Interpretation in the Intuitionistic Fuzzy Triangle of the Results, Obtained by the InterCriteria Analysis

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Abstract

The present research is a consequent step in an ongoing research of a novel approach for decision support, called *InterCriteria Analysis*, which aims at identification of specific correlations between criteria in a decision making processes, using the concepts of intuitionistic fuzziness and index matrices. The step made here is not a gradual improvement of previous results, but a new way of reading them. It is shown how the results produced by the InterCriteria Analysis approach can be interpreted within the specific triangular geometrical interpretation of IFSs, thus allowing us to order these results according simultaneously to the membership and the non-membership component of the intuitionistic fuzzy pairs.

Keywords: InterCriteria Analysis, Intuitionistic fuzziness, Index matrix, Decision support.

1. Introduction

The results in this work are continuation of a recently started research in the field of intuitionistic fuzzy sets (IFSs) based decision support approach, titled *InterCriteria Analysis* (ICA). The approach is specifically applicable to situations where some of the criteria in the decision making process come at a higher cost than others, for instance are harder, more expensive, more human resource or time consuming to measure or evaluate. Such criteria have been considered unfavourable in ICA, hence if the method identifies certain, high enough, level of correlation between such unfavourable criteria and others that are easier, cheaper or quicker to measure or evaluate, these might be disregarded in the further decision making process.

The approach employs two fundamental concepts: intuitionistic fuzzy sets [1, 2, 5, 6] and index matrices (IMs) [4, 7].

Here we will briefly repeat the theoretical framework of the proposed approach, firstly proposed in [8], by slightly improving the notation from [9]. The approach employs an index matrix M of m rows $\{O_1, \ldots, O_m\}$ and n columns $\{C_1, \ldots, C_n\}$, where for every p, q ($1 \leq p \leq m$, $1 \leq q \leq n$), O_p in an evaluated object, C_q is a evaluation criterion, and $e_{O_pC_q}$ is the evaluation of the p-th object against the q-th criterion, defined as a real number or another object that is comparable according to relation R with all the rest elements of the index matrix M.

$$M = \begin{array}{c|ccccccc}
 & C_1 & \cdots & C_k & \cdots & C_l & \cdots & C_n \\
\hline
O_1 & e_{O_1,C_1} & \cdots & e_{O_1,C_k} & \cdots & e_{O_1,C_l} & \cdots & e_{O_1,C_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
O_i & e_{O_i,C_1} & \cdots & e_{O_i,C_k} & \cdots & e_{O_i,C_l} & \cdots & e_{O_i,C_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
O_j & e_{O_j,C_1} & \cdots & e_{O_j,C_k} & \cdots & e_{O_j,C_l} & \cdots & e_{O_j,C_n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
O_m & e_{O_m,C_1} & \cdots & e_{O_m,C_j} & \cdots & e_{O_m,C_l} & \cdots & e_{O_m,C_n}
\end{array},$$

From the requirement for comparability above, it follows that for each i, j, k it holds the relation $R(e_{O_iC_k}, e_{O_jC_k})$. The relation R has dual relation $\overline{R}$, which is true in the cases when relation R is false, and vice versa.

For the needs of our decision making method, pairwise comparisons between every two different criteria are made along all evaluated objects. During the comparison, it is maintained one counter of the number of times when the relation R holds, and another counter for the dual relation.

Let $S^{\mu}_{k,l}$ be the number of cases in which the relations $R(e_{O_iC_k}, e_{O_jC_k})$ and $R(e_{O_iC_l}, e_{O_jC_l})$ are simultaneously satisfied. Let also $S^{\nu}_{k,l}$ be the number of cases in which the relations $R(e_{O_iC_k}, e_{O_jC_k})$ and its dual $\overline{R}(e_{O_iC_l}, e_{O_jC_l})$ are simultaneously satisfied. As the total number of pairwise comparisons between the object is $m(m-1)/2$, it is seen that there hold the inequalities:

$$0 \leq S^{\mu}_{k,l} + S^{\nu}_{k,l} \leq \frac{m(m-1)}{2}.$$

For every k, l, such that $1 \leq k \leq l \leq m$, and for $m \geq 2$ two numbers are defined:

$$\mu_{C_k,C_l} = 2\frac{S^{\mu}_{k,l}}{m(m-1)},\ \nu_{C_k,C_l} = 2\frac{S^{\nu}_{k,l}}{m(m-1)}.$$

