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Phase Plane Analysis for Vehicle Handling and Stability

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Abstract

Nonlinear stability analysis of phase plane is one of the most classic and effective methods. Based on established two degrees of freedom (2 DOF) vehicle model, combined with magic formula tire mode, phase plane analysis is used to study vehicle motion characteristics in various initial conditions. In addition, equilibrium point and phase plane trajectories changes are analyzed under the condition of traveling in a straight line, circular motion and sine wave input. The results denote that vehicle steady state can be well determined according to analysis changes of phase plane trajectory and state variables. Namely, phase plane analysis can be well used to analyze and evaluate the trend of variables and vehicle motion characteristics.

Keywords: Nonlinear Analysis, Handling and Stability, Phase Plane Analysis, Magic Formula

1. Introduction

Phase plane analysis is one of the most classic and effective methods in nonlinear stability analysis. This method references the idea of nonlinear dynamics. Many dynamic characteristics of system can be obtained through the observation and analysis on the phase trajectory of phase plane[1-5], for example, the changes of system stability region can be represented by the changing trend of phase trajectories, the change of system stability during steady motion can be caused by the change of equilibrium point on phase plane.

Phase plane analysis method can show system dynamic performance characteristics, it is widely used in vehicle handling and stability analysis[5-8]. Different phase plane analysis methods are established by researchers, such as: the yaw plane established by Sacks[9], front and rear side slip angle phase plane established by Pacejka[10], ratio of turning kinetic energy and forward driving kinetic energy phase plane used to evaluate steering performance established by Konghui Guo[11]. Konghui Guo focused on energy phase plane transient response of a variety of tests and results of quasi-steady explanation, evaluation and comparison. Two parameters of phase plane were the dimensionless center of yaw rate and side slip angle. Inagaki[3,4] proposed a phase plane which constitutes a state variable with vehicle body side slip angle and its changing rate($\beta-\dot{\beta}$) and vehicle body side slip angle and yaw rate ($\beta-r$),and studied the dynamic characteristics of 2 DOF vehicle system during its steering by using phase plane analysis method based on nonlinear tire model (magic formula).Young Eun Ko and Jang Moo Lee[4] identified stability region of phase plane trajectory in $\beta-r$ phase plane with different longitudinal speed and front wheel angle input based on 2 DOF vehicle model and topological theory. A. Stotsky and X. Hu[1] established the equation of 2 DOF steering dynamic system to determine the system's Lyapunov function, and obtained the analytical expression of state variables (yaw rate and steering wheel angle) in border of stability region by using the expression of the cubic term tire force. John Samsundar[4] obtained the analytical expression of the stable region of the steering dynamic system by using the Lyapunov second method and the expression of cubic term tire force, and obtained

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conservative and elliptical stability boundary of the system by numerical simulation. Shuming Shi and Zhenyong Mao[12-13] obtained the tire lateral force using linear equations fitted by first order Taylor expansion formula express nonlinear expression. The scope of tire force is extended to the whole nonlinear region, which can meet vehicle in high speed and emergency steering.

Other research content in the phase plane analysis of vehicle handling and stability is equilibrium point. Nonlinear system[14] generally has more than one equilibrium point. The results of Inagaki[3,4] show that when vehicle dynamic system parameters (longitude speed and front wheel angle) change, system equilibrium point moves along with corresponding changes. However, when front wheel angle increases to a certain extent, a stable focus point and an unstable saddle point disappear, another unstable saddle point is left. Inagaki[3] and Ono[15,16] obtain the same conclusion, vehicle system has three equilibrium points, one point is stable focus point, and the other two are unstable saddle points. Catino[17] uses MATCONT software also obtains similar conclusion. Young[5] and Nguyen[18] also obtained a similar bifurcation diagram, but their analysis is local effective. Shuiwen Shen[19] extended simplified magic formula tire mode to the whole nonlinear region, obtains three singular points, a stable focus and two unstable saddle points using geometric method. System stable motion characteristics can be obtained by analyzing the change of stable equilibrium point. The border of system stable region can be obtained by analyzing the distribution of unstable equilibrium point.

Therefore, in order to enhance the understanding of vehicle planar motion stability, improve traffic safety[20,21], based on 2 DOF vehicle model, phase plane analysis is used to study vehicle motion characteristics under the condition of traveling in straight line, circular motion and sine wave input motion by using magic formula tire model in this paper.

