Bit Error Rate Performance Bounds of SC-FDE under Arbitrary Multipath Mobile Communication Channels in Transportation

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Abstract

This paper focuses on the bit error rate (BER) performance of intelligent transportation communication system under multipath mobile frequency-selective channels. After presenting the theoretical BER performance formula of single carrier frequency domain equalization (SC-FDE) in the form of double integral, a closed-form approximate upper bound of BER performance was derived, as well as specific analytical expressions for approximate BER performance bounds under typical modulations. Analysis and simulation show that the derived approximate BER bound highly approaches the theoretical one. The bound also indicates that the research about the preferable equalization methods is still necessary.

Keywords: transport system communication; wireless channel; frequently-selective; BER performance bound; SC-FDE

1. Introduction

Modern driver's information processing¹, digital driving in intelligent transport system², transport control, transport safety³ and inter-car communication can be greatly enhanced by modern wireless communication technique. The key challenge to the wireless connection conveying messages in the transport control mechanism is the double-selective (both frequency-selective and time-selective) nature of the highly variable multipath wireless channel. Research about the theoretical performance under multipath broadband channel is critical to design and analysis of the mobile communication systems. The single carrier frequency

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domain equalization (SC-FDE)⁴ is getting more and more widely used in mobile communication. SC-FDE avoids OFDM (Orthogonal Frequency Division Multiplexing) shortages such as high crest factor, phase noise sensitivity and carrier offset sensitivity. Therefore, it is indispensable to study the theoretical performance of SC-FDE.

In Ref. 5, Tse comprehensively analyzes the capacity limitation of the wireless communication system adopting water flooding principle under frequencyselective and time-varying channel with large length in his book. In Ref. 6 Shi comprehensively explores the capacity of SC-FDE by the linear equalization method. However, there are few research effects about the bit error rate (BER) of SC-FDE, especially its BER performance under multi-path channels. The equalization of SC-FDE is conducted in the frequency domain in which the BER capacity of sub-channels could be analyzed. However, the existing theories of the channel capacity are of the time domain in which the transmission of the signal modulation is accomplished. These lead to the difficulty in analyzing the BER of SC-FDE.

There are many studies about the BER capacity of the single carrier (SC) system in time domain. Ref. 7 derives the SC system's BER capacity under various modulations for multiple-path rice flat fading channels. Taking the three-path channel applied in 802.16d within WiMAX⁸ as a special case, Ref. 9 provides the BER capacity bound and its approximate value and illustrates that the SC BER capacity closely approaches its theoretical bound when applying sphere decoder (SD) based on the approximate maximum likelihood detection to detect signal by visual test and verification. Starting from analyzing the SC's BER capacity within different frequency-selective channels, this paper goes off the beaten track to study in the time domain the SC-FDE BER theoretical capacity under arbitrary multipath frequency-selective channels which means the first path is of the straight ray path and the others are Rayleigh-distributed scattering paths. This assumption well describes the situation of the frequency-selective channel under most application environment.

The BER capacity could be the SC-FDE' BER capacity bound assuming that the SC-FDE adopting some ideal equalization method could completely remove the received inter symbol interference (ISI) and collect symbol energy of every path. As an obvious role, the BER capacity bound of SC-FDE is equal to that of the SC system. Therefore, in this paper the results discussed about the SC-FDE's BER capacity bound within the arbitrary multi-path frequency-selective channel can be applied to the SC system.

There are five sections in this paper. Section 2 introduces the applied system model. Section 3 deduces the SC-FDE's BER theoretical general formula and its approximate boundary for specific modulation systems under arbitrary multipath frequency-selected channels and provides an explicit expression of the BER approximate boundary specific to MPSK (M-ary Phase Shift Keying) and MQAM (M-ary Quadrature Amplitude Modulation), taking the QPSK(Quadrature Phase Shift Keying) and 16QAM(16-ary Quadrature Amplitude Modulation) as two typical cases of the modulation system. Section 4 shows the numerical validation of the SC-FDE BER capacity theoretical value and its approximate boundary and their practical simulation and analyzes the validated and simulated results. Section 5 provides the main conclusions of this paper.

