Contextual bipolar queries: "or if impossible" operator case

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Abstract

Bipolar queries consist of negative and positive conditions specifying the features that are making data sought, respectively, rejected and accepted. A possible strategy to evaluate such bipolar queries is to combine matching degrees of both conditions using the *and possibly* operator. We studied this type of aggregation in our earlier work and afterwards proposed the concept of contextual bipolar queries which are based on a context-dependent version of the *and possibly* operator. Here we propose a further extension of this concept which is based on the *or if impossible* operator. We show some properties and possible interpretations of the proposed approach.

Keywords: flexible querying, bipolarity, context

1. Introduction

In the paper we further develop the idea of the *contextual bipolar queries* introduced in [1] and applied in [2, 3]. The aim is to better represent the users preferences and intentions during the querying of a database.

The approach proposed may be seen as a continuation of the line of flexible fuzzy queries [4] which were meant to make querying more human consistent via including linguistic terms as the first class citizens of the query syntax and modeling them using elements of fuzzy logic. This is often exemplified with a reference to the realm of a real-estate agency database (offer) querying and we will keep the tradition of using this setting here to illustrate the concepts of the known and newly proposed approaches. Thus, using a flexible fuzzy query a customer of a real-estate agency may, for example, state that he or she is looking for a *cheap* house or apartment. This is usually much more natural and convenient than setting a precise threshold on the acceptable (intended, preferable) price of an apartment sought.

A further step towards a better representation of the user preferences and intentions may be *bipolar queries.* The concept has been originally introduced by Dubois and Prade [5] and then further developed by many authors including Dubois and Prade themselves [6, 7, 8, 9], Kacprzyk and Zadrożny [10, 11, 12, 13], De Tré and his team [14, 15, 16], Bosc and Pivert [17, 18, 19], Lietard with collaborators [20, 21],etc. The idea behind this concept is to consider independently the negative and positive aspects of (conditions on) the data sought. In the most general case, cf. Zadrożny and Kacprzyk [11] for a recent comprehensive exposition, both negative and positive evaluations of data are to be treated separately and with the equal importance. However, in the literature the most popular approach to bipolar queries consists in a specific interpretation of these conditions as, respectively, a *constraint* and a *wish.* Thus, the primary role is played by the (complement of) the negative evaluation and the satisfaction of positive condition is somehow secondary. In our approach to such a constraint and wish interpretation of bipolar queries we have advocated the *required/desired semantics* [10, 11] which is meant as a fuzzy extension of the seminal work of Lacroix and Lavency [22]. It is based on the combination of negative and positive evaluations via the *and possibly* operator which admits (complement of) the negative condition somehow more important but with respect to the *context* of the whole database. In [1] we proposed an important extension of this approach which relates the importance of both conditions to a *local context* of a given tuple under evaluation. We have shown many possible interpretations of such a context and their practical relevance.

In this paper, we propose another approach to combination of the negative and positive conditions which promotes the latter but, still, with respect to, either, the whole database or a local context of a tuple under consideration. The proposed approach is inspired by our previous work on the *and possibly* operator and on the concept of the *or else* operator introduced by Lietard et al. [20]; cf. also [9, 18].

The structure of the paper is as follows. In the next section we briefly remind the essence of the the concept of a bipolar query and its contextual version. Next section contains the main contribution of this paper, i.e. the introduction of our version of the *or if impossible* operator and based on it a new way of bipolar queries aggregation as well as an extension of the concept of contextual bipolar queries. We provide a preliminary study of the properties of introduced queries and illustrate them on examples. We conclude summarizing the content of the paper

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and planning for the further research.

