \cap P_j = \emptyset$ if $i \neq j$ (non overlapping).
- $\cup_{i=1}^k P_i = N$ (Covering).

has been easily translated into a fuzzy framework as a set of fuzzy classes $\mu_{P_1}, \dots, \mu_{P_k} N \rightarrow [0, 1]$ satisfying the Ruspini condition $\forall x \in N, \sum_{i=1}^k \mu_{P_i}(x) = 1$.

Nevertheless, a crisp image segmentation solution also requires of the connectivity of its elements (see Definition 2.3). In [3] it can be found different ways to translate the crisp connectivity concept into a fuzzy framework, but it is a difficult issue to know (at least in a crisp way) if a fuzzy partition is connected or not. How to translate the connectivity concept to this aim is a question that merits to be explored in a future.

Taking into account previous difficulties as well as the fact that crisp image segmentation permits two characterizations (as a partition of the set of nodes or as a set of breaking edges), we adopt the second one to characterize a fuzzy image segmentation.

That is, it may be reasonable to formally define the concept of fuzzy segmentation through the fuzzyfication of the edge-based segmentation concept introduced in Definition 2.4:

Definition 3.1. Given a network image $N(I) = \{G = (V, E); D\}$, we will say that the fuzzy set $\tilde{B} = \{(e, \mu_B(e)), e \in E\}$ produces a fuzzy image segmentation if and only if for all $\alpha \in [0, 1]$ the crisp set $B(\alpha) = \{e \in E : \mu_B(e) \geq \alpha\}$ produces an image segmentation in the sense of Definition 2.4.

In the above definition, the membership function of the fuzzy set \tilde{B} for a given edge represents the degree of separation between these two adjacent pixels in the segmentation process.

Remark 4. This definition extends Definition 2.4:

$$\tilde{B} = B \iff \mu_B(e) = 1 \forall e \in B$$

We would like to stress that human beings make fuzzy segmentation in a natural way. This fact can be noticed in some human segmentation test images where the thickness of the lines we draw varies according to how clear a border is. For example, in Figure 4, the color that is used to separate the objects in the segmentation shows gradations of black. In this way, human segmentation does not correspond to a crisp image segmentation (see Figure 5), where its visualization should show segmentation with a unique intensity for all segmentation lines.



Figure 3: Real image [20].

Remark 5. In this section we defined the concept of a fuzzy image segmentation based on the boundary fuzzy class over the edge set. In this sense, the membership function of this class represents, for a particular edge, the degree of separation between two adjacent pixels in the segmentation process. It is very important to distinguish between the concept of fuzzy boundary and the classical concept of dissimilarity between two adjacent pixels. We like to emphasize that these two concepts are related but not equivalent. Dissimilarity measures between two adjacent pixels use to take only into account the spectral information of each pixel (or node), without considering the segmentation process. For example, the measure associated with each edge in edge detection or fuzzy edge detection problems [21] represents

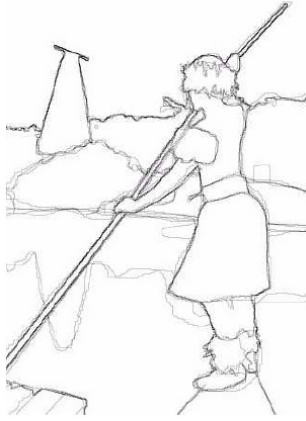


Figure 4: Human segmentation [20].

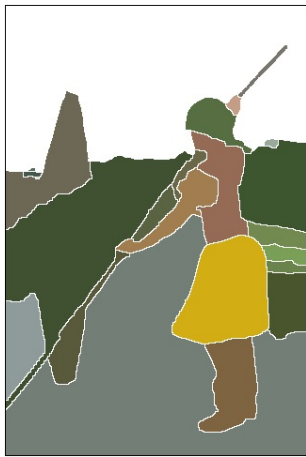


Figure 5: Image segmentation.

the dissimilarity (in terms of spectral information) between these two pixels, that does not necessarily meets the idea of boundary between regions.

Usually, a distance or dissimilarity measure between adjacent pixels is not enough to build a fuzzy segmentation. Consequently, the problem of finding a fuzzy image segmentation is not a trivial task. Nevertheless, as we have mentioned at the beginning, there is a strong relationship between the fuzzy image segmentation problem and the hierarchical image segmentation problem. Thus, the construction of a fuzzy image segmentation can be based on the construction of a hierarchical segmentation of the image network [9]. Thereby it will be possible to build a hierarchical image segmentation solution from a fuzzy image segmentation and vice-versa.