Notations: Vectors are denoted by lowercase bold letters, while matrices are written in uppercase boldface letters. The superscripts T and H stand for, respectively, the transposition and conjugate transposition of the matrix or vector. A*B represents convolution between A and B. N is a set of positive integers, while $i \in \mathbb{N}$ means that i is an element of set N. x~CN (0, N₀) means x is a random normal variable with zero mean and variance N₀ . $E(\bullet)$ computes the expectation of an input random entity. $\|\bullet\|$ returns the Frobenius norm of a matrix or vector.

2. System Model

In this section, an ideal SC-FDE system model of equalization will be derived from the channel impulse response (CIR).

Given

An ideal continuous time domain CIR model is shown as follows:

$$h(t) = a_0 \delta(t) + hs(t) = a_0 \delta(t) + \sum_{i=1}^{M-1} a_i e^{-j2\pi(\tau_i/\tau_0)} \delta(t - \tau_i)$$
(1)

In (1), $\delta(t)$ denotes impulse function and a_i is the strength of the Rayleigh flat fading of each scattering

path. τ_0 is the symbol spacing, while τ_i stands for the multipath time delay. Then hs(t) describes the (M-1) scattering paths. When the multipath delay is integer

times of τ_0 , i.e., $\tau_i = l\tau_0$, $l \in \mathbb{N}$, we get $e^{-j2\pi(\tau_i/\tau_0)} = 1$.

Then we derive the discrete time tap delay line multipath channel model as follows:

$$h(m) = a_0 \delta(m) + \sum_{i=1}^{M-1} a_i \delta(m - m_i)$$
(2)

In Eq. (2), m_i denotes the Rayleigh multipath time delay of path *i*.

The objective here is to analyze the SC-FDE BER capacity boundary of arbitrary multipath frequency-selective channel. For simplicity, it is assumed that the symbol spacing is equal to each adjacent delay time difference of multipaths ^[6]. Let \mathbf{h}_M be the multipath channel vector, where $\mathbf{h}_M = [h_0, h_1, \dots, h_{M-1}]^T$. Given **s** is the transmitted impulse sequence and **v** is the Gaussian white noise sequence. We can formulate the received impulse sequence **r** as follows:

$$\mathbf{r} = \mathbf{s} * \mathbf{h}_M + \mathbf{v} \tag{3}$$

Eq. (3) can be rewritten as follows:

$$\begin{bmatrix} r_{0} \\ r_{1} \\ \vdots \\ r_{M-1} \end{bmatrix} = \begin{bmatrix} h_{M-1} & \cdots & h_{0} & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & h_{M-1} & \cdots & h_{0} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & h_{M-1} & \cdots & h_{0} \end{bmatrix}$$

$$\mathbf{H}$$

$$\mathbf{$$

In Eq.(4), \mathbf{s}_{2M-1} is $\left[s_{-(M-1)}, \dots, s_{-1}, s_0, s_1, \dots, s_{M-1}\right]^T$, which denotes the arbitrary (2M-1) transmitted symbols. **H** is the $M \times (2M - 1)$ dimensional channel matrix. $\mathbf{v}_M = \left[v_0, v_1, \dots, v_{M-1}\right]^T$ denotes the additive white Gaussian noise (AWGN) noise vector, where $v_i \sim C\mathcal{N}(0, N_0)$, $i = 0, 1, \dots, M - 1$ with N_0 denotes the noise variance, i.e., the AWGN Unilateral power spectral density. According to Eq. (4), we can see that the inter-symbol-interference (ISI) can not be negligible in the received impulse sequence \mathbf{r}_{M} .

Denote $\hat{\mathbf{s}}_{2M-1}$ be the estimation of ISI by the ideal equalization with the help of perfect channel state information, where $\hat{\mathbf{s}}_{2M-1} = [\hat{s}_{-(M-1)}, \cdots, \hat{s}_{-1}, \hat{s}_0, \hat{s}_1, \cdots, \hat{s}_{(M-1)}]^T$. Then the received symbol vector can be written as (5). In (5), $[w_0, w_1, \cdots, w_{M-1}]^T$ means the combination item of the residual interference and noise. Define \mathbf{w}_M as $[w_0, w_1, \cdots, w_{M-1}]^T$. The residual interference could be completely removed after the ideal equalization so as to have $\mathbf{w}_M = \mathbf{v}_M$.

$$\mathbf{r}_{M}' = \begin{bmatrix} r_{0}' \\ r_{1}' \\ \vdots \\ r_{M-1}' \end{bmatrix} = \mathbf{r}_{M} - \mathbf{H}\hat{\mathbf{s}}_{2M-1} = \begin{bmatrix} h_{0} \\ h_{1} \\ \vdots \\ h_{M-1} \end{bmatrix} s_{0} + \begin{bmatrix} w_{0} \\ w_{1} \\ \vdots \\ w_{M-1} \end{bmatrix}$$
(5)

