Fuzzy Reliability Evaluation of CNC System Based on Petri Nets

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Abstract. ACNC system is the major part of numerical control Machining Tools, so its reliability is of great important. The failure of CNC system is rare and affected by many factors. In order to estimate the reliability indices more actually, Petri Nets(PN), in stead of FAT, are used to get all the minimal sets effectively and the lambda-Tau method for repairable systems is used too. Furthermore fuzzy sets are applied in calculating the results and a fuzzy evaluating model is set in this paper.

Introduction

A numerical control machining tools is the basic equipment for advanced manufacturing technologies, such as computerized integrated manufacturing, flexible manufacturing systems and so on. The computer numerical control (CNC) system plays a major actor in it. If we want to estimate its reliability indices more accurately by conventional approach only, a large mount of data is required. However, it is usually impossible to obtain such a large quantity of databases. Sometimes these databases are collected in different conditions and affected by different factors. In order to give some solution to these problems, this paper will use a novel approach, which is base on previous reliability, statistical methodology and fuzzy sets to estimate its three reliability indices.

Petri Net model of CNC system

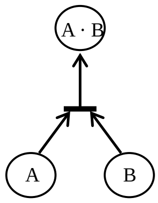
The Petri Net theory is a graphical method using some basic symbols for describing relations between conditions and events. PN have two types of nodes named place P and transition T. Arcs A connect these nodes, i.e., arcs connects transitions to places or places to transitions. The basic symbols are defined as follows:

O： Place, drown as a circle

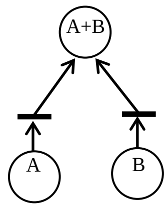
▬： Transition, drawn as a bar

→ ： Arc, drawn as an arrow

Fig.1 illustrates the logic transitions of PN.



A. and change



B. or change

Fig.1 PN model logic change

In the PN model of the CNC system,there are three layers: the top place, the middle place, the basic place. The top place is called “abnormal work”,the middle place (M) ,such as “failure of software（M01）”, “temperature over high(M02) ”,… .and the basic place (B), such as “power interference（B01）”, “circuit break(B02) ”,…. Following the principle of PN, we can get the PN model of the CNC system.(omitted). From the PN model, it's minimal cut sets can be derived. All the minimal cut sets is:{B02、B05}、{B02、B06}、{B03、B05}、{B03、B06}、{B05、B07}、{B06、B07}{B01}、{B04}、{B08}、{B09}…、{B45}

Fuzzy Evaluation of Reliability Indices

$\lambda-\tau$ methodology.

The $\lambda-\tau$ methodology of repairable system uses the failure rate and repair time of basic event to determine the two concepts of top event in the fault tree method . Its basic expression is shown in table 1 .

Table 1 Basic expression of $\lambda-\tau$ methodlogy

Gate	2-Input	3-Input	n-Input
λ_{AND}	$\lambda_1 \cdot \lambda_2[\tau_1+\tau_2]$	$\lambda_1 \cdot \lambda_2 \cdot \lambda_3[\tau_1\tau_2+\tau_1\tau_3+\tau_2\tau_3]$	$\prod_{j=1}^{n}\lambda_j\left[\sum_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\tau_j\right]$
τ_{AND}	$\frac{\tau_1\cdot\tau_2}{\tau_1+\tau_2}$	$\frac{\tau_1\cdot\tau_2\cdot\tau_3}{\tau_1\cdot\tau_2+\tau_1\cdot\tau_3+\tau_2\cdot\tau_3}$	$\frac{\prod_{i=1}^{n}\tau_i}{\sum_{j=1}^{n}\left[\prod_{\substack{i=1\\ i\neq j}}^{n}\tau_j\right]}$
λ_{OR}	$\lambda_1+\lambda_2$	$\lambda_1+\lambda_2+\lambda_3$	$\sum_{i=1}^{n}\lambda_i$
τ_{OR}	$\frac{\lambda_1\tau_1+\lambda_2\tau_2}{\lambda_1+\lambda_2}$	$\frac{\lambda_1\tau_1+\lambda_2\tau_2+\lambda_3\tau_3}{\lambda_1+\lambda_2+\lambda_3}$	$\frac{\sum_{i=1}^{n}\lambda_i\tau_i}{\sum_{j=1}^{n}\lambda_j}$

This method is effective under following conditions:

1. Events are independent, that is, the basic events of the model are not repeated .
2. The basic event failure rates, λ, are very small (preferable≤10-3h-1);
3. The product of τ and λ is very small(preferable≤0.1);
4. The negative exponential distribution must be applied in the quantitative analysis and failures occur independently.

