Distinct 8-QAM+ Perfect Arrays

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Abstract. This paper investigates the construction of 8-QAM+ perfect arrays with at least an odd integer in their sizes, and gives the sufficient condition producing distinct 8-QAM+ perfect arrays. For different choices of odd integers in a given size of at least an odd integer, the resultant distinct 8-QAM+perfect arrays have their own sizes. For each choice, the number of the resultant distinct 8-QAM+perfect arrays is exactly equivalent to this chosen odd integer. This paper can provide more candidates for the applications of 8-QAM+ perfect arrays in communications, radar, and so forth.

Introduction

Perfect arrays have the property of impulse-like autocorrelation, which results in the fact that perfect arrays are widely applied to communications, radar, time-frequency-coding, map matching, synchronization, built-in tests of VLSI-circuits, and so on [1]. There are a large number of perfect binary, ternary, and ployphase array [1]. However, 8-QAM+ perfect arrays are rare, whose main reason lies at the late introduction of 8-QAM+ constellation [2]. In 2010, Boztas and Parampalli introduced the definition of 8-QAM+ signal set [2]. From that time on, the sequences over 8-QAM+ constellation are rapidly developed. Perfect and odd perfect 8-QAM+ sequences are given in 2012 and 2014 , respectively [3] [4]. 8-QAM+ complementary sequences are presented in 2012 and 2014, respectively [5] [6]. ZCZ 8-QAM+ sequences appeared in 2012 [7]. Recently, 8-QAM+ perfect arrays are proposed in 2014 and 2015 [8] [9].

The aim of this paper is to investigate 8-QAM+ perfect arrays, to increase their family size, and to reflect the state-of-the-art of this area. A sufficient condition producing distinct 8-QAM+ perfect arrays will be introduced. The number of the resultant 8-QAM+ perfect arrays will be improved.

The rest of this paper will organized as follows. In Section 2, we will review some necessary concepts and the known results on the constructions of 8-QAM+ perfect arrays. In Section 3, a sufficient condition producing distinct 8-QAM+ perfect arrays will be given. The examples will appear in Section 4. Finally, we conclude this paper in Section 5.

Necessary Concepts and Known Constructions of 8-QAM+ Perfect Arrays

In a mathematical term, an array A of n - dimension with the size $N_1 \times N_2 \times \cdots \times N_n$ is

$$A = \left[a_{i_1,i_2,\cdots,i_n} \mid 0 \le i_k < N_k, 1 \le k \le n \right], \tag{1}$$

whose periodic autocorrelation is defined by

$$R_{A,A}(\tau_1, \tau_2, \cdots, \tau_n) = \sum_{i_1=0}^{N_1-1} \sum_{i_2=0}^{N_2-1} \cdots \sum_{i_n=0}^{N_n-1} a_{i_1,i_2,\cdots,i_n} \left[a_{i_1+\tau_1, i_2+\tau_2, \cdots, i_n+\tau_n} \right]^*, \tag{2}$$

where the symbol x^* denotes the complex-conjugate of x, and the subscript addition $i_k + \tau_k$ $(1 \le k \le n)$ is performed modulo N_k.

If the autocorrelation of an array A of n - dimension satisfies

$$R_{A,A}(\tau_1,\tau_2,\cdots,\tau_n)=\begin{cases}>0 & (\tau_1,\tau_2,\cdots,\tau_n)=(0,0,\cdots,0)\\ 0 & (\tau_1,\tau_2,\cdots,\tau_n)\neq(0,0,\cdots,0)\end{cases}, \quad (3)$$

we refer to this array as perfect array.

Let the symbol T denote the n-dimensional cyclical shift operator, in other words, for a given array, say A in (1), and given n integers μ_k's $(1\le k\le n)$, under this operator a new array $T(\mu_1,\mu_2,\cdots,\mu_n)A$ can be given by

$$T(\mu_1,\mu_2,\cdots,\mu_n)A=\left[a_{i_1+\mu_1,i_2+\mu_2,\cdots,i_n+\mu_n}\mid 0\le i_k<N_k,1\le k\le n\right], \quad (4)$$

where the subscript addition $i_k+\mu_k$ $(1\le k\le n)$ is counted modulo N_k.

Let A and B be two arrays with the same size. If there exist n integers μ_k's $(1\le k\le n)$ so that

$$T(\mu_1,\mu_2,\cdots,\mu_n)A=B, \quad (5)$$

we say these two arrays to be equivalent, or else, to be distinct.