2. Bipolar queries and their contextual version

2.1. Motivation

Some studies [6] as well as the real-life experience show that the human judgment is usually based on the evaluation of the *positive* and *negative* sides of alternatives under consideration. These two separate evaluations may be internally combined before they are expressed but often they are present in the mind of a user, decision maker etc. at the beginning of the judgment process. Thus, it may be worthwhile to make it possible for a user querying a database to express his requirements as to the data sought in line with this observation. Namely, it may be convenient for the user to specify separately a list of features making data attractive and another list of features making data unwanted. Each tuple (we assume here the relational data model terminology) is evaluated against these two conditions and its overall matching is expressed by two numbers. This is applicable already in the crisp context but is especially interesting if the conditions involve fuzzy predicates. In the latter case, we have two matching degrees of a tuple, against the positive and negative condition respectively. These matching degrees may be used to order tuples in the response to a query, e.g., using the lexicographic order taking into account first one of the matching degrees and then the other one. Another approach is to aggregate both matching degrees to obtain one overall matching degree. We are following the latter approach here and adopt a specific aggregation scheme described in the next subsection.

In what follows, we assume query conditions in a simple, practical form of Boolean expressions involving the attributes of a table schema, comparison operators and logical connectives.

2.2. Bipolar queries with the "and possibly" operator

We identify the bipolar query with a pair of conditions, denoted (C, P), where C corresponds to the negative condition (precisely speaking to its complement) while P denotes the positive condition. Both conditions are assumed to be fuzzy in general, i.e., they involve fuzzy predicates referring to the attributes of data under consideration. For example, "price IS low" may form a part of a positive condition, where "price" refers to an attribute while "low" is a *linguistic term* whose meaning is equated with a fuzzy set appropriately defined in the domain of the attribute.

Thus, conditions are satisfied to a degree, a number from the interval $[0, 1]$.

As mentioned earlier, we follow an approach in which matching degrees of both conditions C and P are explicitly aggregated. The aggregation scheme adopted is based on a special interpretation of the conditions. Namely, satisfying the condition C is *required* while satisfying P is merely *desired*. The semantics of this aggregation scheme may be informally represented by the usage of the "and possibly" operator as in, e.g., a hotel accommodation query:

`Price` is *low* <u>and possibly</u> `location` is *close* to the railway station (1)

where linguistic terms are italicized while attributes are shown in the typewriter style font. Thus, the accommodation sought is required to be cheap and – but only *if possible* – is desired to be conveniently located. The "possibility" referred to in this interpretation should be understood as follows: the possibility of satisfying both conditions is admitted if and only if there is in the catalog (database) a hotel room which is both cheap and conveniently located. Then, the rooms of the interest for the user have to satisfy both conditions and the query (1) reduces to the classical conjunction:

`Price` is *low* <u>and</u> `location` is *close* to the railway station

On the other hand, if there is no such a room in the whole database then it is *not possible* to satisfy both conditions as they interfere with each other and the desired condition is dropped leading to the query:

`Price` is *low*

The above description is accurate only for crisp conditions C and P and the fuzzy case calls for a special treatment which has been proposed in [10] and may be formally stated as follows. First, the generic form of bipolar query, under the required/desired semantics considered here, is:

$$C \text{ and possibly } P \tag{2}$$

Next, in the crisp case it is interpreted, following the seminal work of Lacroix and Lavency [22], as the following logical formula:

$$C(t) \text{ and possibly } P(t) \equiv C(t) \wedge (\exists s (C(s) \wedge P(s)) \Rightarrow P(t)) \tag{3}$$

which may be adopted in the fuzzy case and interpreted as [23, 10, 11]:

$$\text{truth}(C(t) \text{ and possibly } P(t)) = \min(\mu_C(t), \max(1 - \max_{s \in R} \min(\mu_C(s), \mu_P(s)), \mu_P(t))) \tag{4}$$

where R denotes the whole dataset (relation) queried. The value of

$$\max_{s \in R} \min(\mu_C(s), \mu_P(s)), \tag{5}$$

which, for notational convenience, is denoted as $\exists CP$, expresses the truth of $\exists s(C(s) \wedge P(s))$ and may be treated as a measure of the interference of P with C: the lower it is the more difficult are both conditions to be satisfied simultaneously. It should be stressed that (3) may be translated into (4) in many different ways employing various sets of fuzzy logical connectives. Above, the standard minimum and maximum operators as well as the related S-implication operator are employed due to their distributivity and idempotency. We will use them also throughout the paper leaving the analysis of the behavior of the newly introduced concepts under other sets of connectives for the future research and referring interested reader to e.g., [11] for such an analysis concerning the "and possibly" operator.