The pair, constructed from these two numbers, plays the role of the intuitionistic fuzzy evaluation of the relations that can be established between any two criteria C_k and C_l. In this way the index matrix M that relates evaluated objects with evaluating criteria can be transformed to another index matrix M^* that gives the relations among the criteria:

$$
M^* = \begin{array}{c|ccc}
 & C_1 & \dots & C_n \\
\hline
C_1 & \langle \mu_{C_1,C_1}, \nu_{C_1,C_1} \rangle & \dots & \langle \mu_{C_1,C_n}, \nu_{C_1,C_n} \rangle \\
\vdots & \vdots & \ddots & \vdots \\
C_n & \langle \mu_{C_n,C_1}, \nu_{C_n,C_1} \rangle & \dots & \langle \mu_{C_n,C_n}, \nu_{C_n,C_n} \rangle
\end{array}.
$$

From practical considerations, it has been more flexible to work with two index matrices M^μ and M^ν, rather than with the index matrix M^* of IF pairs.

The final step of the algorithm is to determine the degrees of correlation between the criteria, depending on the user's choice of μ and ν. We call these correlations between the criteria: 'positive consonance', 'negative consonance' or 'dissonance'. Let $\alpha, \beta \in [0; 1]$ be the threshold values, against which we compare the values of μ_{C_k,C_l} and ν_{C_k,C_l}. We call that criteria C_k and C_l are in:

- (α, β)-positive consonance, if $\mu_{C_k,C_l} > \alpha$ and $\nu_{C_k,C_l} < \beta$;
- (α, β)-negative consonance, if $\mu_{C_k,C_l} < \beta$ and $\nu_{C_k,C_l} > \alpha$;
- (α, β)-dissonance, otherwise.

We will note that so far several consequent steps have been made in the development of the ICA method, mainly related to determining the threshold values α, β. The progress in this leg of our research has been described in details consequently in [9, 10, 11, 12]. In all these parts of the research, we have illustrated the approach with data from the World Economic Forum's annual Global Competitiveness Reports (GCRs), for the 28 EU member states from years 2008–2009 to 2013–2014, taking as a motivation the WEF's general address to policy makers to '*identify and strengthen the transformative forces that will drive future economic growth*' [13].

We will immediately note, however, that the conscious and pragmatic choice at these initial steps of research to distribute the produced results in two index matrices M^μ and M^ν, rather than work with an IM of IF pairs M^*, lead us for a long time to working with these values separately, giving priority to one of components, usually the membership one, while here for the first time we show how to handle both components simultaneously, and expect to have this new consideration leading to a significant improvement of the results of the ICA approach.

This new aspect of the research on ICA came with the realization that we can handle both components of the IF pair, if we interpret them as point in the specific triangular geometrical interpretation of IFS.

2. Introducing the Intuitionistic Fuzzy Triangular Geometrical Interpretation

The basic idea of ICA is related to analysing the evaluations of a set of objects against a set of criteria, aiming at the discovery of certain correlations between the criteria themselves in terms of IF pairs. The final result of the application of the approach is an $n \times n$ index matrix M^* of IF pairs, or, alternatively, a pair of $n \times n$ index matrices M^μ and M^ν for the membership and non-membership parts of the IF pairs, respectively. The IM M^* is particularly notable for having the truth $\langle 1, 0 \rangle$ along the main diagonal and $\langle \mu_{C_k,C_l}, \nu_{C_k,C_l} \rangle = \langle \mu_{C_l,C_k}, \nu_{C_l,C_k} \rangle$, making it interchangeable whether we prefer to work with the triangle above or below the main diagonal.