The 8-QAM+ constellation means the signal set [2]:

$$\Omega_8^+=\{0,-1,1,-j,j,-1-j,1+j,1-j,-1+j\}. \quad (6)$$

Based on the direct product $\{-1,0,1\}\times\{-1,0,1\}$, the 8-QAM+ constellation can be produced by the following mappings: [3]

$$\begin{aligned}\pi:\{-1,0,1\}\times\{-1,0,1\}&\to\Omega_8^+\\ (a,b)&\to\pi(a,b)\end{aligned}, \quad (7)$$

where

$$\pi_1(a,b)=aj+b \quad \text{and} \quad \pi_2(a,b)=aj-b. \quad (8)$$

By the aforementioned mappings, a ternary array can be converted an 8-QAM+array [8] [9]. The construction methods referred to in this paper are given by the following two Theorems.

Theorem 1 [8] [9]: Let A be the perfect ternary array (PTA) with the size $N_1\times N_2\times\cdots\times N_n$, and η_k and δ_k $(1\le k\le n)$ are the integers satisfying $-N_k+1\le\eta_k,\delta_k\le N_k-1$. If all the integers N_k's $(1\le k\le n)$ are even, and we have

$$\eta_k\equiv\delta_k \pmod{N_k/2} \quad (1\le k\le n), \quad (9)$$

the 8-QAM+ array $Q=\left[q_{i_1,i_2,\cdots,i_n}\mid 0\le i_k<N_k,1\le k\le n\right]$ with the size $N_1\times N_2\times\cdots\times N_n$ is perfect, where

$$q_{i_1,i_2,\cdots,i_n}=\pi_1(a_{i_1+\eta_1,i_2+\eta_2,\cdots,i_n+\eta_n},a_{i_1+\delta_1,i_2+\delta_2,\cdots,i_n+\delta_n}). \quad (10)$$

Theorem 2 [8] [9]: Let A and B be two PTAs with the same size $N_1\times N_2\times\cdots\times N_n$, where at least an integer in the set $\{N_k\mid 1\le k\le n\}$ is odd, say, N_r. Set the integers η_k's, δ_k's, ζ_k's, and λ_k's $(1\le k\le n)$ satisfy $-N_k+1\le\eta_k,\delta_k,\zeta_k,\lambda_k\le N_k-1$. Construct the 8-QAM+ array P below.

$$\begin{cases}P=\left[p_{i_1,i_2,\cdots,i_{r-1},i_r',i_{r+1},\cdots,i_n}\mid 0\le i_k<N_k,1\le k\le n,i_r'=2i_r+t,t\in\{0,1\}\right]\\ p_{i_1,i_2,\cdots,i_{r-1},i_r',i_{r+1},\cdots,i_n}=\pi_1(a_{i_1+\eta_1,\cdots,i_{r-1}+\eta_{r-1},i_r+\eta_r,i_{r+1}+\eta_{r+1},\cdots,i_n+\eta_n},b_{i_1+\delta_1,\cdots,i_{r-1}+\delta_{r-1},i_r+\delta_r,i_{r+1}+\delta_{r+1},\cdots,i_n+\delta_n})\text{ if } i_r'=2i_r\\ p_{i_1,i_2,\cdots,i_{r-1},i_r',i_{r+1},\cdots,i_n}=\pi_2(a_{i_1+\zeta_1,\cdots,i_{r-1}+\zeta_{r-1},i_r+\zeta_r,i_{r+1}+\zeta_{r+1},\cdots,i_n+\zeta_n},b_{i_1+\lambda_1,\cdots,i_{r-1}+\lambda_{r-1},i_r+\lambda_r,i_{r+1}+\lambda_{r+1},\cdots,i_n+\lambda_n})\text{ if } i_r'=2i_r+1\end{cases}. \quad (11)$$

If we have

$$\begin{cases}\delta_k-\eta_k\equiv\lambda_k-\zeta_k & \bmod N_k\ (1\le k\le n)\\ \zeta_k-\delta_k\equiv\eta_k-\lambda_k & \bmod N_k\ (1\le k\le n,k\neq r)\\ \zeta_r-\delta_r\equiv\eta_r-\lambda_r+1 & \bmod N_r\end{cases}, \quad (12)$$

the 8-QAM+ array P with the size $N_1\times\cdots\times N_{r-1}\times 2N_r\times N_{r+1}\times\cdots\times N_n$ is perfect.