2. Establishment of vehicle nonlinear model

2.1. *Vehicle model establishment*

2 DOF vehicle model is classical system equation used in handling and stability commonly. In this system equation, longitudinal vehicle speed is assumed as a constant, and tire longitudinal force and air resistance are ignored，as shown in Fig. 1.

$$\begin{cases} m(\dot{v}_y + v_x r) = 2F_{sf}\cos\delta_f + 2F_{sr} \\ I_z \dot{r} = 2F_{sf}\cos\delta_f l_f - 2F_{sr} l_r \end{cases} \quad (1)$$

Where, m—vehicle mass 1640kg; v_y—vehicle lateral velocity m/s; v_x—vehicle longitudinal velocity m/s; r—vehicle yaw velocity rad/s; δ_f—front wheel steering angle rad; F_{sf}—front wheel lateral force N, F_{sr}—front wheel lateral force N, I_z—vehicle yaw moment of inertia 2900$kg{\cdot}m^2$; l_f—distance between front axle and centroid 1.1m; l_r—distance between rear axle and centroid 1.4m.

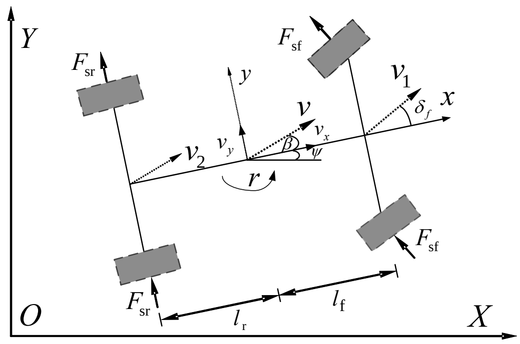


Fig. 1. Vehicle Model

Side slip angles of both front and rear tires are given as follows:

$$\alpha_f = \delta_f - \arctan\left(\frac{v_y + r\cdot l_f}{v_x}\right)$$
$$\alpha_r = -\arctan\left(\frac{v_y - r\cdot l_r}{v_x}\right) \quad (2)$$

2 DOF vehicle system dynamics equations can be simplified as follows:

$$\dot{v}_y = \frac{F_{sf}\cos\delta_f + F_{sr}}{m} - v_x r$$
$$\dot{r} = \frac{F_{sf} l_f \cos\delta_f - F_{sr} l_r}{I_z} \quad (3)$$

2.2. *Tire model*

Lateral tire force has important impact on the vehicle nonlinear dynamics system stability analysis. The common tire force models mainly include Konghui Guo's Unityre model[22], Pacejka's magic formula (Magic Formula)[23,24], tire model established by Gim

G[25,26], and tire model established by Fiala E[27], where the magic formula model is formulated:

$$F_s = D\sin\left(C\arctan\left(B\alpha - E\left(B\alpha - \arctan B\alpha\right)\right)\right) \quad (4)$$

Where B, C, D, E is coefficients, F is the lateral tire force, and α is side slip angle.

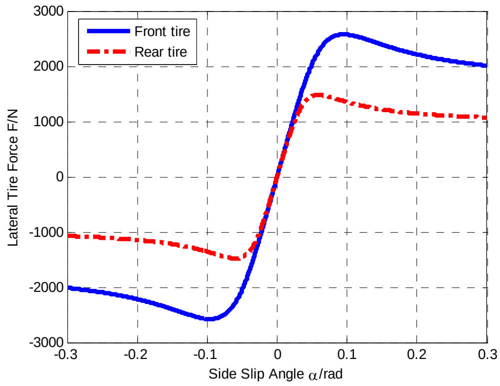


Fig. 2. Lateral tire force

Table 1 Magic formula model parameters

Tire	Coefficient			
	B	C	D	E
Front tire	11.275	1.56	2574.7	-1.9990
Rear tire	18.631	1.56	1749.7	-1.7908

The coefficients are listed in Table 1[28], while Fig. 2 graphically shows the relationship between the lateral tire forces and side slip angles for the respective front and rear tires.

3. Phase plane Portraits of 2DOF

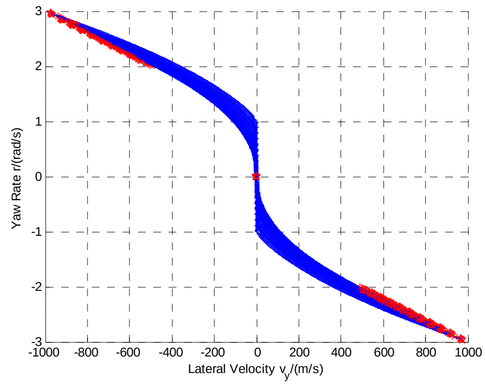
In 2 DOF vehicle model, front steering wheel angle is used as external input variable, it is an important factor in system equation. When front steering wheel angle input different values, vehicle dynamics performance is very different, such as if the front steering wheel angle is different, then trajectory and equilibrium point have different values. Vehicle dynamics is analyzed with different front wheel angles (e.g. zero, a non-zero constant, sine wave input).