This simple observation now naturally brings about the idea that the IF pairs in the IM M^* can be treated as coordinates of points in the plane, and plotted respectively over the IF triangle first proposed in 1989 by Atanassov [3], with coordinates (0; 0) for the complete uncertainty, (1; 0) for the truth, and (0; 1) for the falsity.

Let us here again illustrate the proposed idea with our results from the application of ICA approach over data from WEF's annual GCRs, for the 28 EU member states from year 2008–2009 to year 2013–2014. Everywhere below the labels from '1.' to '12.' have the meaning of the 12 criteria in the WEF's methodology [13], called 'pillars of competitiveness', where: '1. Institutions'; '2. Infrastructure'; '3. Macroeconomic stability'; '4. Health and primary education'; '5. Higher education and training'; '6. Goods market efficiency'; '7. Labor market efficiency'; '8. Financial market sophistication'; '9. Technological readiness'; '10. Market size'; '11. Business sophistication'; '12. Innovation'.

M^μ	1	2	3	4	5	6	7	8	9	10	11	12
1		0.84	0.69	0.76	0.79	0.83	0.6	0.83	0.82	0.5	0.79	0.8
2	0.84		0.63	0.75	0.75	0.74	0.53	0.74	0.78	0.58	0.83	0.81
3	0.69	0.63		0.62	0.64	0.66	0.65	0.65	0.69	0.43	0.65	0.67
4	0.76	0.75	0.62		0.78	0.72	0.55	0.7	0.72	0.52	0.76	0.77
5	0.79	0.75	0.64	0.78		0.75	0.62	0.73	0.76	0.56	0.77	0.8
6	0.83	0.74	0.66	0.72	0.75		0.63	0.82	0.8	0.51	0.79	0.76
7	0.6	0.53	0.65	0.55	0.62	0.63		0.66	0.61	0.39	0.56	0.59
8	0.83	0.74	0.65	0.7	0.73	0.82	0.66		0.82	0.48	0.73	0.75
9	0.82	0.78	0.69	0.72	0.76	0.8	0.61	0.82		0.55	0.82	0.81
10	0.5	0.58	0.43	0.52	0.56	0.51	0.39	0.48	0.55		0.65	0.6
11	0.79	0.83	0.65	0.76	0.77	0.79	0.56	0.73	0.82	0.65		0.86
12	0.8	0.81	0.67	0.77	0.8	0.76	0.59	0.75	0.81	0.6	0.86	

Table 1. Discovered membership values with the application of ICA for the year 2008–2009, where here and in Table 3 below the darker shades the higher degree of membership

M^ν	1	2	3	4	5	6	7	8	9	10	11	12
1		0.11	0.24	0.14	0.14	0.08	0.28	0.12	0.12	0.46	0.15	0.13
2	0.11		0.3	0.16	0.19	0.17	0.37	0.22	0.18	0.38	0.13	0.14
3	0.24	0.3		0.26	0.26	0.21	0.2	0.27	0.22	0.49	0.27	0.24
4	0.14	0.16	0.26		0.11	0.14	0.29	0.2	0.17	0.38	0.14	0.11
5	0.14	0.19	0.26	0.11		0.13	0.23	0.2	0.16	0.38	0.16	0.13
6	0.08	0.17	0.21	0.14	0.13		0.21	0.09	0.1	0.4	0.11	0.13
7	0.28	0.37	0.2	0.29	0.23	0.21		0.21	0.26	0.5	0.31	0.26
8	0.12	0.22	0.27	0.2	0.2	0.09	0.21		0.13	0.48	0.22	0.2
9	0.12	0.18	0.22	0.17	0.16	0.1	0.26	0.13		0.4	0.12	0.12
10	0.46	0.38	0.49	0.38	0.38	0.4	0.5	0.48	0.4		0.31	0.34
11	0.15	0.13	0.27	0.14	0.16	0.11	0.31	0.22	0.12	0.31		0.08
12	0.13	0.14	0.24	0.11	0.13	0.13	0.26	0.2	0.12	0.34	0.08	

Table 2. Discovered non-membership values with the application of ICA for the year 2008–2009, where here and in Table 4 the darker shades the higher degree of non-membership.