Theorem 3 [8] [9]: The equation system (11) has the solution:

$$\begin{cases} \zeta_k \equiv \eta_k \pmod{N_k} & \forall \eta_k \ (1 \le k \le n, k \ne r) \\ \lambda_k \equiv \delta_k \pmod{N_k} & \forall \delta_k \ (1 \le k \le n, k \ne r) \\ \zeta_r \equiv \eta_r + \frac{N_r+1}{2} \pmod{N_r} & \forall \eta_r \\ \lambda_r \equiv \delta_r + \frac{N_r+1}{2} \pmod{N_r} & \forall \delta_r \end{cases}. \tag{13}$$

Production Procedure of New Arrays

In this section, we investigate the construction method producing distinct 8-QAM+ perfect arrays, based on Theorem 2. Assume we need such arrays with the size $N_1 \times \cdots \times N_{r-1} \times 2N_r \times N_{r+1} \times \cdots \times N_n$, where N_r is odd number. The distinct 8-QAM+ perfect arrays can be educed by following the given steps below.

Step 1: Choose two PTAs A and B with the size $N_1 \times \cdots \times N_{r-1} \times N_r \times N_{r+1} \times \cdots \times N_n$;

Step 2: Initialization. Arbitrarily choose the integers η_k 's, δ_k 's, ζ_k 's, and λ_k 's $(1 \le k \le n, k \ne r)$;

Step 3: Initialization. Arbitrarily choose the integers δ_k 's $(1 \le k \le n)$;

Step 4: Initialization. Determine the integer λ_r by $\lambda_r \equiv \delta_r + \frac{N_r+1}{2} \pmod{N_r}$;

Step 5: Iteration times: $t \leftarrow 1$;

Step 6: Set $\eta_r = t-1$;

Step 7: Determine the integer ζ_r by $\zeta_r \equiv \eta_r + \frac{N_r+1}{2} \pmod{N_r}$;

Step 8: Produce 8-QAM+ array P, denoted by P_t, by Eq. (11) in Theorem 2;

Step 9: If $t \le N_r - 1$, turn to Step 6 with $t \leftarrow t+1$, or else turn to Step 10;

Step 10: End.

Summarizing the above, we can obtain N_r distinct 8-QAM+ perfect arrays. Incidentally, for the reader who wants to know the detailed derivation of this procedure, please refer to [8].

Examples

Consider the production of 8-QAM+ perfect arrays with the size $2 \cdot 3 \times 7$. Choose the PTA with the size 3×7 below.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}. \tag{14}$$

Note that $r = 1$. In accordance with our steps, we initialize that $(\delta_1, \delta_2) = (1, 0)$, $(\eta_2, \zeta_2) = (0, 0)$, and $(\lambda_1, \lambda_2) = (0, 0)$. Now, we perform our procedure as follows.

(1) When the iteration times $t = 1$, we have $(\eta_1, \zeta_2) = (0, 2)$. The resultant 8-QAM+ perfect array is

$$P_0 = \begin{bmatrix} j & 1 & 1 & -1+j & 1 & -1+j & -1+j \\ -1 & j & j & -1-j & j & -1-j & -1-j \\ 0 & 1+j & 1+j & -1-j & 1+j & -1-j & -1-j \\ j & -1 & -1 & 1+j & -1 & 1+j & 1+j \\ 1 & j & j & 1-j & j & 1-j & 1-j \\ 0 & -1+j & -1+j & 1-j & -1+j & 1-j & 1-j \end{bmatrix}. \tag{15}$$

(2) When the iteration times $t = 2$, we have $(\eta_1, \zeta_2) = (1, 0)$. The resultant 8-QAM+ perfect array is given by

$$P_1 = \begin{bmatrix} 0 & 1+j & 1+j & -1-j & 1+j & -1-j & -1-j \\ -1+j & 0 & 0 & -1+j & 0 & -1+j & -1+j \\ 0 & 1+j & 1+j & -1-j & 1+j & -1-j & -1-j \\ 0 & -1+j & -1+j & 1-j & -1+j & 1-j & 1-j \\ 1+j & 0 & 0 & 1+j & 0 & 1+j & 1+j \\ 0 & -1+j & -1+j & 1-j & -1+j & 1-j & 1-j \end{bmatrix}. \quad (16)$$