3.1. *Phase plane portraits with the variation of initial longitudinal velocity*

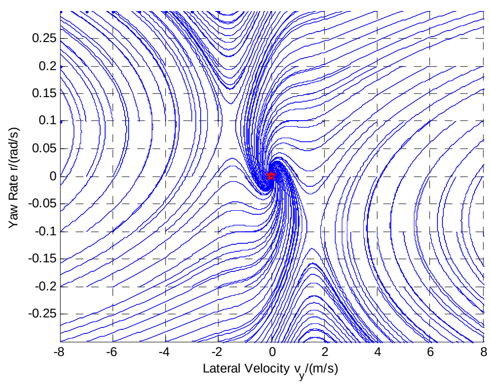
Fig. 3 and Fig. 4 show the vehicle trajectories, where Fig.（a）of Fig. 3 and Fig. 4 are global view of v_y-r phase plane trajectories and Fig.（b）of Fig. 3 and Fig. 4 are local view of it. In those figures v_y and r taking their initial value from the interval of (–10,10) *m/s* and (–1,1) *rad/s* and the initial longitudinal velocity being 25*m/s* and 35*m/s*, respectively. Simulation time is 20*s*.

There are three equilibrium points in phase plane, one is stable equilibrium point, and the others are unstable equilibrium point[29-33]. There is a strip stable region in phase plane, and point in this region could return to stable equilibrium point after a certain time, but the outside can't return to the stable region[29,33].

These results are used to distinguish the stable region and unstable region of 2 DOF vehicle model. It can be seen from the phase plane, the middle part of vehicle is stability region. In this region, variables can return to the final point of the final steady state that is the value of lateral velocity and yaw acceleration is zero. Moreover, the range of stable region is gradually shrinking while longitudinal velocity is increasing.

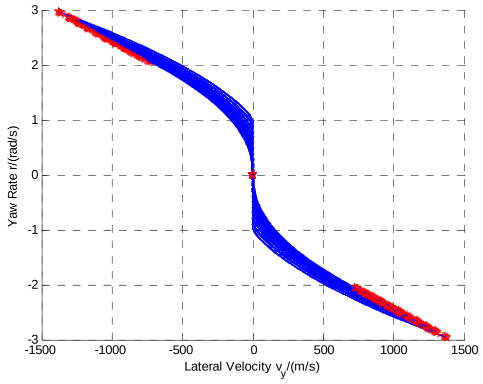


（a）Global view of v_y-r phase plane trajectories

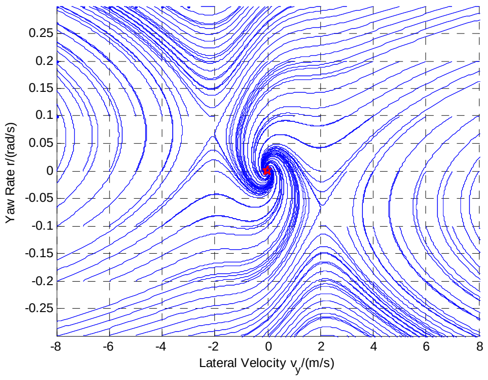


（b）Local view of v_y-r phase plane trajectories

Fig. 3. Lateral tire force



(a) Global view of v_y-r phase plane trajectories



(b) Local view of v_y-r phase plane trajectories

Fig. 4 v_y-r phase plane with initial longitudinal velocity being v_x=*35m/s*

3.2. *Phase plane portraits with the variation of front wheel steering angle*

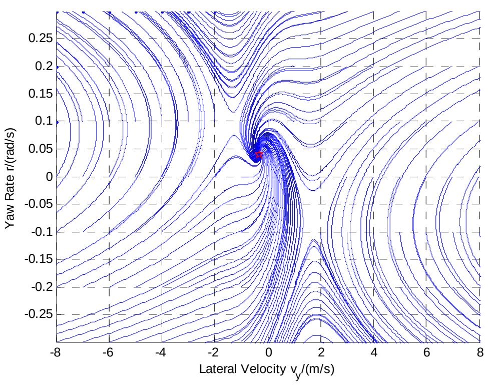


Fig. 5. Local phase plane trajectories onto v_y-r plane with δ_f being 0.01 *rad*

Vehicle movement will transit to steady state circular motion when front wheel steering angle is non-zero constant value. Vehicle dynamics performance will be analyzed in different initial states.

