Optimal Distribution of Medicine in Ebola

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Abstract. First, we use the adjusted SEIR model as the infection model in the chosen area. We choose Sierra Leone, the country where Ebola is the most serious, as the study case to give detailed analysis. Next, we build a multi-step medicine shipping model. First we use the GRA (Grey Relationship Analysis) method to quantify the urgency of salvage of each area, combining the network analysis to determine the priority weight of each area. Then with these results we look into the transportation system of each area to establish the three-level delivery system.

1. Background

The most widespread epidemic of Ebola virus disease (commonly known as "Ebola") in history is currently ongoing in several West African countries.[1]It has caused significant mortality, with reported case fatality rates of up to 71%. It began in Guinea in December 2013 and then spread to Liberia and Sierra Leone.A small outbreak of twenty cases occurred in Nigeria and one case occurred in Senegal, both now declared disease-free. [2]

2. The Medicine Delivery Model

2.1 The Epidemic Model under Medication Supply

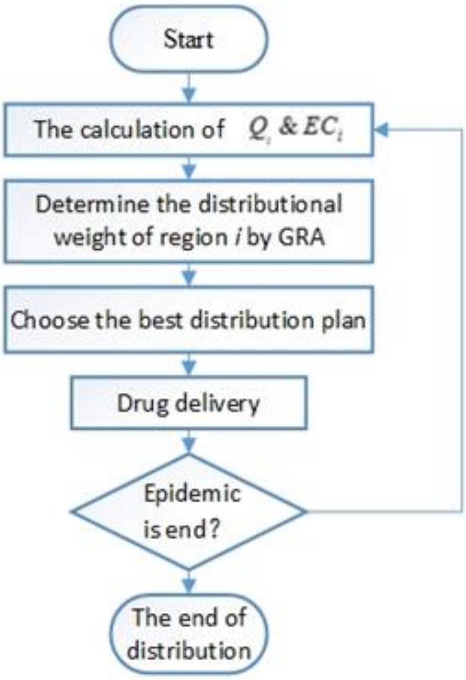
We suppose the medicine is given periodically in each area and our model can be reflected by the Figure. And we can found the function as follow.

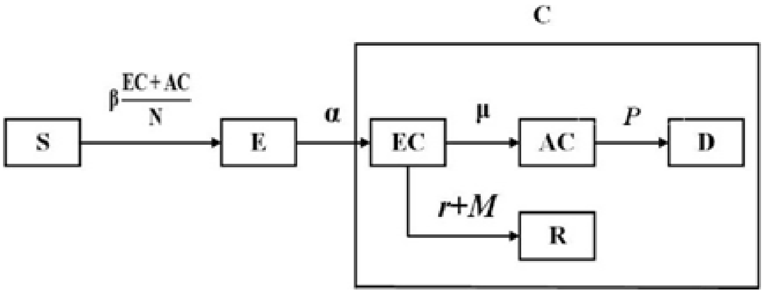
$$\begin{cases} \dfrac{dS}{dt} = -\dfrac{\beta \times S\left(EC + AC\right)}{N} \\ \dfrac{dE}{dt} = \dfrac{\beta \times S\left(EC + AC\right)}{N} - \alpha \times E \\ \dfrac{dEC}{dt} = \alpha \times E - \left(\mu + r\right) \times EC \\ \dfrac{dAC}{dt} = \mu \times EC - P \times AC & t \neq kT, k = 0,1,2,3\ldots \\ \dfrac{dD}{dt} = P \times AC \\ \dfrac{dR}{dt} = r \times EC \\ \dfrac{dC}{dt} = \alpha \times E \end{cases}$$

$$\begin{cases} EC(\mathrm{t}^{+}) = \left(1 - M\right) \times EC(\mathrm{t}) \\ R(\mathrm{t}^{+}) = R(\mathrm{t}) + M \times EC(\mathrm{t}) \end{cases} \quad t = kT, k = 1,2,3\ldots$$

Here, S, E, EC, AC, D and R denote the number of susceptible, exposed, not advanced, advanced, dead and recovered individuals at time t. And T is equal to 7.Nevertheless, C denotes the cumulative number of infected cases from the time of the onset of the outbreak. And the transmission rate is modeled as a function of time by Bashar. After the government take action, the value of β varies from β0 to β1 as follows:

$$\beta = \begin{cases} \beta_0 & t < \tau \\ \beta_1 + (\beta_0 - \beta_1)e^{-q(t-\tau)} & t \ge \tau \end{cases}$$





2.2 Urgency Degree in Each Area

We use the index the urbanization rate reflects an area’s economic development and the other indexes reflect the severity of disease. And we use the GRA to do this work.

$$\xi_i(k) = \frac{\min_s \min_t |l_0(t) - l_s(t)| + \rho \max_s \max_t |l_0(t) - l_s(t)|}{|l_0(k) - l_s(k)| + \rho \max_s \max_t |l_0(t) - l_s(t)|} \quad \mathrm{Re} = \frac{\sum_{k=1}^{5} \xi_i(k)}{5}$$

Here l_i denotes the i-th row in normalization matrix and l_0 is a row vector. And ρ is the distinguishing coefficient, and typically ρ = 0.5.

