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Introducing Interpolative Boolean algebra into Intuitionistic fuzzy sets

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Abstract

In this paper, we introduce Interpolative Boolean algebra (IBA) as a suitable algebra for intuitionistic fuzzy sets (IFSs). IBA is [0,1]-valued realization of Boolean algebra, consistent with Boolean axioms and theorems. IFS theory takes into account both membership and non-membership function, so it can be viewed as a generalization of the traditional fuzzy set theory. We propose a realization of IFS conjunction and disjunction operations based on IBA. This may be viewed as a generalized framework for IFS-IBA calculus. Finally, we investigate the validity of the laws of contradiction and excluded middle in our approach.

Keywords: Interpolative Boolean algebra, Intuitionistic fuzzy sets, IFS operations, law of excluded middle, law of contradiction

1. Introduction

Human reasoning is naturally supported by both logic and intuition. In the presence of imperfect and imprecise information it is challenging to reason and make good decisions.

To elaborate uncertainty involved in fuzzy decision making there are many approaches and modeling tools. Intuitionistic fuzzy sets (IFSs) have been introduced by Atanassov [1] and it is considered as a generalization of traditional fuzzy sets. IFS theory offers more expressive power by incorporating not only degree of membership, but also degrees of non-membership and non-determinacy (uncertainty). Operations and relations over IFSs extend the relations and operations over fuzzy sets [2]. Further, the intuitionistic fuzzy (IF) propositional calculus has been introduced [3].

Numerous extensions and generalizations of basic IFS operators were proposed. Dancev [4] presented a generalized operation that extends original union, intersection, etc. Lemnaouar and Abdelaziz [5] introduced Lukasiewicz-Moisil algebra into IFS. The main idea in [6] was to extend the concepts of t-norm/t-conorm, as a generalization of operations of intersection/union, to IFS theory. Furthermore, the authors formulated IF negation as an extension of traditional fuzzy negation. Since it was introduced 20 years ago, only classical negation and implication were applied on IFS. It is considered as one of the crucial drawbacks in this approach [7]. Consequently, more than 140 operators of implication and 35 operators of negation over IFSs are proposed in literature, supporting different properties and domains. Bustince et al. [8] studied the manner in which to construct different Atanassov intuitionistic fuzzy connectives starting from an operator. Further, they examined the law of excluded middle (LEM) and the law of contradiction (LC) with respect to the proposed connectives.

The laws of excluded middle and contradiction are the fundamental laws of thought and a property of Boolean algebra. In the case of conventional fuzzy logic, these laws are not satisfied [9, 10]. Furthermore, some authors consider them as redundant and constraints on reasoning [11]. Since fuzzyfication in IFS theory holds the idea of intuitionism (e.g. see [12]), IFS does not follow the LEM. On the other hand, this law is thoroughly investigated in the context of IFS, using different forms of LEM and various negation operators [7, 13]. It is argued that no negation satisfies the LEM in the tautological form and that none of proposed negations satisfies more than two forms of LEM. Conventional IFS negation neither satisfies LC. That is one of the reasons for terminological debate regarding the term 'intuitionistic' in Atanassov IFS [14].

Interpolative Boolean algebra (IBA) is a theoretical framework introduced by Radojevic [15]. It is a consistent [0,1]-realization of Boolean algebra. Logical functions are uniquely mapped to generalized Boolean polynomials (GBP), securing Boolean laws. Values are introduced after the GBP transformation procedure [16]. IBA may be utilized as a natural framework for consistent fuzzy logic in the sense of Boole [17].

In this paper, we aim to propose a logical consistent approach, in the sense of Boole, to IFSs by combining IFS with IBA. The concept of intuitionistic fuzzy sets is fully retained, while IBA-based algebra, adapted for IF calculus, is introduced as a generalization of operations. More precisely, operations of conjunction and disjunction are realized using GBP analogous to classical intuitionistic fuzzy operations, while the negation operator is chosen among the existing once. Further, we study if LEM and LC are satisfied in our approach.

The paper is organized as follows. Section 2 provides the basic concepts of IFS theory, operations over IFS and their variants. In Section 3 the brief overview of IBA is given. In Section 4 the proposed realization of IFSs using IBA is introduced. Special attention is dedicated to the laws of excluded middle and contradiction. Finally, Section 5 concludes the paper.