According to the maximum ratio combining (MRC) rule, an optimal diversity gain merger will be derived from (5) as follows:

$$\hat{s}_0 = \mathbf{h}_M^H \mathbf{r}_0' / \left\| \mathbf{h}_M \right\|^2 = s_0 + \mathbf{h}_M^H \mathbf{v}_M / \left\| \mathbf{h}_M \right\|^2 = s_0 + \varepsilon_0 \quad (6)$$

In Eq. (6), \hat{s}_0 is the estimation of the transmitted symbol s_0 , and $\varepsilon_0 = \mathbf{h}_M^H \mathbf{v}_M / \|\mathbf{h}_M\|^2$ is the residual noise. As the sum of statistics-independent Gaussian random variables is still a Gaussian random variable ¹⁰, it could be easily inferred that $\varepsilon_0 \sim \text{CN} (0, N_0 / \|\mathbf{h}_M\|^2)$.

3. The SC-FDE BER Capacity Boundary

In this section, we will demonstrate the deduction of the SC-FDE BER capacity boundary of arbitrary multipath frequency-selective channel. Based on the BER general formula and the distribution of the multipath channel, we obtain the theoretical expression and its approximate analytic expression of the SC-FDE BER capacity of arbitrary multipath frequency-selective channel. Then the correspondent BER expressions for the traditional modulations are derived.

3.1. The general theoretical results of the BER boundary

Proakis¹⁰ presented the BER formula of Gaussian channels with various modulations. It can be found that most of the BER P_b can be expressed as follows:

$$P_{b} = f(M_{0})Q(\sqrt{\frac{E_{b}}{N_{0}}g(M_{0})})$$
(7)

In (7), M₀ denotes the modulation order. $f(M_0)$ and $g(M_0)$ are functions only related to the modulation order. Function Q(x) is $\int_x^{\infty} (1/\sqrt{2\pi}) \exp(-t^2/2) dt$. E_b denotes the bit energy. The BER general formula can be derived by applying Eq. (6) with Eq. (7):

$$P_{b} = f(M_{0})Q(\sqrt{g(M_{0})\frac{E_{b} \|\mathbf{h}_{M}\|}{N_{0}}})$$

$$= f(M_{0})Q(\sqrt{g(M_{0})\frac{E_{b}\sum_{i=0}^{M-1}|h_{i}|^{2}}{N_{0}}})$$
(8)

We denote $x = |h_0|^2$ and $y = \sum_{i=1}^{M-1} y_i = \sum_{i=1}^{M-1} |h_i|^2$ as the neuron of the first path channel and other path channels.

power of the first path channel and other path channels, respectively. Then (8) can be modified as:

$$P_{b}(x,y) = f(M_{0})Q(\sqrt{g(M_{0})\frac{E_{b}(x+y)}{N_{0}}})$$
(9)

The first path channel has a Rice flat fading within a range of Rice distribution. Its power, i.e., $x = |h_0|^2$, has a probability density function (PDF) of non-central χ^2 distribution with double degrees of freedom¹⁰:

$$p(x) = \frac{1}{2\sigma^2} \exp\left(-\frac{x+s^2}{2\sigma^2}\right) I_0(\frac{\sqrt{xs}}{\sigma^2})$$
(10)

In Eq. (10), s^2 and $2\sigma^2$ represent the direct radiation power component and the scattering power component, respectively. The expectation of *x*, e.g. E(x) is $s^2 + 2\sigma^2$. Here we define $K = s^2/2\sigma^2$ as the direct radiation parameter of the Rice flat fading. $I_0(z)$ is $\sum_{k=0}^{\infty} \left(\frac{z}{2}\right)^{2k} / (k!)^2$, the first kind of modified zero order Bessel function.

The other path channels are all the independent Rayleigh flat fading channels with their ranges satisfying Rayleigh distribution and powers, i.e., $y_i = |h_i|^2$. The range PDF is the central χ^2 distribution with double degrees of freedom as follows:

$$p(y_i) = \frac{1}{\gamma_i} \exp\left(-\frac{y_i}{\gamma_i}\right)$$
(11)

In Eq. (11), γ_i denotes the scattering power of the i-th path satisfying $\gamma_i = E(\gamma_i)$, $i = 1, \dots M - 1$. Then we obtain $\psi_{\gamma_i}(jv) = (1 - jv\gamma_i)^{-1}$, i.e., the eigen function corresponding to Eq. (11).

