A CT-AHP based Quantitative Assessment Approach to Decision Support

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Keywords: Analytic Hierarchy Process; Calibration Training; reliability factor; quantitative assessment

Abstract. The quantitative analysis approach is a powerful tool for auxiliary decision-making, especially suitable for the semi-structured decision problem. In our studies, we propose a new CT-AHP based quantitative assessment approach with a multi-step process, in which AHP, calibration training, reliability factor are cooperatively used to make quantitative evaluation result more scientific. The breakthrough choosing problem is selected as an example to explain the application of this approach. The results shown that the conclusion drawn by domain experts is consistent with the CT-AHP based approach proposed in this paper.

Introduction

The proposed quantitative assessment approaches can be divided into three types: one is the traditional approach of operational research, this kind of approach applies to the decision problem of benign structure, can give a sense of the optimal solution. Decision-making problems[1,2] of technical level and some tactical decisions, often adopt this approach. Secondly, the approach of artificial intelligence. Currently expert system and broader knowledge base system is mainly used for decision-making and artificial intelligence technology. Thirdly, it is judgement analysis approach. Decision-makerselects one from multiple alternatives solutions to form a set of concepts and logic approachaccording to his own judgement and preference. Decision-maker has to judge each possible outcome of alternative likelihood and preference degree. It becomes a powerful tool for auxiliary decision-making, especially suitable for the semi-structured decision problem.

This paper presents a new quantitative assessmentapproach, belonging to the category of judgement analysis approach. According to the characteristics of the quantitative decision, the approach is put forward based on the thorough analysis of the deficiency of the traditional AHP[3,4] (Analytic Hierarchy Process). AHP is a combination of qualitative and quantitative analysis, modeling the human evaluation, is proposed by R.L.Saaey, a professorof the University of Pittsburgh. It’s designed to solve complex system evaluation.It can introduce quantitative analysis in complex decision-making process, and make full use of the evaluators in comparing two given preference information analysis and decision support. This approach not only effectively absorbs the results of qualitative analysis, but also brings out the advantages of quantitative analysis.However, in the process of the use of AHP, whether to establish hierarchy or to construct judgement matrix, people's subjective judgement has enormous influence on the preference of the results. When evaluators put forward a set of illogical data or implicit contradictory data because of inconsiderate, the results of AHP will deviate from the actual.In order to make quantitative evaluation result more scientific,taking into account both the practical experience of experts and reducing the subjective arbitrariness, this paper puts forward a CT-AHP(Calibration Training-AHP)based quantitative assessment approach,adopting calibration training combined with reliability factor, whichboth reflects subjective information of decision makers to some extent, and can make use of the relationship between the matrix and the weight coefficient.

Approach

The approachcombines qualitative indexes and quantitative indexes, subjective weights and objective weights, by minimizing subjective randomness of traditional AHP approach. So it can make decision more reasonably. Specific steps are as follows:

Step 1: Calibration training

Choosing a certain number of domain experts (if there are N experts), listing all the target object and the associated uncertainty proposed by every expert. Carrying out calibration training for experts on the basis of 90% confidence, let each expert evaluate the target and related uncertain factors at a 90% confidence. Then an equivalent gambling test method is used to test the evaluation ability of each expert in order to raise the calibration ability of experts.

Step 2: To establish a quantitative decision model and find out the factors of high information value

Organizing trained experts to research target and related uncertain factors, constructing quantitative decision-making model. The complex problem is decomposed into multiple factors based on this model.Calculating theinformation value of each target and uncertain factor through expert grading and sorting to find out the factors containing high information value.

Step 3: Methodizingthe factors containing high information valueobtained at step 2 and establishing hierarchical structure model is established.

The factors containing high information valueis divided into three layers refer to AHP approach, they are the target layer, the norm layer and the measure layer.

The number of hierarchy levels is related to the complexity and detailed degree of problems. Generally the level number is not restricted. Thedominated factor number in each level is generally no more than 9, too much will bringjudgement difficult.

Step 4: Constructingsubjective judgement matrix at all levels

First of all, comparing the importance of various factors on the same level of relative to its upper factors, subjective judgement matrix is constructed. To facilitate the comparison between various factors and obtain the quantitative judgement matrix, using scale 1 ~ 5 as the factors of judgement matrix scale.