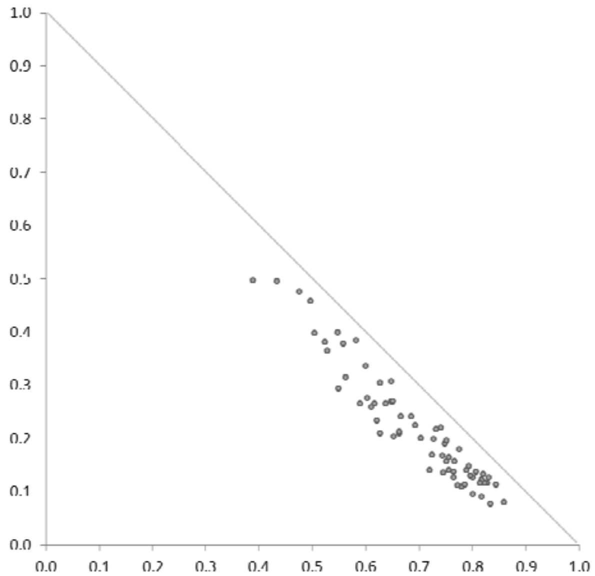

Figure 1. The IF data from Tables 1 and 2 (year 2008–2009), plotted as coordinates of points on the IF triangle.

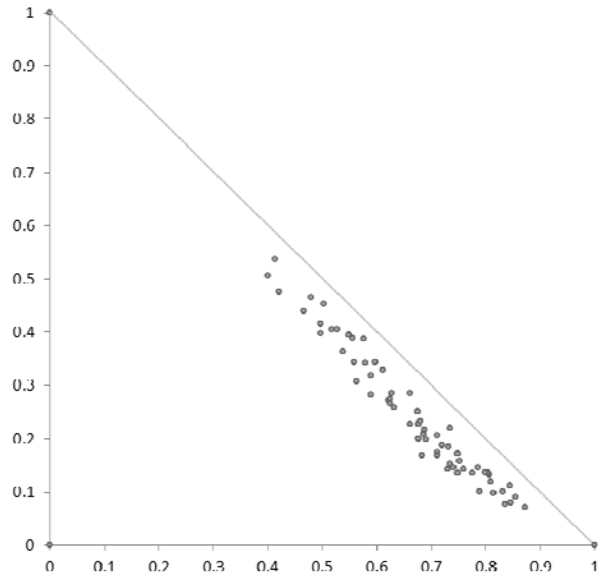

Figure 2. The IF data from Tables 3 and 4 (year 2013–2014), plotted as coordinates of points on the IF triangle.

M^μ	1	2	3	4	5	6	7	8	9	10	11	12
1		0.74	0.58	0.72	0.81	0.84	0.73	0.75	0.85	0.5	0.8	0.84
2	0.74		0.48	0.66	0.75	0.68	0.54	0.59	0.79	0.66	0.8	0.8
3	0.58	0.48		0.42	0.52	0.56	0.63	0.67	0.55	0.41	0.55	0.56
4	0.72	0.66	0.42		0.73	0.68	0.59	0.56	0.68	0.5	0.71	0.69
5	0.81	0.75	0.52	0.73		0.74	0.62	0.63	0.78	0.58	0.81	0.85
6	0.84	0.68	0.56	0.68	0.74		0.75	0.71	0.79	0.47	0.76	0.75
7	0.73	0.54	0.63	0.59	0.62	0.75		0.74	0.69	0.4	0.62	0.62
8	0.75	0.59	0.67	0.56	0.63	0.71	0.74		0.71	0.5	0.69	0.68
9	0.85	0.79	0.55	0.68	0.78	0.79	0.69	0.71		0.53	0.81	0.83
10	0.5	0.66	0.41	0.5	0.58	0.47	0.4	0.5	0.53		0.61	0.6
11	0.8	0.8	0.55	0.71	0.81	0.76	0.62	0.69	0.81	0.61		0.87
12	0.84	0.8	0.56	0.69	0.85	0.75	0.62	0.68	0.83	0.6	0.87	

Table 3. Discovered membership values with the application of ICA for the year 2013–2014.