4. Visualizing an image segmentation

In this section, we first address the problem of visualizing a crisp image segmentation. Secondly, we deal with the problem of how to visualize a fuzzy image segmentation. At the end, some fuzzy seg-

mented images obtained through a hierarchical image segmentation procedure are shown.

4.1. Visualization of a segmented image

Usually, a crisp image segmentation is visualized as a picture in which the pixels that belong to the same region are drawn in a color that is obtained as an aggregation of the pixels that belong to this region (see picture of figure 5). Therefore, the regions of the segmented image can be seen easily. Nevertheless, this approach presents some inconveniences:

- not all images are three dimensional so they cannot be visualized,
- if the number of homogeneous regions is high the segmented image is quite identical to the original,
- and two adjacent but different incorrectly segmented regions could appear as only one in the visualization.

Another way to visualize an image segmentation is by means of the contour of the regions that has been obtained. In this approach, the pixels are classified into black and white pixels. The white pixels represent the boundary pixels (those pixels that are between different homogeneous regions) and the black pixels represent the core of the homogeneous regions. This way of visualization is more common in edge detection or contour problems (that are different problems since they are only interested in showing spectral differences between adjacent pixels). In the picture of figure 4, we can see that a gradation of these white and black classes is possible as is done in the visualization of fuzzy edge detection solutions.

Based on the fact that the first approach looks insufficient to visualize fuzzy image segmentations solutions, because does not permit a gradation in the visualization process, clearly it is better to use the second approach in order to visualize crisp image segmentations, hierarchical image segmentations and our fuzzy image segmentations.

Taking into account these considerations, first is necessary to know how to classify the pixels, within a given crisp image segmentation S , into such black and white classes. For this, if we denote by $S(p)$ the region to which the pixel p belongs, we propose to classify the set of pixels $V = \text{black} \cup \text{white}$ as follows:

Definition 4.1. Given a network image $N(I) = \{ G = (V, E); D \}$, and given a segmentation $S = \{ R_1, \dots, R_k \}$, we define the white and black class as:

- $\text{black} = \{ p \in V \text{ such that } S(p) = S(p') \forall e = (p, p') \in E \}$
- $\text{white} = \{ p \in V \text{ such that there exist } e = (p, p') \in E \text{ with } S(p) \neq S(p') \}$.

where $S(p)$ is the region to which the pixel p belongs.

It can be visualized that one pixel is coloured white if it has at least one neighbor in other region. On the other hand, the pixels that are coloured black are those that are rounded by pixels of the same region.

It is important to emphasize that image segmentation and edge detection are different problems, but it is possible to get an edge detection solution through an image segmentation solution. That is, edge detection is defined as a partition of the set of nodes into the object and the boundary. Moreover, an edge detection solution should satisfy some desirable properties but in general, not necessarily disconnects the image into homogeneous regions, so the image segmentation solution obtained from an edge detection solution is usually not suitable [22]. Since the opposite is true, this is why it is not trivial how to go from a suitable image segmentation output into a suitable edge detection solution (although this implication is easier than the other).

4.2. Visualization of a fuzzy segmented image

The issue dealt in this subsection is how to visualize a fuzzy image segmentation. We must determine how to build the fuzzy class *White* over the nodes set. In this paper, we establish that the *Black* fuzzy class is built as a negation of the *White* class.

Let \tilde{B} be a fuzzy image segmentation, with membership function $\mu_B : E \rightarrow [0, 1]$, one natural way to build the membership functions μ_{white} and μ_{black} is the following:

Definition 4.2. Given a network image $N(I) = \{ G = (V, E); D \}$, and given a fuzzy image segmentation \tilde{B} with membership function $\mu_B : E \rightarrow [0, 1]$, we define the white and black fuzzy classes in the following way:

- $\mu_{white}(p) = \text{MAX}\{\mu_B(e), \text{for all } e = (p, q) \in E\} \forall p \in V.$
- $\mu_{black}(p) = N(\mu_{white}(P)) = 1 - \mu_{white}(P) \forall p \in V.$