Fig. 5 and Fig. 6 show local phase plane trajectories on v_y-r plane when lateral velocity is 25*m/s* Simulation time is 20*s*. Front wheel steering angle δ_f is 0.01*rad* and 0.05*rad*, respectively.

When front wheel steering angle is small (e.g. 0.01*rad*), the system has stable equilibrium point, however when the angle increases to a certain extent(e.g. 0.05*rad*), system's stable equilibrium point disappears and it is in unstable state of motion.

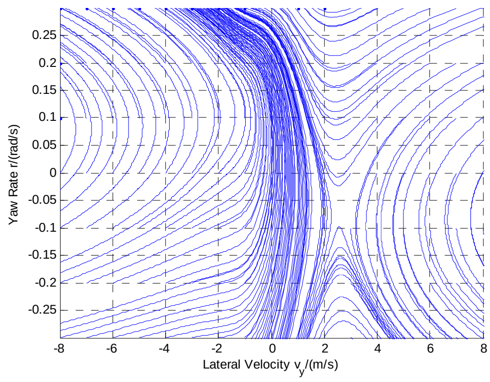


Fig. 6. Local phase plane trajectories onto v_y-r plane with δ_f being 0.05 *rad*

3.3. *Front wheel steering angle sine wave input*

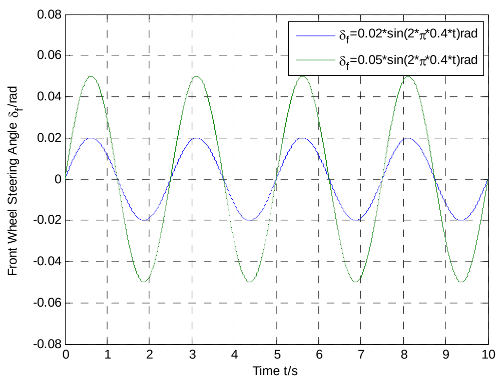


Fig. 7 Front wheel steering angle sine wave input

Fig. 7 represents the results of front wheel steering angle sine wave inputs. All of frequencies of the sine waves are 0.4*Hz*, and the amplitudes are 0.02*rad* and 0.05*rad*, respectively.

Fig. 8 and Fig. 9 show phase plane trajectories with the amplitude of δ_f being 0.02*rad* and 0.05*rad* respectively, and the lateral velocity is 25*m/s*.

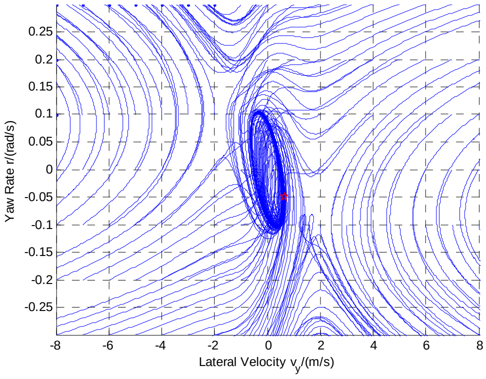


Fig. 8 Local view of v_y-r phase plane trajectories with the amplitude of δ_f being 0.02*rad*

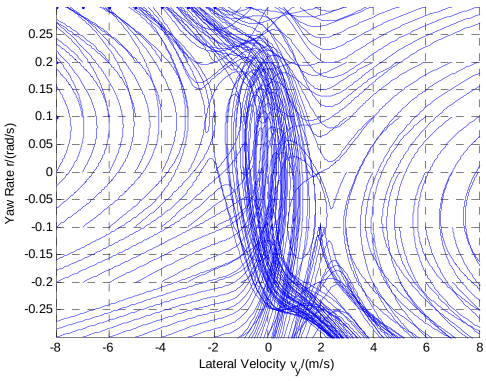


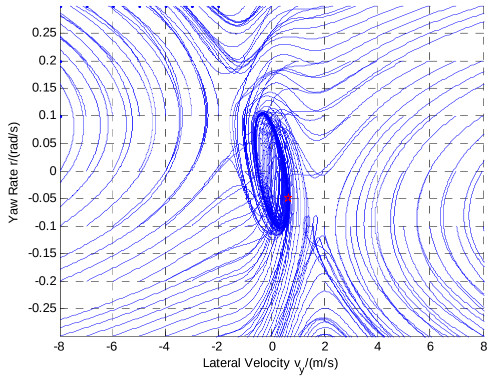
Fig. 9 Local view of v_y-r phase plane trajectories with the amplitude of δ_f being 0.05*rad*