2. Intuitionistic fuzzy sets

Intuitionistic fuzzy set theory has been introduced in [1]. It is a generalization of the traditional fuzzy set theory: it takes into account both membership and non-membership of an element to certain set. Furthermore, IFS theory supports the concept of uncertainty. It is possible that an expert can better describe the values for which he is not sure whether they belong or does not belong to a particular set. Thus, the richer semantic description is provided. Formal definition of IFS is given as follows.

Definition 1 [2]**.** An intuionistic fuzzy set A in a universe E is an object:

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in E\} \quad (1)$$

where functions $\mu_A(x): E \to [0,1]$ and $\nu_A(x): E \to [0,1]$ define the degree of membership and the degree of non-membership of the element x to the IFS A. For every $x \in E$:

$$0 \le \mu_A(x) + \nu_A(x) \le 1 \qquad (2)$$

In a case when $\mu_A(x) + \nu_A(x) = 1$ IFS is reduced to a traditional fuzzy set. Otherwise, there is some uncertainty or the degree of non-determinacy $\pi(x) > 0$ of the membership of element $x \in E$ to the set A:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \qquad (3)$$

IFS provides to an expert greater descriptive power compared to traditional fuzzy set and leaves the possibility to define the margin of error. Arbitrary IFS A is presented in Fig. 1 [2].

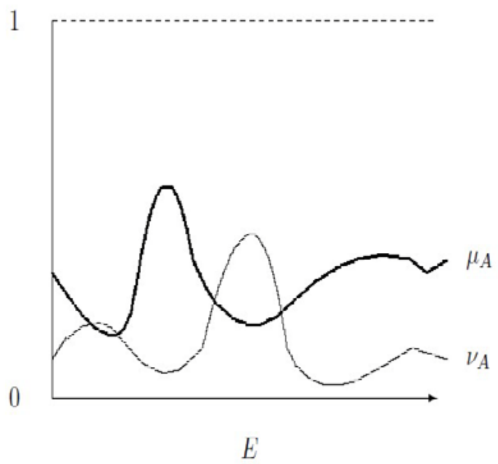

Fig. 1: Intuitionistic fuzzy set *A*.

The degrees of membership and non-membership are measured on the left and right vertical axes, respectively.

2.1. Operations over IFS

Operations over IFSs extend the definitions of the operations over conventional fuzzy sets. The basic operations over two IFSs A and B in their originally proposed form are given:

$$\begin{aligned}
A \cap B &= \{(x, \min(\mu_A(x), \mu_B(x)), \\
&\qquad \max(\nu_A(x), \nu_B(x))) \mid x \in E\} \\
A \cup B &= \{(x, \max(\mu_A(x), \mu_B(x)), \\
&\qquad \min(\nu_A(x), \nu_B(x))) \mid x \in E\} \quad (4) \\
A \to B &= \{(x, \max(\nu_A(x), \mu_B(x)), \\
&\qquad \min(\mu_A(x), \nu_B(x))) \mid x \in E\} \\
\overline{A} &= \{(x, \nu_A(x), \mu_A(x)) \mid x \in E\}
\end{aligned}$$

Numerous extensions, generalizations and additional operators are proposed (e.g. [2, 4, 6, 18]). Negation and implication are operators that the most attention is devoted.

2.2. Intuitionistic fuzzy propositional calculus and negation

The intuitionistic fuzzy propositional calculus has been introduced in [3]. Analogous to the IFS theory, the truth-value of the variable x is represented by the ordered couple:

$$V(x) = \langle a_x, b_x \rangle \qquad (5)$$

while the logical operations of conjunction, disjunction and negation are defined as follows:

$$\begin{aligned}
V(x \wedge y) &= \langle \min(a_x, a_y), \max(b_x, b_y) \rangle \\
V(x \vee y) &= \langle \max(a_x, a_y), \min(b_x, b_y) \rangle \quad (6) \\
V(\neg x) &= \langle b_x, a_x \rangle
\end{aligned}$$

Conventional IFS negation follows strong double negation law, which is rejected by intuitionism [14]. Due to its non-intuitionistic nature, there is an ongoing research on various negation operators. The main results are presented in [13]. Negations are usually studied in the respect of the following properties:

$$A \to \neg\neg A \qquad (7)$$

$$\neg\neg A \to A \qquad (8)$$

$$\neg\neg\neg A = \neg A \qquad (9)$$

More than 35 negations are proposed [7]. Those operators come down on five distinctive intuitionistic fuzzy negations given in Table 1.