The joint eigen function of (M-1) independent variables of central χ^2 distribution is shown as follows.

$$\psi_{y}(jv) = \prod_{i=1}^{M-1} (1 - jv\gamma_{i})^{-1}$$
. (12)

Eq. (12) can be reformulated into a form of sum by partial fraction decomposition:

$$\psi_{y}(jv) = \sum_{i=1}^{M-1} \frac{(-\gamma_{i})^{M-2}}{\prod_{\substack{j=1\\j\neq i}}^{M-1} (\gamma_{j} - \gamma_{i})} \frac{1}{(1 - jv\gamma_{i})}$$
(13)

The PDF of y listed below can be deduced by applying the Fourier transformation with Eq. (13):

$$p(y) = \sum_{i=1}^{M-1} \frac{-(-\gamma_i)^{M-3}}{\prod_{\substack{j=1\\j\neq i}}^{M-1} (\gamma_j - \gamma_i)} \exp(-\frac{y}{\gamma_i})$$
(14)

By integrating (9), (10) and (14), we obtain the BER theoretical value of SC-FDE within arbitrary multipath channel:

$$\overline{P}_{b} = = \int_{0}^{\infty} \int_{0}^{\infty} f(M_{0}) Q(\sqrt{g(M_{0})} \frac{E_{b}(x+y)}{N_{0}}) p(x) p(y) dx dy \quad (15)$$

Eq. (15) provides a double integral expression of the SC-FDE BER capacity. The BER value can be derived by numerical integration of (15). With the aids of the inequation¹¹ $(Q(\sqrt{a+b}) \le Q(\sqrt{a})\exp(-b/2))$ and the discussion of the Rice and Rayleigh flat fading in the relevant references, here we will derive the approximate upper boundary of the SC-FDE BER capacity within arbitrary multipath frequency-selective channel.

Applying Eq. (15) with the above inequation, we formulate the following BER as:

$$\overline{P}_{b} \leq \underbrace{\int_{0}^{\infty} \mathcal{Q}(\sqrt{g(M_{0})\frac{E_{b}x}{N_{0}}})p(x)dx}_{\Phi} \times \underbrace{\int_{0}^{\infty} f(M_{0})\exp\left(-g(M_{0})\frac{E_{b}y}{2N_{0}}\right)p(y)dy}_{\Theta}$$
(16)

In Eq. (16), the parameters Φ and Θ are interpreted as follows:

$$\Phi = \frac{1}{2\sigma^2} \int_0^\infty Q(\sqrt{g(M_0) \frac{E_b x}{N_0}}) \exp\left(-\frac{x+s^2}{2\sigma^2}\right) I_0(\frac{\sqrt{xs}}{\sigma^2}) dx \quad (17)$$

and

$$\Theta = f(M_0) \sum_{i=1}^{M-1} \int_{0}^{\infty} \exp\left(-g(M_0) \frac{E_b y}{2N_0}\right) \frac{-(-\gamma_i)^{M-3}}{\prod_{\substack{j=1\\j\neq i}}^{M-1} (\gamma_j - \gamma_i)} \exp\left(-\frac{y}{\gamma_i}\right) dy$$

$$= f(M_0) \sum_{i=1}^{M-1} \frac{-(-\gamma_i)^{M-3}}{\prod_{\substack{j=1\\j\neq i}}^{M-1} (\gamma_j - \gamma_i)} \int_{0}^{\infty} \exp\left(-\left(g(M_0) \frac{E_b}{2N_0} + \frac{1}{\gamma_i}\right) y\right) dy$$
(18)

Then the analytic solution of the (17) is solved as follows⁹:

$$\Phi = \frac{\exp\left(-\frac{s^2}{2\sigma^2}\right)}{4\sigma^2} \sum_{n=0}^{\infty} \frac{s^{2n}}{\left(2\sigma^2\right)^{n-1} n!} \left[1 - \sum_{k=0}^n \mu\left(\frac{1-\mu^2}{4}\right)^k \binom{2k}{k}\right]$$
(19)

In Eq. (19), μ is $\sqrt{\frac{g(M_0)E_b\sigma^2/N_0}{(1+g(M_0)E_b\sigma^2/N_0)}}$. μ has

various expressions for different modulation methods. The expressions of μ and Φ is decided by the modulation method, which will be shown lately.