For three-layer structure, there are two types of judgement matrix: the target-norm judgement matrix and the norm-measurejudgement matrix. The former is mainly used for the calculation of the relative weight of each index of the norm layer, the latter is used to calculate the relative weights ofeach measure under a certain criterion.

The form of two types of matrix are the same. It can be expressed as follows:

$$\boldsymbol{A}^{\mathbf{k}} = \begin{bmatrix} \mathbf{a}_{1,1}^{\mathbf{k}} & \mathbf{a}_{1,2}^{\mathbf{k}} \cdots & \mathbf{a}_{1,n}^{\mathbf{k}} \\ \mathbf{a}_{2,1}^{\mathbf{k}} & \mathbf{a}_{2,2}^{\mathbf{k}} \cdots & \mathbf{a}_{2,n}^{\mathbf{k}} \\ \vdots & \vdots \ \cdots & \vdots \\ \mathbf{a}_{n,1}^{\mathbf{k}} & \mathbf{a}_{n,2}^{\mathbf{k}} \cdots & \mathbf{a}_{n,n}^{\mathbf{k}} \end{bmatrix} (1)$$

Where $A^k$is the subjective judgement matrix given by the *kth* expert, namely $A^k = \left[a_{ij}^k\right]_{n\times n}$.

Step 5: Comprehensive judgement matrix of all layers is structured

Next first calculate reliability factor of each expert judgement matrix, which is measured by the discrete degree of each judgement matrix. If the discrete degree of an expert judgement matrix and other experts judgement matrix is smaller, then the expert judgement matrix is considered relatively reliable, should be given greater weight.

In order to make the deviation among the judgement matrix comparable, it’s important to standardize the expert's subjective judgement matrix. The transformation is as follows:

$$Z_{ij}^k = \frac{a_{ij}^k - \overline{a_j}}{\sigma_j}, i, j = 1,2,\cdots,n; (2)$$

Where $\overline{\boldsymbol{a_j}}$ is the average of the index:

$$\overline{a_j} = \frac{\sum_{i=1}^n a_{ij}^k}{n} (3)$$

And $\boldsymbol{\sigma_j}$ is the mean square errorof the index:

$\sigma_j^2 = \frac{\sum_{i=1}^{n}(a_{ij}^k - \overline{a_j})^2}{n-1}$ (4)

The standardize matrix is obtained:

$$\boldsymbol{Z^k} = [z_{ij}^k]_{n\times n} = \begin{bmatrix} \boldsymbol{z_{1,1}^k} & \boldsymbol{z_{1,2}^k}\cdots & \boldsymbol{z_{1,n}^k} \\ \boldsymbol{z_{2,1}^k} & \boldsymbol{z_{2,2}^k}\cdots & \boldsymbol{z_{2,n}^k} \\ \vdots & \vdots\ \ldots & \vdots \\ \boldsymbol{z_{n,1}^k} & \boldsymbol{z_{n,2}^k}\cdots & \boldsymbol{z_{n,n}^k} \end{bmatrix} (5)$$

Computing the summation of the distance between the standardized subjective judgement matrix given byexpert *k* and other experts:

$d^k = \sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{z=1}^{p} d(z_{ij}^k, a_{ij}^z)$=$\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{z=1}^{p}\left\| z_{ij}^k, a_{ij}^z \right\|$ (6)

The reliability factor of judgement matrix given by expert k is as follows:

$\mu^k = \frac{1/d^k}{\sum_{z=1}^{p}(1/d^z)}$ (7)

Comprehensive judgement matrix is:

$$S = \frac{\frac{1}{d^1}}{\sum_{z=1}^{p}\left(\frac{1}{d^z}\right)} \times \begin{bmatrix} \mathbf{z_{1,1}^1} & \mathbf{z_{1,2}^1}\cdots & \mathbf{z_{1,n}^1} \\ \mathbf{z_{2,1}^1} & \mathbf{z_{2,2}^1}\cdots & \mathbf{z_{2,n}^1} \\ \vdots & \vdots\ \ldots & \vdots \\ \mathbf{z_{n,1}^1} & \mathbf{z_{n,2}^1}\cdots & \mathbf{z_{n,n}^1} \end{bmatrix} + \frac{\frac{1}{d^2}}{\sum_{z=1}^{p}\left(\frac{1}{d^z}\right)} \times \begin{bmatrix} \mathbf{z_{1,1}^2} & \mathbf{z_{1,2}^2}\cdots & \mathbf{z_{1,n}^2} \\ \mathbf{z_{2,1}^2} & \mathbf{z_{2,2}^2}\cdots & \mathbf{z_{2,n}^2} \\ \vdots & \vdots\ \ldots & \vdots \\ \mathbf{z_{n,1}^2} & \mathbf{z_{n,2}^2}\cdots & \mathbf{z_{n,n}^2} \end{bmatrix}$$