Notation	Negation
$\neg_1 \langle a,b \rangle$	$\langle b,a \rangle$
$\neg_2 \langle a,b \rangle$	$\langle 1-sg(a), sg(a) \rangle$
$\neg_3 \langle a,b \rangle$	$\langle b, a.b+a^2 \rangle$
$\neg_4 \langle a,b \rangle$	$\langle b, 1-b \rangle$
$\neg_5 \langle a,b \rangle$	$\langle \overline{sg}(1-b), sg(1-b) \rangle$

Table 1: List of the distinctive intuitionistic fuzzy negations.

Function sg is defined as:

$$sg(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases} \qquad (10)$$

Details about the operator $a.b$ can be found in [2].

In this paper, we will pay a special attention to the negation operator defined in following manner:

$$V(\neg x) = \langle b_x, 1-b_x \rangle \qquad (11)$$

This operator satisfies Properties 1 and 3 (Eqs. 7 and 9), while Property 2 (Eq. 8) is not satisfied [7]. Consequently, we can say that this negation does not satisfy the double negation rule and that it has intuitionistic nature. This negation will be used as a negation in the context of the proposed approach.

3. Interpolative Boolean algebra

In this section, we briefly present the basic concepts of Interpolative Boolean Algebra (IBA) [15]. IBA is a consistent generalization of classical Boolean algebra in the sense that all elements of Boolean algebra $BA(\Omega)$ have their [0,1]-valued realization and it preserves all Boolean axioms. Having distinguished the symbolic and valued level in IBA, Radojevic [16] focuses on the structure instead on the values as in conventional multi-valued algebras. On the symbolic IBA level the laws of Boolean algebra (excluded middle, contradiction, etc.) are value-independent. Each IBA element is considered independently of its realization, and it represents the analyzed object. Values of IBA elements are introduced on the valued level, keeping the properties of Boolean logic in [0,1] interval.

IBA framework is technically based on generalized Boolean polynomials. GBP consists of elements of Boolean algebra as variables and standard +, standard - and generalized product (GP) as operators. GP operator is a subclass of t-norms; it can be any function that satisfies all four axioms of t-norms (commutativity, associativity, monotonicity, boundary condition) defined in [19] and the additional axiom - non-negativity condition [16]. Accordingly, GP on valued level may be any operator from the following interval:

$$\max(a+b-1,0) \le a \otimes b \le \min(a,b) \, (12)$$

Generalized product distributes over addition and subtraction on the symbolic level.

Any logical function is uniquely mapped to a particular GBP. The transformation procedure of Boolean functions into corresponding GBPs is based on the principle of structural functionality [16]. Structural functionality is a value irrelevant, algebraic principle suggesting the necessity of structural mapping before introducing the values. Combined/compound elements $F(p_1,\ldots,p_n)$, $G(p_1,\ldots,p_n) \in BA(\Omega)$ are transformed in the following manner:

$$\begin{aligned} &(F \wedge G)^{\otimes} = F^{\otimes} \otimes G^{\otimes}, \\ &(F \vee G)^{\otimes} = F^{\otimes} + G^{\otimes} - (F \wedge G)^{\otimes}, \quad (13) \\ &(\neg F)^{\otimes} = 1 - F^{\otimes} \end{aligned}$$

while primary variables $\Omega = \{p_1, \ldots p_n\}$ are transformed as follows:

$$\begin{aligned} &(p_i \wedge p_j)^{\otimes} = \begin{cases} p_i \otimes p_j, & i \ne j \\ p_i, & i = j \end{cases}, \\ &(p_i \vee a_j)^{\otimes} = p_i + p_j - (p_i \wedge p_j)^{\otimes}, (14) \\ &(\neg p_i)^{\otimes} = 1 - p_i \end{aligned}$$

The transformation rule that ensures idempotence (see the first line in Eq. 14) has a crucial role for securing Boolean frame. More on GBPs, transformation procedure and its realization may be found in [17, 20].

Once the transformation procedure has been completed, the valued level is introduced, i.e. elements of IBA are valued in a [0,1] interval, and suitable operator for GP is selected. In [21, 22], authors suggest that the nature of the $BA(\Omega)$ elements determines which operator is suitable in particular situation. For example, min function should be utilized for elements of the same/similar nature. For the elements that are independent/different by nature standard product should be used.