2.3. Contextual bipolar queries with the "and possibly" operator

Bipolar query "C and possibly P", as discussed above, is satisfied (to a high degree) by a tuple t only if either of the two conditions holds:

1. it satisfies (to a high degree) both conditions C and P, or
2. it satisfies C and there is no tuple in the whole database which satisfies both conditions. (6)

In [1] we introduced the concept of *contextual bipolar queries* which is based on a modified version of the "and possibly" operator. The generic form of the contextual bipolar query is as follows:

$$C \text{ and possibly } P \text{ with respect to } W \tag{7}$$

where predicates C and P should be interpreted, as previously, as representing the required and desired conditions, respectively. The predicate W denotes the context of a tuple t for which the query is evaluated, i.e., a part of the database somehow, via W, related to t:

$$Context(t) = \{s \in R : W(t,s)\} \tag{8}$$

where R, as previously, denotes the whole database.

The formula (7) is interpreted as:

$$\begin{aligned} &C(t) \text{ and possibly } P(t) \text{ with respect to } W \equiv \\ &\quad C(t) \wedge (\exists s(W(t,s) \wedge C(s) \wedge P(s)) \Rightarrow P(t)) \end{aligned} \tag{9}$$

The applicability of such a contextual bipolar query may be illustrated with the following example. We plan to visit all capitals of the US states and we are looking for a hotel in each one. We would like a moderately priced room and, if possible *in the neighborhood*, at one of our preferred hotel chains. Then, the contextual bipolar query yielding a list of preferred hotels may be expressed as:

Find a *moderately priced* hotel room in each US state capital city which, if *in its neighbourhood* there exists a hotel of our *preferred chain* offering such rooms, is in such a hotel. (10)

This query matches the generic form (9) with the following instantiation of the predicates therein:

- C is a conjunction of two predicates: *moderately priced*, represented by a fuzzy set properly defined in the domain of the `price` attribute, and another predicate corresponding to the location of the hotel in one of the cities of interest;
- P is a predicate defining the set of preferred hotel chains; e.g., represented by the following membership function:

$$\begin{aligned} &\mu_P(\text{Sheraton}) = 1.0, \\ &\mu_P(\text{Holiday Inn}) = 0.9, \\ &\mu_P(\text{Hilton}) = 0.8; \end{aligned} \tag{11}$$

- W is a predicate defining the neighborhood of a hotel which may be defined, e.g., in terms of the distance d in kilometers between the hotels represented by tuples t and s as follows:

$$\mu_W(t,s) = \begin{cases} 1 & \textit{if } d < 1.5 \\ \frac{2}{3}d - 1 & \textit{if } 1.5 \le d \le 3 \\ 0 & \textit{if } d > 3 \end{cases}$$

In [1] we study several possible interpretations of the context in the contextual bipolar query. We will remind some of them while introducing a new type of contextual bipolar queries in the next section.

3. Bipolar queries with the "or if impossible" operator

3.1. The concept

The "and possibly" operator was introduced in order to combine the matching degrees of the positive and negative conditions present within a bipolar query. As discussed earlier, the negative condition, or more precisely its complement, is represented by the condition C while the positive condition is directly represented by the condition P in (2).

The combination of the positive and negative conditions using the "and possibly" operator favors in a sense the negative condition, represented in (2) by its complement, C. This may be best demonstrated by looking at the value of the $\exists CP$ indicator defined in (5). Namely, only for $\exists CP = 0$ both conditions are equally important and then the expression "C and possibly P" boils down to the classical conjunction $C \wedge P$. For all other values of $\exists CP \in [0,1)$ the positive condition is considered as less important in the combination. This is true for any interpretation of fuzzy implication appearing in (3) via an operator which is monotonically non-increasing in the first argument, what is usually assumed [24]. In fact, as shown by Yager [25, 26] (cf. also [23]), the expression "C and possibly P" may be seen as a weighted aggregation of conditions C and P where the weight of C is equal 1 and the weight of P is "dynamic", depending on the content of database, equal to $\exists CP$, which belongs to the interval $[0, 1]$.