When front wheel steering angle is sine wave input, vehicle system's equilibrium position is not a fixed point, but a trajectory. If vehicle system is stable, state variables and front wheel angle change are consistent and show a periodic. There is an equilibrium position and stability region when amplitude of front wheel steering angle is small (e.g. A=0.02*rad*). But amplitude of front wheel steering angle increases to a certain extent (e.g. A=0.05*rad*), vehicle system will not exist. Equilibrium position and the corresponding stability region will disappear.

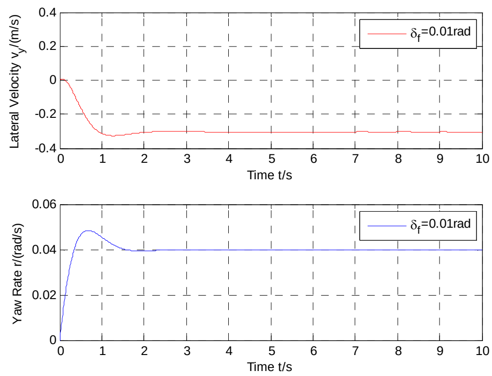
4. Analysis of Results

4.1. *Front wheel steering angle constant inputs*

A trajectory on the phase plane is analyzed for the sake of analysis the change of phase trajectory and corresponding vehicle motion. Vehicle initial state is v_x=25*m/s*, v_y=r=0 with different front wheel steering angle inputs.



(a) v_y-r phase plane trajectories

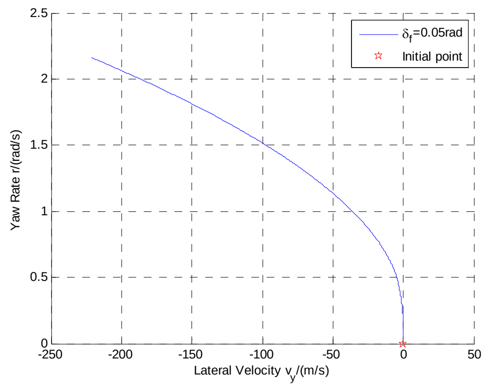


(b) state variables

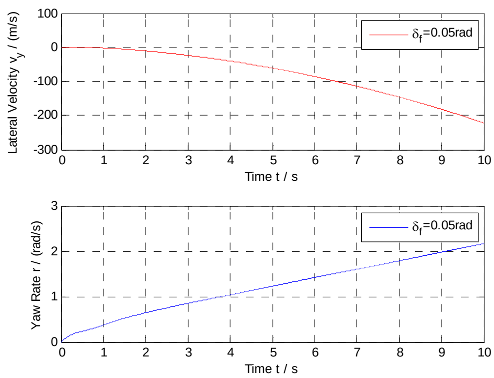
Fig. 10 Phase plane trajectories and state variables with front wheel steering angle being 0.01*rad*

Fig. 10 and Fig. 11 show phase plane trajectories and state variables with front wheel steering angle being 0.01*rad* and 0.05*rad* respectively and the lateral velocity is 25*m/s*.

The results show 2 DOF vehicle model stable and unstable phase plane and state variable time domain characteristic. When the steering system is stable (e.g. δ_f =0.01*rad*), the system can return to the stable equilibrium point, and the final state variables v_y and r return to the constant stability values. When the steering system is unstable (e.g. δ_f =0.05*rad*), state variables v_y and r grow rapidly and vehicle system loses stability.