In this evaluation, all the restrictions are all right.

Fuzzy extension of failure rate and repair time.

From the data collected, the failure rate and repair time of the basic event can be gained(shown in table 2.

We extend these data to triangular fuzzy numbers because under some weak assumptions, these specific membership functions immediately comply with the relevant optimization criteria. For example, a triplet（λ_{i1}, λ_{i2}, λ_{i3}）defines the triangular fuzzy number of failure rate, the parameter λ_{i2} is the crisp value of basic event i, "λ_{i1}"and"λ_{i3}"are the lower and upper bounds of the available area for the evaluation data. From our engineering experience, we choose a ±15% spread of the crisp values to obtain the lower and upper limits.Following this method, the triplet（τ_{i1}, τ_{i2}, τ_{i3}）of repair time can be obtained.

The calculation of fuzzy numbers must use α-cut, $A^{(\alpha)}$, which is defined below: $A^{(\alpha)}=\{x|A(x)\geq\alpha\}$ $\alpha\in[0,1]$. The α-cut defines the interval of confidence for the triangular number can be written as $A_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}]$, The four operations are given below:

Suppose $A_\alpha = [a_1^{(\alpha)}, a_3^{(\alpha)}]$, $B_\alpha = [b_1^{(\alpha)}, b_3^{(\alpha)}]$

1.Plus $\tilde{A} + \tilde{B} = [a_1^{(\alpha)} + b_1^{(\alpha)} , a_3^{(\alpha)} + b_3^{(\alpha)}]$

2.minus $\tilde{A} - \tilde{B} = [a_1^{(\alpha)} - b_3^{(\alpha)} , a_3^{(\alpha)} + b_1^{(\alpha)}]$

3.multiply $\tilde{A} \cdot \tilde{B} = [a_1^{(\alpha)} \cdot b_1^{(\alpha)} , a_3^{(\alpha)} \cdot b_3^{(\alpha)}]$

4.divide $\tilde{A} \div \tilde{B} = [a_1^{(\alpha)} \div b_3^{(\alpha)} , a_3^{(\alpha)} \div b_1^{(\alpha)}]$ $0 \notin [b_1^{(\alpha)}, b_3^{(\alpha)}]$

Table 2 The failure rates and repair time of event happened

Name and Code	Failure rate(λ)	Repair time (τ)
Failure of lithium battery (B07)	$\lambda_1 = 1.460\times10^{-4}$	$\tau_1 = 1.67$
Circuit break(B02)	$\lambda_2 = 9.970\times10^{-5}$	$\tau_2 = 1.50$
Failure of function module (B03)	$\lambda_3 = 1.108\times10^{-4}$	$\tau_3 = 2.17$
Failure of button (B26)	$\lambda_4 = 8.852\times10^{-5}$	$\tau_4 = 3.50$
Failure of X spindle drive module (B30)	$\lambda_5 = 1.040\times10^{-4}$	$\tau_5 = 4.33$
Failure of power module (B14)	$\lambda_6 = 1.089\times10^{-4}$	$\tau_6 = 4.67$
Failure of CPU (B23)	$\lambda_7 = 8.788\times10^{-5}$	$\tau_7 = 4.50$
Failure of CRT (B24)	$\lambda_8 = 9.551\times10^{-5}$	$\tau_8 = 3.67$
Failure of blowing machine (B09)	$\lambda_9 = 1.018\times10^{-4}$	$\tau_9 = 3.67$
Blocking of filter fishnet(B10)	$\lambda_{10} = 7.976 \times10^{-5}$	$\tau_{10} = 1.67$
Failure of RAM+5 connection (B05)	$\lambda_{11} = 1.073\times10^{-4}$	$\tau_{11} = 1.33$

The calculation of AND-transition.

In the Petri nets model of this CNC system, there are six AND-transitions. For example, using α =0,0.1,0.2…1.0, we calculate the and-transition for events “B02”and “B05”. its calculation result is tabulated in table 3.

Table 3 The AND-transition result of B02 and B05

α	Failure rate		Repair time	
	Lower limit	Upper limit	Lower limit	Upper limit
1.0	3.02748E-08	3.02748E-08	0.704947	0.704947
0.9	2.89328E-08	3.07220E-08	0.673849	0.737314
0.8	2.76310E-08	3.11550E-08	0.643966	0.771009
0.7	2.63689E-08	3.15731E-08	0.615243	0.806094
0.6	2.51458E-08	3.19758E-08	0.587633	0.842637
0.5	2.39611E-08	3.23623E-08	0.561089	0.880707
0.4	2.28142E-08	3.27322E-08	0.535566	0.920382
0.3	2.17045E-08	3.30848E-08	0.511023	0.961741
0.2	2.06314E-08	3.34195E-08	0.487420	1.004870
0.1	1.95943E-08	3.37357E-08	0.464722	1.049862
0.0	1.85925E-08	3.40327E-08	0.442891	1.096815

The calculation of OR-transition.