Here we propose another operator which is also dependent on the content of the database but favors the positive condition P:

$$C \textit{ or if impossible } P \tag{12}$$

The use of such an operator may be motivated with the following example of a query involving bipolarity of preferences (the structure of the conditions will be simplified to better show the intended semantics). Let us assume that we are looking for a hotel in a given city (e.g., San Francisco) and a positive, appreciated feature of the hotel is its belongingness to one of the *preferred hotel chains* while the negative feature, which we would like to avoid, is location in an *unsafe* area of the city. Both aspects have a gradual nature and may be conveniently represented by fuzzy sets in the domains of the attributes `chain` and `location`, respectively. The concept of the "preferred hotel chain" may be represented as in (11) and the concept of the "unsafe area" of a given city may be defined similarly via a fuzzy set of neighbourhoods such as Castro, Fisherman's Wharf, Mission Dolores etc., where the membership degree represents a (subjective) assessment of how unsafe it is. Then, the query:

`Chain` is among *preferred* chains
or if impossible `location` is *safe* (13)

Thus, referring to (12), the condition P corresponds now to belonging to a preferred hotel chain (to a degree) while the condition C corresponds to the hotel location safety status and is represented by a fuzzy set being a complement of the fuzzy set defined in the space of San Francisco's neighborhoods, mentioned above. A tuple (hotel) satisfies such a bipolar query (to a high degree) if either of the two conditions holds:

1. it satisfies (to a high degree) condition P, i.e., belongs to a preferred chain, or
2. it satisfies C, i.e., is located in a safe area, and there is no tuple in the whole database which satisfies condition P. (14)

The semantics of the "or if impossible" operator may be formally expressed as follows:

$$P(t) \text{ or if impossible } C(t) \equiv P(t) \vee (\neg \exists s P(s) \wedge C(t)) \tag{15}$$

which may be adopted in the fuzzy case and interpreted as:

$$\text{truth}(P(t) \text{ or if impossible } C(t)) = \max(\mu_P(t), \min(1 - \max_{s \in R} \mu_P(s), \mu_C(t))) \tag{16}$$

The value of

$$\max_{s \in R} P(s), \text{ denoted as } \exists P \tag{17}$$

which expresses the truth of $\exists s P(s)$, may be treated as a measure of the possibility to satisfy the condition P: the lower it is the more difficult to satisfy this condition it is.

Thus, we obtain a different way of combining positive and negative conditions in the framework of the bipolar query. The former way, based on the "and possibly" operator may be characterized as appropriate for the user who is primarily concerned with avoiding the choice of an alternative featuring negative traits. The new way, proposed here, is more appropriate for the user more concerned with the positive traits of the chosen alternative.

It should be noticed that we do not assume any special relation between P and C, e.g., the subsumption (or set inclusion when treating the predicates extensionally) which is sometimes assumed in the literature, in particular in case of the "and possibly" operator (or its counterparts). Namely, sometimes it is argued [9] that if something is desired (P) it has to be first not rejected (C), and thus $P \subseteq C$. However, the user may not be aware of the "conflict" between his or her desires and constraints. Then, if such a conflict exists it is suggested to replace P with $P \cap C$. We do not follow this strategy and leave full freedom for the user to express his or her preferences. In case of the "or if impossible" operator it is suggested that changing C with $C \cup P$ should not change the meaning of the query. It is true as long as the fuzzy logical connectives are represented by the idempotent and distributive operators.