$$+\cdots \frac{\frac{1}{d^p}}{\sum_{z=1}^{p}\left(\frac{1}{d^z}\right)} \times \begin{bmatrix} \mathbf{z_{1,1}^p} & \mathbf{z_{1,2}^p}\cdots & \mathbf{z_{1,n}^p} \\ \mathbf{z_{2,1}^p} & \mathbf{z_{2,2}^p}\cdots & \mathbf{z_{2,n}^p} \\ \vdots & \vdots\ \ldots & \vdots \\ \mathbf{z_{n,1}^p} & \mathbf{z_{n,2}^p}\cdots & \mathbf{z_{n,n}^p} \end{bmatrix} = \sum_{z=1}^{p}(\mu^z \times \boldsymbol{Z^z}) = [s_{ij}]_{n\times n} (8)$$

To be sure, each element in the comprehensive judgement matrix needs to be translated into integer:

Step 6: Hierarchical single sorting and consistency check

Calculate the product of element per linefor comprehensive judgement matrix*S*:

$M_i = \prod_{j=1}^{n} s_{ij}, i = 1,2,\cdots n$(9)

To calculate the nth root ofM_i, and carryon the normalization processing:

$\overline{w_i} = \sqrt[n]{M_i}, i = 1,2,\cdots n$ (10)

$w_i = \overline{w_i}/(\sum_{i=1}^{n}\overline{w_i}), i = 1,2,\cdots n$ (11)

Calculate the maximum eigenvalue ofjudgement matrix*S*:

$\lambda_{max} = \sum_{i=1}^{n}\frac{(SW)_i}{nw_i}$(12)

Where$(SW)_i$is the *ith* element of vector*SW*.

Next, the consistency test of the judgement matrix is made as follows:

①Calculating consistency index:

$CI = \frac{\lambda_{\max} - n}{n-1}$ (13)

②Looking for corresponding average random consistency index from following table:

Table 1. the value of average random consistency index

n	1	2	3	4	5
RI	0	0	0.58	0.90	1.12

To calculate the average value of the greatest characteristic root，and make following definition:

$RI = \frac{\lambda'_{\max} - n}{n-1}$ (14)

③Calculating consistency proportion:

$CR = \frac{CI}{RI}$ (15)

when $CR<0.10$, the consistency of judgement matrix is acceptable, otherwise the judgement matrix is revised.

Step 7: Hierarchical whole sorting and consistency check

The last step is to aggregate relative weights of each layer to get synthesis weights vector, especially the sorting weights of each measure at the lowest layer to the target, thus carries on the measure selection.

Results and Discussion

The best breakthrough choosing problem is selected as an example to explain the application of this approach.

Five experts are chose to discuss and list all the indicators to measure target objects, including (1) the superior intention; (2) the impact on the global; (3)the tactical principle; (4) to complete the task at the corresponding levels; (5) facilitate hidden form; (6) easy to impact and breakthroughs; (7)vitals defense against the enemy; (8)It's easy to decollate and surround enemy;(9)the terrain condition; (10)to choose the best breakthrough, a total of 10 indicators.

Carrying out calibration training for experts on the basis of 90% confidence, then an equivalent gambling test method is used to test the evaluation ability of each expert.Organize the trained experts to focus on the above indicators, then calculate the information value of each target and uncertain factor through expert grading and sorting to find out the factors containing high information value. Finally four most information value indicators are determined, including: (1)the superior intention; (3)the tactical principle; (9)the terrain condition; (10)to choose the best breakthrough.

The hierarchical structure model is established, in order to simplify the example, column only three layers.

First putthe breakthrough choosing evaluation target on the highest layer.