4. Introducing Interpolative Boolean algebra into intuitionistic fuzzy sets

In this section we aim to propose IBA-based approach to IFS theory. IBA-IFS hybridization maintains descriptiveness and applicability of IFS and structural functionality paradigm used in IBA.

According to transformation rules (Eqs. 13 and 14) we introduce generalizations of conjunction and disjunction in intuitionistic fuzzy propositional calculus. This operation may be viewed as analogous to the conventional IF operation. As a negation operator we utilize previously analyzed negation given in Eq. 11. It has intuitionistic nature and it appears appropriate in the context of the proposed approach. IBA-IFS operations of conjunction, disjunction and negation are defined in the following manner:

$$V(x \wedge y)^{\otimes} = \langle a_x \otimes a_y, b_x + b_y - b_x \otimes b_y \rangle$$
$$V(x \vee y)^{\otimes} = \langle a_x + a_y - a_x \otimes a_y, b_x \otimes b_y \rangle \quad (15)$$
$$V(\neg x)^{\otimes} = \langle b_x, 1 - b_x \rangle$$

The IBA transformation rules are extended with the following:

$$(a_x \wedge b_x)^{\otimes} = a_x \otimes b_x = 0 \qquad (16)$$

where a_x and b_x refers to the truth-value of the variable x (or degrees of membership and non-membership of element x, from the perspective of IFS). This rule follows from:

$$\begin{aligned} &0 \le a_x + b_x \le 1 \\ &0 \le b_x \le 1 - a_x \\ &0 \le a_x \otimes b_x \le a_x \otimes (1 - a_x) \\ &0 \le a_x \otimes b_x \le a_x - a_x \otimes a_x \\ &0 \le a_x \otimes b_x \le a_x - a_x \\ &0 \le a_x \otimes b_x \le 0 \end{aligned} \qquad (17)$$

Rule presented in Eq. 16 is a direct consequence of the principle of structural functionality applied on IFS. Therefore, this is directly transferred to the proposed IFS-IBA calculus. Graphical interpretation of this rule from the perspective of IFS will be the subject of future work.

The proposed operations can be considered as a generalized framework over IFSs. Conventional IF conjunction and disjunction given in Eq. 6. can be obtained as a special case when min function is used as generalized product ($\otimes := \min$). Taking into account that in IBA the operator realization depends on the nature of variables, application of different operators may improve the existing results. Also, it may increase the interpretability of the aggregation process.

4.1. IBA-IFS and the laws of excluded middle and contradiction

In general, the law of excluded middle is not followed in IFS theory since the idea of intuitionism is carried out through fuzzyfication. On the other hand, this fundamental law of thought is thoroughly studied in IFS theory [2, 8, 13]. Various negation operators are investigated in the perspective of LEM.

The validity of the law of excluded middle in a context of IFS is studied in four forms: 1) tautology form; 2) intuitionistic fuzzy tautology (IFT)-form; 3) modified LEM in tautology form; 4) modified LEM in IFT-form [2]. LEM for IFS is defined as follows:

$$\langle a,b \rangle \vee \neg \langle a,b \rangle = \langle p,q \rangle \qquad (18)$$

where $\langle p,q \rangle = \langle 1,0 \rangle$ for tautology form, while $1 \ge p \ge q \ge 0$ for IFT-form. Modified LEM is studied in the following form:

$$\neg\neg \langle a,b \rangle \vee \neg \langle a,b \rangle = \langle p,q \rangle \qquad (19)$$

where $\langle p,q \rangle = \langle 1,0 \rangle$ for tautology form, while $1 \ge p \ge q \ge 0$ for IFT-form.

Analogous to LEM, the law of contradiction will be investigated in conventional and modified form and discussed from the perspective of tautology and IFT.