3.2. Related work

As mentioned in section 1, the concept of bipolar query has been introduced by Dubois and Prade [5]. In this original approach the resulting couple of evaluation degrees, against the positive (P) and the complement of the negative (C) condition is made operational via the usage of lexicographic order. Namely, the tuples are ordered in the response to a bipolar queries first according to the degree of matching of C and only in case of the draws the matching degree of P intervenes. This line of reasoning has been further developed by Dubois and Prade, also following the work of Lietard et al. [20] or Bosc and Pivert [18]. In particular, in [9] an interesting discussion of the "and if possible" and "or at least" operators, introduced in [20] (the latter under the name "or else"), which are somehow related to our "and possibly" and "or if impossible" operators. Namely, analysis of both operators as, respectively, weighted conjunction and weighted disjunctions is provided and many interesting properties are shown which will be of use in what follows.

It is worth stressing the differences between these two pairs of operators: "(and if possible, or else)" introduced by Lietard et al. [20] and our "(and possibly, or if impossible)". The most important is the truth-functionality of the former operators what is not the case of our operators. Another, more subtle, difference consists in the interpretation of the role of conditions C and P. In our approach, "C and possibly P" is meant to be satisfied as defined in (6). In particular, if there exists in the database a tuple satisfying both conditions C and P then a tuple satisfying just C is rejected – its matching degree is equal to 0. On the other hand, in the another approach it is clearly stated [9] that the satisfaction of C is sufficient to accept a tuple and a possible satisfaction of P makes it only "better". This another semantics is also stated as "C or even better $C \wedge P$" [18].

Moreover, as already mentioned earlier, in cases of both operators, it is assumed in [9] that conditions C and P (using our notation) are in such a relation that in case of the "and if possible" operator condition P may be equivalently replaced with $P \wedge C$ while in case of the "or at least" C may be replaced with $C \vee P$. While this is the case also for our operators when conditions are crisp it is not anymore so for fuzzy setting, in general.

Bosc and Pivert [18] propose some properties which should be satisfied by the operators "and if possible" and "or else". They propose their own definitions for these operators and show they possess the postulated properties. Their definitions of these operators are as follows (using our notation):

$$\text{truth}(C(t) \text{ and if possible } P(t)) = \min(\mu_C(t), k\mu_C(t) + (1-k)\mu_P(t)) \quad (18)$$

$$\text{truth}(P(t) \text{ or else } C(t)) = \max(\mu_P(t), k\mu_P(t) + (1-k)\mu_C(t)) \quad (19)$$

They also show that their proposed operators are related to each other by the De Morgan laws like the classical conjunction and disjunction, i.e.,

$$\neg(\neg C(t) \text{ and if possible } \neg P(t)) \equiv C(t) \text{ or else } P(t) \quad (20)$$

Interestingly enough, Bosc and Pivert show that the "and possibly" operator defined by (4) also enjoys the postulated properties. Moreover, assuming the De Morgan laws they also derive the "or else" operator corresponding to (4) obtaining the following formula:

$$\text{truth}(P(t) \text{ or else } C(t)) = \max(\mu_P(t), \min(\max_{s \in R} \min(\mu_C(s), \mu_P(s)), \mu_C(t)))$$

Thus the obtained "or else" operator is different from our "or if impossible" operator defined by (16). Moreover, our operators "and possibly" and "or if impossible" are not related by the De Morgan laws. On the other hand, it may be easily shown that for the representation of the conjunction and disjunction by, respectively the minimum and maximum operators, the following relation between our "and possibly" (to be denoted as: $\wedge_p$) and "or if impossible" (to be denoted as: $\vee_i$) operators holds [9]:

$$C(t) \wedge_p P(t) \equiv (C(t) \wedge P(t)) \vee_i C(t) \quad (21)$$

$$P(t) \vee_i C(t) \equiv (P(t) \vee C(t)) \wedge_p P(t) \quad (22)$$

It has to be stressed that for other interpretations of fuzzy logical connectives some properties may not hold anymore due to the lack of the idempotence and/or distributivity of other t-norm/t-conorms.