M^ν	1	2	3	4	5	6	7	8	9	10	11	12
1		0.22	0.39	0.19	0.13	0.08	0.19	0.17	0.09	0.45	0.14	0.11
2	0.22		0.47	0.23	0.17	0.23	0.36	0.32	0.15	0.29	0.13	0.14
3	0.39	0.47		0.48	0.4	0.34	0.29	0.25	0.39	0.54	0.39	0.39
4	0.19	0.23	0.48		0.14	0.17	0.28	0.31	0.2	0.4	0.17	0.2
5	0.13	0.17	0.4	0.14		0.15	0.27	0.26	0.13	0.34	0.10	0.08
6	0.08	0.23	0.34	0.17	0.15		0.13	0.17	0.1	0.44	0.14	0.16
7	0.19	0.36	0.29	0.28	0.27	0.13		0.15	0.21	0.51	0.27	0.28
8	0.17	0.32	0.25	0.31	0.26	0.17	0.15		0.21	0.42	0.22	0.23
9	0.09	0.15	0.39	0.2	0.13	0.1	0.21	0.21		0.4	0.12	0.10
10	0.45	0.29	0.54	0.4	0.34	0.44	0.51	0.42	0.4		0.33	0.34
11	0.14	0.13	0.39	0.17	0.10	0.14	0.27	0.22	0.12	0.33		0.07
12	0.11	0.14	0.39	0.2	0.08	0.16	0.28	0.23	0.10	0.34	0.07	

Table 4. Discovered non-membership values with the application of ICA for the year 2013–2014.

Taking the results produced by the ICA, with the IF pairs distributed in two index matrices M^μ and M^ν, collected respectively in Tables 1, 2 for the year 2008–2009 and in Tables 3, 4 for the year 2013–2014, we plot them onto the IF interpretational triangle, respectively in Figures 1, 2, noting the quite similar form and location of both plotted sets, which though not presented here for the rest years in the period, has shown to repeat there, as well.

As above, from the point of view of the ICA method, our aim in this case study is to discover which criteria are in correlation, or in *positive consonance* per the adopted terminology, which means to compare these IF pairs against two threshold values: the constants α for the membership, and β for the non-membership, as illustrated on Fig. 3 (based on Fig. 2).

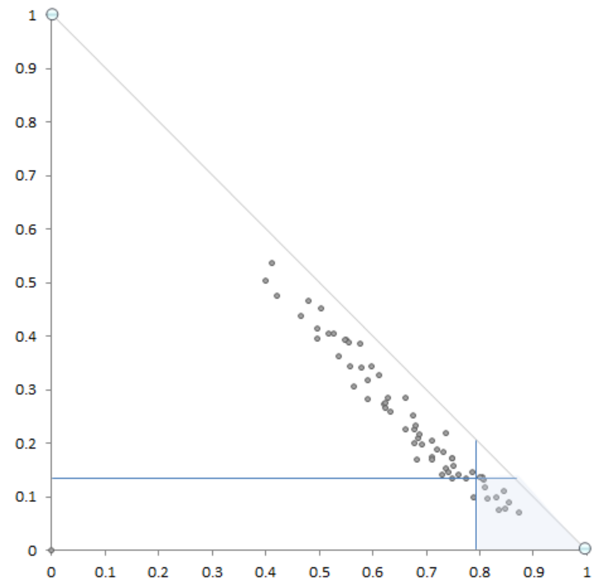

Figure 3. Example: With thresholds $\alpha = 0.796$ and $\beta = 0.134$, 9 points fall in the trapezoidal cut-out of the IF triangle.

Following the definition of the (α, β)-positive consonance, we are interested in the points which fall within the trapezoidal cut-out, formed between axis x, the hypotenuse, and the lines $x = \alpha$ and $y = \beta$.