Through the definite integration to the exponential function in (18), we can obtain the following:

$$\Theta = f(M_0) \sum_{i=1}^{M-1} \frac{2(-\gamma_i)^{M-2} N_0}{\prod_{\substack{j=1\\j\neq i}}^{M-1} (\gamma_j - \gamma_i) (g(M_0) E_b \gamma_i + 2N_0)}$$
(20)

Combining (16), (19) and (20), the approximate SC-FDE BER capacity boundary within arbitrary multipath frequency-selective channel is obtained as:

$$\overline{P}_{b} \leq \frac{f(M_{0}) \exp\left(-\frac{s^{2}}{2\sigma^{2}}\right)}{2\sigma^{2}} \times \sum_{i=1}^{M-1} \frac{(-\gamma_{i})^{M-2} N_{0}}{\prod_{\substack{j=1\\j\neq i}}^{M-1} (\gamma_{j} - \gamma_{i}) (g(M_{0}) E_{b} \gamma_{i} + 2N_{0})} \times \sum_{n=0}^{\infty} \frac{s^{2n}}{(2\sigma^{2})^{n-1} n!} \left[1 - \sum_{k=0}^{n} \mu \left(\frac{1-\mu^{2}}{4}\right)^{k} \binom{2k}{k}\right]$$
(21)

3.2. The BER capacity boundary for the traditional modulation method

In Section 3.1, we discussed the integration expression and approximate boundary of the SC-FDE BER capacity within arbitrary multi-path frequency-selective channel. In this section we will analyze the approximate BER capacity boundary for the traditional modulation method.

As MPSK ($M_0 \le 4$) and MQAM have similar BER expressions as Eq. (7), we will discuss the approximate BER capacity boundary for the MPSK and MQAM. MPSK and MQAM are more popular in the broadband wireless communication area.

(i) The approximate BER capacity boundary of MPSK

The BER expression of MPSK ($M_0 \le 4$) P_b is

 $Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ within Gaussian channel. Here, we assume

that $M_0 = 2, 4$, $f(M_0) = 1$, $g(M_0) = 2$ and μ

is
$$\sqrt{\frac{2E_b\sigma^2}{N_0+2E_b\sigma^2}}$$
 in Eq. (19). From above, we obtain the

approximate SC-FDE BER capacity boundary within arbitrary multipath frequency-selective channel by MPSK ($M_0 \le 4$) modulation as Eq. (22):

$$\overline{P}_{b,MPSK} \leq \frac{\exp\left(-\frac{s^2}{2\sigma^2}\right)}{4\sigma^2} \sum_{i=1}^{M-1} \frac{(-\gamma_i)^{M-2} N_0}{\prod_{\substack{j=1\\j\neq i}}^{M-1} (\gamma_j - \gamma_i) (E_b \gamma_i + N_0)}$$
(22)
$$\times \sum_{n=0}^{\infty} \left[\frac{s^{2n}}{(2\sigma^2)^{n-1} n!} \Psi_1(n)\right]$$

In Eq. (22), $\Psi_1(n)$ is defined in (23).

$$\Psi_{1}(n) = 1 - \sum_{k=0}^{n} \sqrt{\frac{2E_{b}\sigma^{2}}{N_{0} + 2E_{b}\sigma^{2}}} \left(\frac{1 - \frac{2E_{b}\sigma^{2}}{N_{0} + 2E_{b}\sigma^{2}}}{4} \right)^{k} \binom{2k}{k}$$
(23)

(ii) The approximate BER capacity boundary of MQAM

The BER expression¹⁰ of MQAM within Gaussian channel is Eq. (24).

$$P_{b} \cong \frac{2\sqrt{M_{0}-1}}{\sqrt{M_{0}}\log_{2}\sqrt{M_{0}}} Q\left(\sqrt{\frac{3(\log_{2}M_{0})E_{b}}{(M_{0}-1)N_{0}}}\right).$$
(24)

Here, we define the following functions and variable.