(3) When the iteration times $t=3$, we have $(\eta_1, \zeta_2) = (2,1)$. The resultant 8-QAM+ perfect array is equivalent to

$$P_2 = \begin{bmatrix} 0 & 1+j & 1+j & -1-j & 1+j & -1-j & -1-j \\ -1 & j & j & -1-j & j & -1-j & -1-j \\ j & 1 & 1 & -1+j & 1 & -1+j & -1+j \\ 0 & -1+j & -1+j & 1-j & -1+j & 1-j & 1-j \\ 1 & j & j & 1-j & j & 1-j & 1-j \\ j & -1 & -1 & 1+j & -1 & 1+j & 1+j \end{bmatrix}. \quad (17)$$

The autocorrelation functions of all the arrays P_0 - P_2 are depicted in Fig.1 below. Apparently, it is an impulse-like function as promised in our procedure.

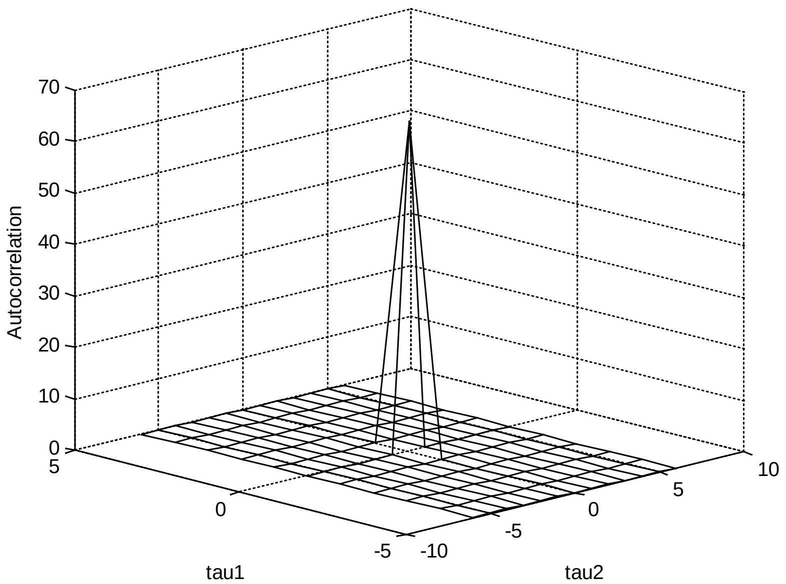

Fig.1 Autocorrelation of the resultant 8-QAM+ perfect arrays.

Conclusion

This paper gives the production procedure of 8-QAM+ perfect arrays, and some examples show its validation. However, the flaw of this procedure lies at the expansion of the size of the resultant arrays, in comparison with the size of the arrays employed. It is our future work to explore new method so as to construct the 8-QAM+ perfect arrays with the size unaltered.

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References

[1] Pingzhi Fan, Michael Darnell. Sequence design for communications applications. John Wiley & Sons INC., 1996, Ch. 12.

[2] Serdar Boztas, Udaya Parampalli. Nonbinary sequences with perfect and nearly perfect autocorrelations. Proceedings International Symposium on Information Theory, Austin, Texas, U.S.A., 2010, 1300-1304.

[3] Fanxin Zeng, et al. Perfect 8-QAM+ sequences. IEEE Wireless Commun. Lett., 2012 1(4) 388-391.

[4] Yubo Li, et al. Odd perfect sequences and sequence sets with zero odd correlation zone over the 8-QAM+ constellation. IEICE Trans. Fundamentals, 2014 E97-A(1) 425-428.

[5] Fanxin Zeng, et al. 8-QAM+ periodic complementary sequence sets, IEEE Commun. Lett., 2012 16(1) 83-85.

[6] Fanxin Zeng, et al. A note on 8-QAM+ sequences, IEICE Trans. Fundamentals, 2104 E97-A(3) 888-893.

[7] Yubo Li, et al. Zero correlation zone sequence sets over the 8-QAM+ constellation. IEEE Commun. Lett., 2012 16(11) 1844-1847.

[8] Fanxin Zeng, et al. Perfect arrays over the 8-QAM+ constellation. IEICE Trans. Fundamentals. 2015 E98-A(4) 1038-1043.

[9] Fanxin Zeng, et al. Design of 8-QAM+ perfect arrays from perfect ternary arrays. Proceedings 2014 7th International Congress on Image and Signal Processing, Dalian, China, 2014, 1185-1190.