In our earlier works [10, 11] we proposed a fuzzy version of the *winnow* operator introduced by Chomicki [27]. This is a unary operator of the relational algebra which selects from a set of tuples R those which are *non-dominated* with respect to a given *preference relation* D, $D \subseteq R \times R$. If two tuples $t, s \in R$ are in relation D, i.e., $D(t, s)$, then it is said that the tuple t *dominates* the tuple s with respect to the relation D. Then the *winnow* operator ω_D is defined as follows

$$\omega_D(R) = \{t \in R : \neg\exists_{s \in R}\ D(s,t)\} \quad (23)$$

Thus, for a given set of tuples it yields a subset of the *non-dominated* tuples with respect to D. Queries employing the *winnow* operator are called *queries with preferences*. Chomicki [27], considering only the crisp case, shows that our bipolar query with the "and possibly" operator (3) is a special case of a query with preferences. Namely, for a given bipolar query (C, P) let us define the following preference relation D

$$D(t,s) \Leftrightarrow P(t) \wedge \neg P(s) \quad (24)$$

Then the bipolar query "C and possibly P" may be expressed as the combination of the classical selection of the relational algebra $\sigma_C(R)$ and the fuzzy *winnow* operators: $\omega_D(\sigma_C(R))$. In [10, 11] we show its fuzzy counterpart.

Thanks to the property (22) we can immediately represent also the bipolar query with the "or if impossible" operator as the following combination of the selection and *winnow* operator:

$$\omega_D(\sigma_{C \vee P}(R))$$

with exactly the same preference relation (24) as in case of the bipolar query with the "and possibly" operator.

4. Contextual bipolar queries with the "or if impossible" operator

The concept of bipolar query "P or if impossible C", introduced in the previous section, is based on the notion of "possibility" to satisfy the condition P. This possibility is considered in the context of the current instance of a database. After its degree is settled as $\exists P$ (17) then the bipolar query $C \vee_i P$ turns into weighted disjunction of conditions C and P [28] as shown in the right hand side of equation (16), i.e.,

$$\max(\mu_P(t), \min(1 - \exists P, \mu_C(t))) \tag{25}$$

This approach, with a slightly different agenda, is discussed already in, e.g., [9, 18]. Things are getting more interesting if the possibility of satisfying P is considered *locally*, in the *context* of a given tuple. Thus, we follow the path similar to the case of the contextual bipolar queries with the "and possibly" operator.

We start with the observation that the bipolar query $P \vee_i C$ is satisfied (to a high degree) by a tuple t only if either of the two conditions holds:

1. it satisfies (to a high degree) condition P, or
2. it satisfies C and there is no tuple in the whole database which satisfies the condition P. (26)

Then, the idea is to relate the possibility of satisfying P to some context W of a tuple t rather than to the whole database. The generic form of the contextual bipolar query with the "or if impossible" operator is as follows:

$$P \text{ and if impossible } C \text{ with respect to } W \tag{27}$$

where predicates P and C are meant as previously, as representing the positive condition and the complement of the negative condition, respectively. The predicate W denotes as previously the context of a tuple t with respect to which the impossibility of satisfying P is considered:

$$Context(t) = \{s \in R : W(t, s)\} \tag{28}$$

where R denotes the whole database.

The formula (27) is interpreted as:

$$P(t) \text{ or if impossible } C(t) \text{ with respect to } W \equiv \\ P(t) \vee (\neg\exists s(W(t, s) \wedge P(s)) \wedge C(t)) \tag{29}$$

Let us illustrate such a contextual bipolar query with an example similar to the one used in section 3.1. We plan to visit all capitals of the US states and we are looking for a hotel in each one. We would like to stay in each city at one of the preferred hotel chains (as, e.g., we have a discount at them) but if it is *impossible in a given city* as there is no hotel belonging to any of preferred chains there, then we would like to get a moderately priced room. Then, the contextual bipolar query yielding a list of preferred hotels may be expressed as:

> Find a hotel room in each US state capital at a hotel of our *preferred chain* but if such does not exist there then find a *moderately priced* hotel. (30)