In this way, we establish a new procedure for shortlisting the set of top consonance pairs of criteria according to both α and β simultaneously. This is worth noting, because in the previous steps of our research, the initial idea was to shortlist the individual criteria with respect firstly to one of the thresholds (usually the membership), and only then to proceed with respect to the other, using for this purpose first the max-row-aggregation operation of IMs [11] and, then, for comparison, the average-row-aggregation operation [12].

Now, we may prefer to work with this set of shortlisted intercriteria pairs as a whole, or we may still wish to find how to rank them. Ranking the pairs in (α, β)-positive consonance, in this new way, shall again be performed in both dimensions simultaneously. Thus we reach the idea to calculate for each point in the selection its distance from the (1; 0) point, standing in this case for perfect positive consonance between two criteria, which in the general case would only be the positive consonance of any criterion with itself.

The formula for the distance d_{C_i,C_j} of the intercriteria pair (C_i, C_j) to the (1; 0) point is obviously:

$$d_{C_i,C_j} = \sqrt{(1-\mu_{C_i,C_j})^2 + \nu_{C_i,C_j}^2}$$

and the pairs are ordered according to their d_{C_i,C_j} sorted in ascending way.

Back to our example, with (randomly taken) threshold values of $\alpha = 0.796$ and $\beta = 0.134$, we get 9 points in the cut-out, i.e. 9 InterCriteria positive consonance pairs, and we check that they are formed among a set of 6 individual criteria. Calculating the distances for these 9 points to the (1; 0) point, we obtain in Table 5 the following ordering of the pairs and illustrate with Figure 4.

No.*	d_{C_i,C_j}	Criteria in (α, β)-positive consonance		μ_{C_i,C_j}	ν_{C_i,C_j}
❶	0.148	**11 – 12**	*Business sophistication – Innovation*	0.87	0.07
❷	0.170	**5 – 12**	*Higher education and training – Innovation*	0.85	0.08
❸	0.175	**1 – 9**	*Institutions – Technological readiness*	0.85	0.09
❹	0.179	**1 – 6**	*Institutions – Goods market efficiency*	0.84	0.08
❺	0.194	**1 – 12**	*Institutions – Innovation*	0.84	0.11
❻	0.197	**9 – 12**	*Technological readiness – Innovation*	0.83	0.10
❼	0.206	**5 – 11**	*Higher education and training – Business sophistication*	0.82	0.10
❽	0.225	**9 – 11**	*Technological readiness – Business sophistication*	0.81	0.12
❾	0.230	**1 – 5**	*Institutions – Higher education and training*	0.81	0.13

* The precise ordering can depend on the precision of μ and ν, which we work with, in this case two digits after the decimal point.

Table 5. Ordering of the correlating pairs, with respect to the distance from (1; 0) of the points that represent them in the IF interpretational triangle.

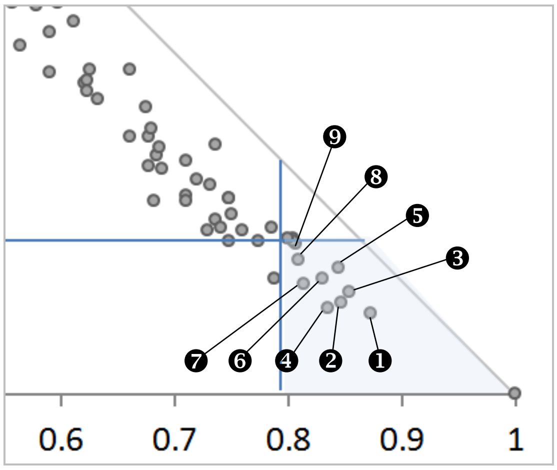

Figure 4. Close-up of the 9 points in the IF triangle cut-out.

Having selected and ordered the *pairs of criteria*, we may further want on this basis to order the *individual criteria*; but this is a matter of future research.

3. Comparison with Previous Results and Discussion

Plotting the intercriteria correlations as points in the IF triangle gives us the possibility to rank and work with the strongest *pairs of criteria*, while in the hitherto steps of our research we have ranked and worked with the *individual criteria*, ordered according to one of the components in the pair, namely the membership component. Generally, the results are not expected to be much different, given that we process the same data, once taken as two index matrices M^μ and M^ν, and then – as one IM of IF pairs M^*.