(a) v_y-r phase plane trajectories

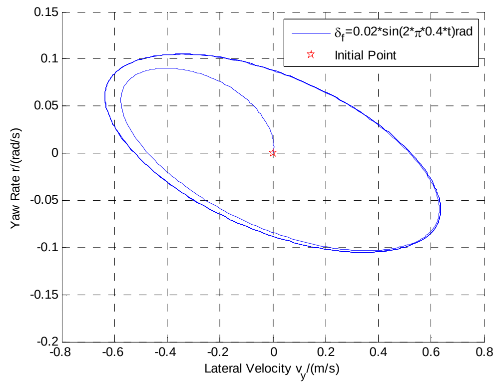


(b) state variables

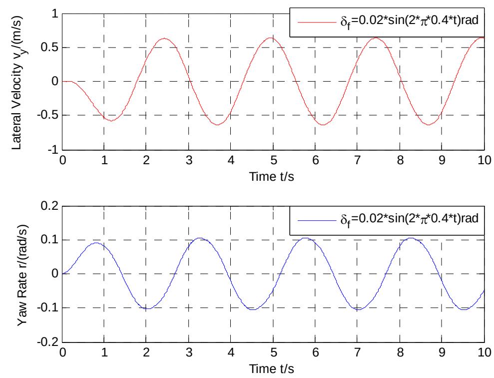
Fig. 11 Phase plane trajectories and state variables with front wheel steering angle being 0.05*rad*

4.2. *Front wheel steering angle sine wave input*

In order to analyze vehicle movement characteristics of stable and unstable, a comparative analysis is studied with different front wheel angle sine wave inputs when vehicle initial state is v_x=25*m/s*, v_y=r=0.

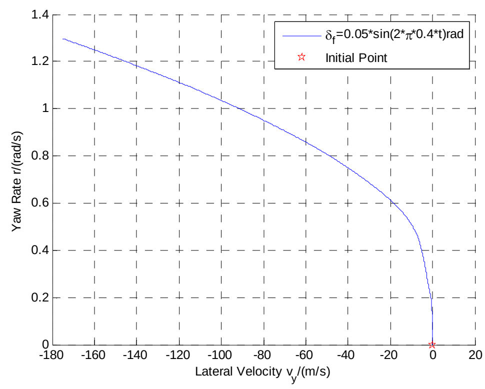


(a) v_y-r phase plane trajectories

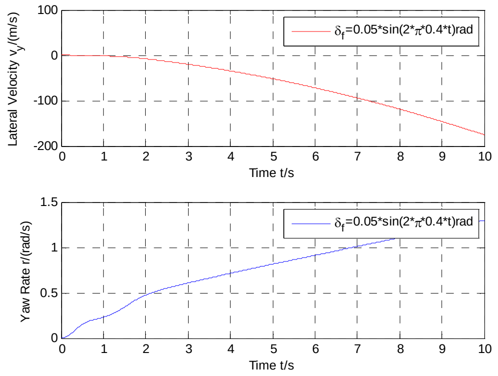


(b) state variables

Fig. 12 Phase plane trajectories and state variables with the amplitude of δ_f being 0.02*rad*



(a) v_y-r phase plane trajectories



(b) state variables

Fig. 13 Phase plane trajectories and state variables with the amplitude of δ_f being 0.05*rad*

The results show 2 DOF vehicle model stable and unstable phase plane and state variable frequency domain characteristic. When steering input is small (e.g. A=0.02*rad*), vehicle system can return to stable equilibrium point, and changes of state variables v_y and r are stability. When the steering input is large (e.g. A=0.05*rad*), state variables v_y and r grow rapidly, the system is unstable. This can be used to judge system stability.

Form the results of phase plane analysis under different various initial conditions (in a straight line, circular motion and sine wave input), 2 DOF vehicle plane phase can analyze vehicle handling and stability characteristics and state variables change. In other words, the proposed model and the used method can solve the vehicle handling and stability problem.

5. Conclusion

(1) Local view of phase plane trajectories reflects the changes of system stability region; it can be used to analyze vehicle handling and stability characteristic under different various initial conditions.

(2) A trajectory on the phase plane can be use to analyze the change of phase trajectory and corresponding vehicle motion. The change of equilibrium point on phase plane reflects system stability change in different motion.

(3) When phase trajectory returns to steady state, the state variables tend to a constant value, and when phase trajectory can't return to steady state, the state variables increase rapidly. Therefore, according to the change of state variables can be used to judge system stability.

Based on the above analysis, 2 DOF vehicle plane phase can analyze vehicle handling and stability characteristics and state variables change. This lays the foundation and theory guideline for the depth study of related vehicle control problem.

There exits some limitations with the proposed method. For example, in 2 DOF vehicle model, longitudinal movement is constrained, this model can only partly express vehicle planar motion dynamic characteristics. So it can't be used to comprehensively and detailed analyze vehicle stability in high speed and emergency steering, which will be the topics of future works.

Acknowledgements

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