$$f(M_0) \text{ is } \frac{2\sqrt{M_0 - 1}}{\sqrt{M_0} \log_2 \sqrt{M_0}}, \ g(M_0) = \frac{3\log_2 M_0}{M_0 - 1} = 1 \text{ and}$$

$$\mu \text{ is } \sqrt{\frac{3(\log_2 M_0)E_b\sigma^2}{((M - 1)N_0 + 3(\log_2 M_0)E_b\sigma^2)}} \text{ in Eq. (19).}$$

From above, we obtain the approximate BER capacity boundary within arbitrary multipath frequency-selective channel by MQAM modulation as follows:

$$\overline{P}_{b,MQAM} \leq \frac{\sqrt{M_{0} - 1} \exp\left(-\frac{s^{2}}{2\sigma^{2}}\right)}{\sigma^{2} \sqrt{M_{0}} \log_{2} \sqrt{M_{0}}} \times \sum_{n=0}^{\infty} \{\Lambda_{1}(n)\} \times \sum_{i=1}^{M-1} \frac{(-\gamma_{i})^{M-2} N_{0}}{\prod_{\substack{j=1\\j\neq i}}^{M-1} \left(\gamma_{j} - \gamma_{i}\right) \left(\frac{3 \log_{2} M_{0}}{M_{0} - 1} E_{b} \gamma_{i} + 2N_{0}\right)}$$
(25)

 $\Lambda_1(n)$ in Eq.(25) is defined in Eq. (26).

$$\Lambda_{1}(n) = \frac{s^{2n}}{\left(2\sigma^{2}\right)^{n-1} n!} \times \left[1 - \sum_{k=0}^{n} \left(\sqrt{\frac{3(\log_{2} M_{0})E_{b}\sigma^{2}}{(M_{0} - 1)N_{0} + 3(\log_{2} M_{0})E_{b}\sigma^{2}}}\Lambda_{2}(k)\right)\right]$$
(26)

 $\Lambda_2(k)$ in Eq.(26) is defined as:

$$\Lambda_{2}(k) = \left(\frac{1 - \frac{3(\log_{2} M_{0})E_{b}\sigma^{2}}{(M_{0} - 1)N_{0} + 3(\log_{2} M_{0})E_{b}\sigma^{2}}}{4}\right)^{k} \binom{2k}{k}$$
(27)

4. Capacity Simulation and Results Analysis

Based on the above theoretical values and approximate boundary of the SC-FDE BER capacity with arbitrary multipath frequency-selective channel, this section will test and verify the results by numerical computation and the Monte-Carlo simulation taking the 6-path channel as an typical case.

There are various models for the arbitrary multipath frequency-selective channel. Among them, the Stanford University Interim Channel Model (SUI) is suitable for the IEEE 802.16 fixed wireless access. We select the modified 6-path SUI-3 model¹² (in Table 1) to simulate and verify the proposed BER performance bounds in this paper.

Table 1 the parameters of the modified6-path SUI-3 channel model

Index	Value
Path Gain (dB)	[0, -3, -5, -7, -10, -12]
Κ	[1,0,0,0,0,0]
First path s^2 , σ^2	0.229, 0.115
$\left[\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\right]$	[0.230,0.145,0.092,0.0459,0.0289]

In Table 1, the path gain and the direct radiation parameter are set by SUI channel model. The first path s^2 , σ^2 and $[\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5]$ are calculated by the known parameters. As described above, we assumed that the SC-FDE symbol spacing is same as the adjacent delay difference of multipath, thus we don't take into account the path delay in the SUI model. Furthermore, we will not think over time-selectivity caused by factors such as movement occurring in the frequency-selective arbitrary multi-path channel. As a result, the tiny Doppler frequency shift of each path is equal to zero in the SUI model.

In the simulation, QPSK and 16QAM are used as the modulation methods. To compare with the theoretical value, we take the SC-FDE BER performance simulation curves employing ZF (Zero Forcing) and MMSE (Minimum Mean Square Error) equalization as the references, respectively. Both of the two SC-FDE methods share the same parameters, including the symbol block length of 64, the circulated prefix length of 8 and the simulation length of 16384 blocks. Figure 1 and 2 illustrate the SC-FDE BER performance curve of

SUI-3 6-path channel model under QPSK and 16QAM modulations, respectively.

In figure 1 and 2, we demonstrate that the SC-FDE BER performance against the Bit signal noise ratio (E_b/N_0) . Both of the QPSK and 16QAM BER theoretical value are calculated by the numerical integration of Eq. (15) and the correspondent BER approximate bounds are the results of (22) and (23). It can be found that the difference between the approximate bounds and the theoretical bounds of the SC-FDE BER performance is only 0.7-0.8dB for QPSK and 16QAM modulation. Consequently it is of value to study the different SC-FDE methods.