We are interested to compare the results here with those reported in [11], obtained with the proposed there algorithm for max-row-aggregation of the M^μ and min-row-aggregation of the M^ν, see Table 3 in [11]. Following the logic from that previous step of our research, we look at the first subtable for M^μ and identify for the threshold $\alpha = 0.796$ which are the individual top-correlating criteria, six in number: '11', '12', '1', '9', '5' and '2', and the number of intercriteria pairs formed between them, 11 in number. Then, in order to determine which is the value of the threshold β, so that the produced sets of positive and negative consonance pairs be as close and consistent as possible, as they can hardly be identical using this algorithm applied to M^ν. We look for the smallest subset of ordered set of criteria in Table 3b, which contains the six outlined criteria above, and see that this happens when β is as large as 0.135, when this smallest subset contains eight (*sic!*) criteria '11', '12', '1', '6', '5', '9', '2' and '7'. Back to the IM M^μ in Table 3a, we note that with respect to membership, criterion '6' appears among the top-correlating criteria when α = 0.788 and criterion '7' appears there with α as low as 0.749. As long as criterion '6' concerned, it appears so early in the results in Table 5 here, not due to a high level of the membership component of its top positive consonance with another criterion (namely, '1'), but alternatively due to the low level of the non-membership component. This circumstance, however, cannot be reflected when we handle both components of the IF pair separately, as has been done by now.

This all comes to demonstrate that using the 'old' approach developed in [11] for consequently handling the membership and the non-membership components of the IF pairs may introduce certain levels of 'noise' (like the inclusion of criteria '2' and '7') and is less accurate than the proposed here new approach, which is inspired by the triangular geometric interpretation of IF sets.

Nevertheless, in other case studies and problems, approached in future with the ICA approach, all proposed algorithms – from the present and previous researches – are worth approbating, in order to compare the results for various fields of application, consult them with experts in the respective areas, and make a better justification of our choice of method of selecting threshold values and selecting the top-correlating criteria.

4. Conclusion

The present work aims to offer and illustrate a new way of processing of the results, produced with the ICA approach for IFS-based identification of possible correlations between criteria in a decision making process. After a series of research steps, where the IF pairs, generated by the ICA, were treated separately and consequently, here we propose a new way for selecting the best InterCriteria correlations (here termed '*consonances*'), which simultaneously takes into account both the membership and the non-membership component of the IF pair. This is done by taking the IF pairs as coordinates of points, plotted onto the IF triangle, which is the IFS-specific way of geometrically interpret IFSs.

Moreover, given the nature of the IF pairs, we speculate that plotting them as coordinates in the IF triangle is the only appropriate way of visualizing them, rather than using the standard (or modified standard) linear interpretation.

For illustration of the proposed algorithm, we make use of previously generated results of the application of the ICA over data from the World Economic Forum's annual Global Competitiveness Reports for 28 EU Member states for the period 2008–2014, and we comment on the differences between the proposed algorithm with previously proposed and explored algorithms. Although the present paper does not aim to provide economic analysis of the results achieved, any interested and knowledgeable reader is welcome to join us in this part of the research.

As a further step of research, we consider applying of the topological operators of *Closure* and *Interior*, and their extensions from [6], and the modified and extended modal operators like D, F, G, H, J, H^*, J^* over the so plotted set of points in the triangle. This application of the hitherto existing theoretical framework of IFSs to this new decision support approach of ICA may prompt some interesting and pragmatic ideas.

The ICA is a novel idea, which has been under active development in the last year, and any particular analysed datasets and results provoke new ideas about new legs of research in this field. New modifications of the algorithms that underlie the calculation and interpretation of the results in ICA are currently an object of discussion and elaboration. These new algorithms will reflect the specifics of the evaluations of the objects against the evaluation criteria, which can be integers, real numbers, symbols (e.g. '+' and '–'), ordered linguistic variable, etc.

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