And we can determine the distribution weight through this method:

$$\omega_i = \frac{\mathrm{Re}_i}{\sum_{i=1}^{13} \mathrm{Re}_i}$$

2.3 Select the Best Distribution Nodes

For the convenience of the analysis, we divide 13 areas: Bombali, Kambia, Koinadugu, Port Loko, Tonkolili, Kailahun, Kenema, Kono, Bo, Bonthe, Moyamba, Pujehun, Western. Particularly, corresponding to region 1, region 2, region 3….. Region 12, region 13.we set Western as the node 1, Kenema as the node 2 and Koinadugu as the node 3.

2.4 Optimal Distribution Function

Suppose the minimum shipments of the medication could satisfy the area's patients, who are in the early stage and that can ultimately reduce the patients there. Which is to say, each area's dosing frequency M should meet the inequality $\frac{d\left(EC(t)+AC(t)\right)EC(t)}{dt}<0$.After the drug taken. We can readily calculate each area's minimum dosing frequency M_i^k, thus obtain the minimum medication Q_i^k of each distribution to each area. Besides, we regard it as an insufficient supply if the amount of delivery is less than the *EC* of a region during medication distribution.

To simplify the model, we suppose the total amount of the medication delivered to Sierra Leone is a constant value U, so we can get relationship between U and: $U_1^k=U-U_2^k-U_3^k$. And the haulage cost c_{ij} is proportional to the haulage time t_{ij}.

Take the cost of distribution from Freetown to the other nodes into account, we gain the total cost function of the *k*th shipping to area i.

$$W_i^k=\sum_{j=1}^{3}\sum_{i=1}^{13}\left(c_{ij}\times\min\left\{1,x_{ij}^k\right\}+H_i^k\times\max\left\{0,t_{ij}-G^k\right\}\right)$$

Where the x_{ij}^k means during the *k*th medication delivery, the traffic volume from the distribution node *j* to the region *i*

We should consider the cost of air transportation

$$WA_j^k=O_j\times\min\left\{0,U_j^k\right\}$$

Then we can get the modified cost function

$$W_i^k=\sum_{j=1}^{3}\sum_{i=1}^{13}\left(c_{ij}\times\min\left\{1,x_{ij}^k\right\}+H_i^k\times\max\left\{0,t_{ij}-G^k\right\}\right)+\sum_{j=2}^{3}\left(\frac{x_{ij}}{U_j^k}\times WA_j^k\right)$$

Besides, some constraints should be meted:

$$\sum_{i=1}^{13}x_{ij}^k\le U_j^k$$

Besides, the coefficient of the penalty function is closely related to the degree of the region's rescue, the coefficient of the penalty function is represented by the priority heavy of each region (before each drug distribution) $H_i^k=\omega_i^k$.

We can calculate the shipping shortages of each area. Since the supply lower than Q_i^k can bring a greater loss,we put more weight on the part below the minimum demand.

$$F_i^k=1.5\times\min\left\{0,Q_i^k-\sum_{j=1}^{3}\sum_{i=1}^{13}x_{ij}^k\right\}+\min\left\{0,EC_i^k-\sum_{j=1}^{3}\sum_{i=1}^{13}x_{ij}^k\right\}$$

Because of the different priorities, we bring in the weight coefficient $\theta_1,\theta_2,$ to combine the two functions and form a composite optimization function. In order to measure them on the same level, we adopt normalization method to normalize the results.

$$\bar{W}_i^k=\frac{W_i^k-W_{i\ \min}^k}{W_{i\ \max}^k-W_{i\ \min}^k}$$

$$\bar{F}_i^k=\frac{F_i^k-F_{i\ \min}^k}{F_{i\ \max}^k-F_{i\ \min}^k}$$

$$WF_i^k=\theta_1\times\bar{W}_i^k+\theta_2\times\bar{F}_i^k$$

Because of the different distribution priority of each area (determined by the rescue urgency), we need to take each area's optimization function's priority weight into account, thus obtain the final optimization distribution function:

$$WF^k=\sum_{i=1}^{13}\left(\omega_i^k\times WF_i^k\right)$$

We stop to distribute the drugs when the number of the infectious verge to 0. The target of the optimization is to minimum the distribution function

3. Model results analysis

From the map of Sierra Leone, we can get each area's corresponding s_{ij} , and consequently get c_{ij} and t_{ij} we make $\Theta 1$ =0.3 and $\Theta 2$=0.7..

When the drug supply is 500 per time, we can get that after 7 times of medication supplement. The Ebola disease trend to die down. After every calculation, the optimal drugs delivery each time is shown in the table.

Region	1	2	3	4	5	6	7	8	9	10	11	12	13
1st	0	0	0	11	0	0	0	0	0	0	19	0	470
2nd	40	0	67	15	14	0	0	0	0	0	21	0	343
3rd	36	19	60	13	16	39	57	13	19	0	17	0	212
4th	44	21	79	7	21	53	61	27	30	0	9	0	148
5th	56	31	93	0	37	54	59	31	40	0	0	8	91
6th	72	11	138	0	53	50	57	32	43	0	0	7	37
7th	115	0	167	0	60	50	31	27	33	0	0	0	17

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References

[1] "Ebola 2014 — New Challenges, New Global Response and Responsibility". The New England Journal of Medicine. Retrieved 18 September 2014.

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