Figure 1 the SC-FDE BER performance of SUI-3 6-path channel model under QPSK modulation



Figure 2 the SC-FDE BER performance of SUI-3 6-path channel model under 16QAM modulation

When it comes to two practical SC-FDEs, there is a big gap between the SC-FDE BER performance using ZF or MMSE equalization and its bounds. This is more evident to the ZF equalization because the ZF noise magnifying effect is apt to lead to error symbol platform. When employing the QPSK modulation, the SC-FDE BER performance approaches more to its bounds than that of the 16QAM modulation does. The reason is that the SC-FDE is more easily affected by the modulation order, i.e., its BER performance gets worse with the higher modulation order.

The gap between the SC-FDE theoretical performance bounds and its practical performance (eg. using MMSE equalization) indicate a possibility in finding a better equalization or symbol detecting method to enhance the BER performance.

5. Conclusions

The safer traffic system need more research on reliable mobile communication, but there are few researches about BER performance of SC-FDE, a key mobile communication technology for transportation system, especially for arbitrary multi-path channels. A BER performance bound under arbitrary multipath mobile communication channels is proposed in this paper.

Based on the former results about the SC BER performance under Rice channel and Rayleigh flat fading channel, we analyzed the BER performance bounds of SC-FDE under arbitrary multi-path frequency-selective channels and derived the general formula in the analytic form. The approximate upper bound of the BER performance can be applied to various typical modulations to obtain the specific analytical BER bounds. Numerical analysis and simulation results show that the proposed BER bounds of SC-FDE highly approach the theoretical one for calculating the BER performance. Furthermore, the bound also indicates that the preferable equalization methods or symbol detecting methods should be studied to narrow the gap between the SC-FDE BER practical performance and its bounds.

References

- W. Wang, Y. Mao, J. Jing, X. Wang, H. Guo, X. Ren and I. Katsushi, Driver's various information process and multi-ruled decision-making mechanism: a fundamental of intelligent driving shaping model, International Journal of Computational Intelligence Systems, 4(3) (2011) 297-305.
- W. Wang, H. Guo, K. Ikeuchi and H. Bubb, Numerical simulation and analysis procedure for digital driving dependability in intelligent transport system, KSCE Journal of Civil Engineering, 15(5) (2011) 891-898.
- W. Wang, F. Hou, H. Tan and H. Bubb, A framework for function allocation in intelligent driver interface design for comfort and safety, International Journal of Computational Intelligence Systems, 3(5) (2010) 531-541.

- F. Pancaldi, G. M. Vitetta and R. Kalbasi, et al. *Single-carrier frequency domain equalization*. Signal Processing Magazine, IEEE, 25 (2008), 37-56.
- D. Tse and P. Viswanath. *Fundamentals of Wireless Communication*. (Cambridge, U.K.: Cambridge Univ. Press 2005), pp.166-227.
- 6. T. Shi, S. Zhou and Y. Yao. *Capacity of single carrier* systems with frequency-domain equalization, Emerging Technologies: Frontiers of Mobile and Wireless Communication, Proceedings of the IEEE 6th Circuits and Systems Symposium on, 2 (2004) 429-432.
- 7. H. Zhang and T. Gulliver. *Error probability for maximum ratio combining multichannel reception of M-ary coherent systems over flat Ricean fading channels*,. IEEE Wireless Communications and Networking Conference 2004, 1 (2004) 306-310.
- 8. IEEE Std. 802.16-2009. *IEEE Standard for Local and metropolitan area networks Part 16: Air Interface for Broadband Wireless Access Systems*. 2009.
- P. Xiao, L.G. Barbero and M. Sellathurai, et al. On the Uncoded BER Performance Bound of the IEEE 802.16d Channel. IEEE Signal Processing Letters, 15 2008,: 561-564.
- 10. J. Proakis. *Digital Communications 4th edn* (McGraw-Hill Press, New York, 2000).
- 11. S. Verdu. *Multiuser Detection 1st edn*, (U.K.: Cambridge Univ. Press, Cambridge, 1998).
- 12. IEEE Std. 802.16a cont. IEEE 802.16.3c-01/29r4. Channel models for fixed wireless applications. Jun. 2003.