Let us notice that the above query (30) may be derived from the same bipolar query which may be assumed behind the query (10) in section 3.1. Namely, this common bipolar query is composed of a *positive* condition (a room at the hotel belonging to one of preferred chains we find as offering a good service for a good price) and a *negative* condition represented by its complement (moderately priced: as we would like not to spend too much and are also afraid that a low price means low quality of service). In (10) we treat the complement of the negative condition as a *constraint* but we also insist on satisfying the positive condition *if possible in a given context*: we reject a moderately priced hotel not belonging to one of the preferred chains if in its neighbourhood there is a hotel satisfying both our conditions. The results returned by the query (10) meet this conditions. On the other hand, in (30) the satisfaction of the positive condition, i.e., belonging to one of the preferred chains, is enough but if it is impossible *in a given context*, i.e., in a given city, then we want the complement of negative condition to be satisfied, at least.

The query (30) matches the generic form of the contextual bipolar query with the "or if impossible operator" (27) with the following instantiation of the predicates therein:

- P is a conjunction of two predicates: one defining the set of preferred hotel chains; e.g., given by (11), and another predicate corresponding to the location of the hotel in one of the cities of interest, i.e., capitals of US states;
- C is a predicate corresponding to the concept of the *moderately priced* room, represented by a fuzzy set appropriately defined in the domain of the `price` attribute, ;
- W is a predicate defining the equivalence relation on the set of the hotels, such that all hotels located in the same capital city belong to the same class, i.e., :

$$\mu_W(t, s) = \begin{cases} 1 & \texttt{t.location=s.location} \\ 0 & \text{otherwise} \end{cases}$$

The context relation W is in example (30) a crisp relation but in general it may be fuzzy.

In [1] we proposed various possible interpretations of the context relation W for the contextual bipolar queries with the "and possibly" operator. These are applicable also in the context of the new type of contextual bipolar queries proposed here. We will briefly remind these interpretations, adopting them for the new setting, when necessary.

The relation W used in (30) is an example of the (crisp) *equivalence relation*, partitioning the set of objects under consideration (here: the hotels in US state capitals). Such a grouping/partitioning may be in general fuzzy as, e.g., when the hotel location is identified with some region lacking a clear-cut definition (e.g., Central Europe). Then an object may be seen as belonging to a partition to a degree, and possibly to many partitions at the same time.

Let us consider another example of a query.

> We are interested in *large* rooms however rooms which are not large but offer the *cheap internet access* are also of interest provided there are no large rooms among the the rooms which are *not much more expensive* than a given room.

Thus, of interest is now a room which:

1. is large (to a high degree), or
2. offers cheap Internet access and there is no another room not much more expensive which is large.

This query also matches the scheme (27) with the following instantiation of the predicates therein:

- P is a predicate defining the concept of the large room;
- C is a predicate defining the concept of the cheap internet access;
- W is a predicate defining order on the hotel rooms such as, e.g.,:

$$\mu_W(t,s) = \begin{cases} 1 & \texttt{(t.price - s.price) > 0.5s.price} \\ 0 & \text{otherwise} \end{cases}$$

5. Concluding remarks

We propose an extension to the earlier introduced by us concept of a contextual bipolar query. Our approach is relevant for the general concept of the bipolar query which is meant as a pair the of positive and negative conditions. We focus on a special approach which consists in an explicit aggregation of the matching degrees of both conditions. We discuss the use of the "or if impossible" aggregation operator which may help to easier express the preferences of a user, together with the earlier introduced "and possibly" operator. The aggregation scheme employed takes into account the possibility of matching a single condition or a conjunction of them. We have proposed both a global approach, where this possibility is referred to the whole database, and a local approach which uses a context of a given tuple for that purpose.

Further research is needed to study the properties of the proposed operators in a more detail. Here we mention some properties but they may be proved for the standard modelling of fuzzy logical connectives of conjunction and disjunction via the minimum and maximum operators, respectively. Some preliminary work on the implementation of the proposed approach has been done but further efforts are required also in this direction. We plan also to study the relevance of the proposed approach in a real setting of the users querying a database.

Possible applications of the considered operators are multitude. Some of them has been already explored in [12, 13] and [2] but this is just the beginning.

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