The second layer is the norm layer, which is the best indicator of measuring target, they are: a. the superior intention; b. the tactical principle; c. the terrain condition.

The third layer is the lowest layer, which lists the alternative schemes, including operational plan 1, operational plan 2 and operational plan 3.

The target layer is expressed as A, the layer 2 from left to right in turn is expressed asB_1,B_2,B_3,the layer 3 from left to right in turn is expressedas theC_1,C_2,C_3. The pairwise comparison results about the importance ofeach criterion (B_1,B_2,B_3) vs. the best breakthrough choosing（A）are given respectively, andthe pairwise comparison results about the importance of each operational plan(C_1,C_2,C_3) vs. the superior intention(B_1), the tactical principle(B_2)and the terrain conditions(B_3) are given respectively. According to the formula (8), comprehensive judgement matrix is calculated respectively as follows:

Table 2.the comprehensive judgement matrix of each criterion (B1,B2,B3) vs. the best breakthrough choosing（A）

A	B_1	B_2	B_3
B_1	1	3	4
B_2	1/3	1	3
B_3	1/4	1/3	1

Table 3. the comprehensive judgement matrix of each operational plan(C1,C2,C3) vs. the superior intention（B1）

B_1	C_1	C_2	C_3
C_1	1	1/3	1/3
C_2	3	1	1/2
C_3	3	2	1

Table4. the comprehensive judgement matrix of each operational plan(C1,C2,C3) vs. the tactical principle（B2）

B_2	C_1	C_2	C_3
C_1	1	3	1/3
C_2	1/3	1	1/5
C_3	3	5	1

Table5. the comprehensive judgement matrix of each operational plan(C1,C2,C3) vs. the terrain conditions（B3）

B_3	C_1	C_2	C_3
C_1	1	2	1/3
C_2	1/2	1	1/5
C_3	3	5	1

To calculate the maximum characteristic roots of comprehensive judgement matrix A, B_1,B_2 and B_3 in Matlab, they are respectively $\lambda_0 = 3.0735, \lambda_1 = 3.0536, \lambda_2 = 3.0385, \lambda_3 = 3.0037$. And the corresponding eigenvectors is $\omega = [0.9027\ 0.3943\ 0.1722]^T$, $\omega_1 = [0.2184\ 0.5201\ 0.8257]^T$, $\omega_2 = [0.3715\ 0.1506\ 0.9161]^T$, $\omega_3 = [-0.3288\ -0.1747\ -0.9281]^T$.Furthermore, the consistency index CI and CR are calculated as follows:

CI =0.0368 0.0268 0.0192 0.0018

CR=0.0634 0.0462 0.0332 0.0032

CR valuesare all less than 1, so the inconsistent degree of the above comprehensive judgementmatrix is within the scope of the permit.

The last step is to aggregate relative weights of each layer to get synthesis weights vector $\omega_S = W*w$, and the resultis as follows:

ans =[0.28700.49880.9468]

Known from the analysis of the above, operational plan 3 is the best way to select the best breakthrough, the second is operational plan 2, the worst is operational plan 1, the ordering weights were 0.9468, 0.4988 and 0.2870 respectively.

In order to further verify the superiority of this approach, we compare it with the traditional AHP approach. Invited five military experts to give anonymous comprehensive score of three kinds of schemes. These experts come from military operational research, military command, and other fields, have rich command or teaching experiences. After comprehensive score, the conclusion drawn bydomain experts is consistent with the CT-AHP based assessment approach proposed in this paper, but the result of the traditional AHP approachis a little different from that of domain experts.

Conclusion

In this paper, the proposed CT-AHP based quantitative assessmentapproachadopts equivalent gambling test method for calibration training and tests each expert evaluation ability, thus effectively promotes the calibration ability of each expert and reduces the subjective randomness and uncertainty. By calculating the discrete degree of thejudgement matrix to measure the reliability of the judgement matrix factor, to get comprehensive judgement matrix with higher credibility after standardize the judgement matrix, lay a foundation for subsequent levels sorting and consistency check. The approach has strong application value and the military economic benefit, not only helps to achieve efficient and correct quantitative decision, but also can be used in complex systems inherent effectiveness evaluation and project management[4], risk evaluation[5], and other fields.

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