Remark 6. Notice that in the above definition, the membership degree of a given pixel p to the fuzzy class boundary-white is the result of the aggregation of the values $\{\mu_{\tilde{B}}(p, q_i), i = 1, \dots, 4\}$, where q_1, \dots, q_4 are the neighbors of the pixel p . In general, the aggregation operator $\phi : [0, 1]^4 \rightarrow [0, 1]$ used in this process should satisfy the following properties:

1. ϕ is symmetric.
2. $\phi(x_1, \dots, x_4) = 0$ if and only if $x_i = 0, \forall i = 1, \dots, 4$ (i.e., a pixel that is always in the same region as its neighbors should have degree zero to the boundary class).
3. $\phi(x_1, \dots, x_4) = 1$ if and only if there exist $i \in \{1, \dots, 4\}$ with $x_i = 1$ (i.e., a pixel that is always in the boundary since it always has

an adjacent pixel in a different region should have degree one to the boundary class).

In order to simplify this visualization process, we have used the *MAX* aggregation operator, but other aggregation functions can be obviously considered.

Next, we show some computational experiences by applying it, which gives a fuzzy segmented image obtained from the hierarchical image segmentation procedure [9, 10] to W/B images (the dissimilarity measure between pixels is calculated by the Euclidean Distance) and RGB-color (the dissimilarity measure is calculated by the CIE76 distance) obtained from the Berkeley database [20]. In them we can see gradations in the lines used to delimit the fuzzy segmented objects in the visualization process, more similar to a human segmentation. That is, the greater the membership functions of a fuzzy boundary, the bigger the white intensity of the end-points of the edge.



Figure 6: Visualization of a fuzzy segmented image.



Figure 7: Original Japan image.

5. Final Comments

We would like to remark that the aim of this paper is to define a new problem that is the fuzzy image segmentation problem. Also discuss the problem of the visualization of an image segmentation, in order to deal with the need of a visualization technique for a proper fuzzy segmentation. As we mentioned at

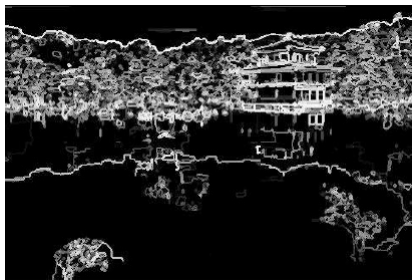


Figure 8: Visualization of a fuzzy segmented image.

the introduction to the extent of our knowledge, the fuzzy image segmentation problem have not been clearly defined so far. Thus, we proposed a way to define a fuzzy image segmentation by a graph approach and a method to visualize the results of such segmentation. Besides it is very important to differentiate with respect to a crisp image segmentation obtained by means of any fuzzy-based technique.

In many papers that use fuzzy techniques to obtain crisp segmentations, the terms fuzzy and image segmentation are strongly connected. Also in numerous papers about classification, a crisp classification is obtained based on fuzzy techniques. In such a classification framework we should distinguish between the term *fuzzy classification* (which produces a set of fuzzy classes, each one with a membership function) and the term crisp fuzzy-based classification (that produces a crisp classification using fuzzy algorithms). But in general, this distinction is difficult to realize in image segmentation, once the concept of *fuzzy image segmentation* has not been clearly defined. The term fuzzy image segmentation has been quite often associated to crisp fuzzy based segmentation.

As we established previously, when we try to segment an image into homogeneous regions, we might be able to identify some regions first than others since their boundaries are noted very clear. The formal definition of the concept fuzzy image segmentation presented in this paper, is a more natural and consistent approach to human segmentation.

To achieve our goal, firstly we have considered that there exists a bijection between the set of all crisp image segmentations and the set of what we have called *boundary links*. A crisp image segmentation is univocally determined by such *boundary links*. For a given segmentation, these links are those that connects pixels of different regions, and thus represent the boundary of the segmentation process. Then, the proposed approach was defined by means of a fuzzy class over the links set, where for a given edge the membership function of this fuzzy class represents the degree of separation between two adjacent pixels in the segmentation process. However, since a dissimilarity measure between adjacent pixels might not be an adequate solution, we took into consideration the bijective relation between the concept of fuzzy image segmenta-

tion and hierarchical image segmentation (see [9, 10] for further details). Some computational experiments reflect the advantages of using fuzzy image segmentation instead of crisp image segmentation.

The issue of how to define a fuzzy image segmentation or establishing which characteristics should be imposed to a fuzzy classification of the nodes, in order to guarantee a suitable fuzzy image segmentation, is a question that should be explored in future works.

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