The validity of all LEM and LC variations will be investigated in the context of the proposed approached. Rules of transformation given in Eqs. 13, 14 and 16 are utilized.

$$\begin{aligned} &V(x \vee \neg x)^{\otimes} = \\ &= (\langle a_x, b_x \rangle \vee \langle b_x, 1 - b_x \rangle)^{\otimes} \\ &= \langle a_x + b_x - a_x \otimes b_x, b_x \otimes (1 - b_x) \rangle \\ &= \langle a_x + b_x, b_x - b_x \otimes b_x \rangle \\ &= \langle 1 - \pi_x, b_x - b_x \rangle \\ &\equiv \langle 1 - \pi_x, 0 \rangle \end{aligned} \qquad (20)$$

$$\begin{aligned} &V(x \wedge \neg x)^{\otimes} = \\ &= (\langle a_x, b_x \rangle \wedge \langle b_x, 1 - b_x \rangle)^{\otimes} \\ &= \langle a_x \otimes b_x, b_x + 1 - b_x - b_x \otimes (1 - b_x) \rangle \\ &= \langle 0, 1 - b_x + b_x \otimes b_x \rangle \\ &= \langle 0, 1 - b_x + b_x \rangle \\ &\equiv \langle 0, 1 \rangle \end{aligned} \qquad (21)$$

As shown, LC in tautology form is satisfied in the proposed approach. Consequently, it is satisfied in IFT-form also. LEM is followed in IFT-form. LEM in tautology form is satisfied only in the case when $\pi_x = 0$, e.i. in the case of traditional fuzzy sets.

Further, we check if the proposed generalization satisfies modified LEM and modified LC.

$$
\begin{aligned}
&V\left(\neg\neg x \vee \neg x\right)^{\otimes} = \\
&= \left(\neg\left\langle b_x, 1-b_x\right\rangle \vee \left\langle b_x, 1-b_x\right\rangle\right)^{\otimes} \\
&= \left(\left\langle 1-b_x, b_x\right\rangle \vee \left\langle b_x, 1-b_x\right\rangle\right)^{\otimes} \\
&= \left(\left\langle 1-b_x, b_x\right\rangle \vee \left\langle b_x, 1-b_x\right\rangle\right)^{\otimes} \\
&= \left\langle 1-b_x + b_x - \left(1-b_x\right)\otimes b_x, b_x \otimes \left(1-b_x\right)\right\rangle \\
&= \left\langle 1-b_x + b_x \otimes b_x, b_x - b_x \otimes b_x\right\rangle \\
&= \left\langle 1-b_x + b_x, b_x - b_x\right\rangle \\
&\equiv \left\langle 1, 0\right\rangle
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
&V\left(\neg\neg x \wedge \neg x\right)^{\otimes} = \\
&= \left(\neg\left\langle b_x, 1-b_x\right\rangle \wedge \left\langle b_x, 1-b_x\right\rangle\right)^{\otimes} \\
&= \left(\left\langle 1-b_x, b_x\right\rangle \wedge \left\langle b_x, 1-b_x\right\rangle\right)^{\otimes} \\
&= \left\langle \left(1-b_x\right)\otimes b_x, b_x + 1 - b_x - \left(1-b_x\right)\otimes b_x\right\rangle \\
&= \left\langle b_x - b_x \otimes b_x, 1 - b_x + b_x \otimes b_x\right\rangle \\
&= \left\langle b_x - b_x, 1 - b_x + b_x\right\rangle \\
&\equiv \left\langle 0, 1\right\rangle
\end{aligned} \tag{23}
$$

Both modified LEM and modified LC are followed in tautology form and, consequently, they are satisfied in IFT-form.

To sum up, the law of contradiction is followed in all four forms studied in IFS. LEM is not satisfied only in tautology form. This may lead us to conclusion that intuitionistic nature of IFS is underlined by using IBA.

5. Conclusion

In general, the law of contradiction is not valid in IFS theory. This fact is considered as one of the main reasons for terminological and theoretical debate regarding the term 'intuitionistic' in IFS, and it is the constant motivation for research.

In this paper, we have introduced concepts of Interpolative Boolean algebra into intuitionistic fuzzy theory. In the proposed approach, IFS holds the idea of intuition and IBA provides suitable algebra. Logical operations of conjunction and disjunction are realized using GBP. Thus, the principle of structural functionality is introduced into IFS, emphasizing the structure and the nature of variables. The conventional IF calculus is obtained as the special case of our approach, when min function is used as generalized product. Further, we study the validity of LC and LEM. We have proved that LC is followed in IFS-IBA approach, supporting the concepts of intuitionism. LEM is satisfied in three of four basic forms. The negation operator used in this framework is the distinctive IF negation. The double negation rule is not preserved for this operator, which is also in accordance with the idea of intuitionism.

For future research we aim to provide a detail analysis of the IFS-IBA approach in terms of various realizations of generalized product. We will be focused on practical problems in order to estimate its applicative value.

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