In order to elaborate the calculation, we can get the triplet of top place using such equation:

$$\lambda_1^{(\alpha)} = \sum_{\substack{i=1 \\ i\neq 2,3,7,11}}^{11} (0.15\alpha + 0.85)\lambda_i$$

$$\lambda_2^{(\alpha)} = \sum_{\substack{i=1 \\ i\neq 2,3,7,11}}^{11} \lambda_i$$

$$\lambda_3^{(\alpha)} = \sum_{\substack{i=1 \\ i\neq 2,3,7,11}}^{11} (-0.15\alpha + 1.15)\lambda_i$$

The complete computer solution is tabulated in table 4. The fuzzy numbers (failure rate and repair time) are shown in Fig.2 and Fig.3.

Table 4 The failure rates and repair time of CNC system

α	Failure rates		Repair time	
	Lower limit	Upper limit	Lower limit	Upper limit
1.0	0.0006483	0.0006483	3.033107	3.033107
0.9	0.0006337	0.0006629	2.899305	3.172370
0.8	0.0006191	0.0006775	2.770729	3.317343
0.7	0.0006045	0.0006921	2.647147	3.468304
0.6	0.0005899	0.0007067	2.528353	3.625530
0.5	0.0005752	0.0007213	2.414141	3.789336
0.4	0.0005606	0.0007359	2.304328	3.960037
0.3	0.0005460	0.0007505	2.198728	4.137990
0.2	0.0005314	0.0007651	2.097178	4.323554
0.1	0.0005168	0.0007797	1.999512	4.517139
0.0	0.0005022	0.0007944	1.905582	4.719155

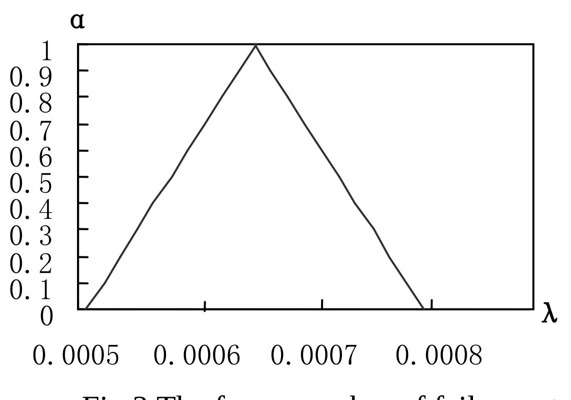


Fig.2 The fuzzy number of failure rate

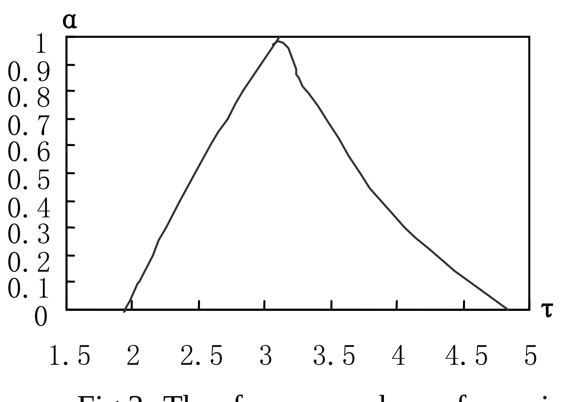


Fig.3 The fuzzy number of repair

Reliability indices.

In many engineering applications of fuzzy set theory, the fuzzy result should be converted to crisp value, which is called defuzzification. A method named “center of area (COA)”can be selected for this defuzzification. The equation of COA is:

$$x^* = \frac{\int_{x_1}^{x_2} xA_{out}(x)dx}{\int_{x_1}^{x_2} A_{out}(x)dx}$$

The result of defuzzification are:

$\lambda = 6.4389 \times 10^{-4}$ $\tau = 4.1726(h)$

Because the distribution of time-between-failures fits the exponential distribution, the indices mean time to failure (MTBF) and mean time to repair (MTTR) can be expressed as:

$$MTTF = \int_0^\infty t\lambda e^{-\lambda t} dt = \frac{1}{\lambda} = 1554.27(h)$$

$$MTTR = \int_0^\infty t\mu\lambda e^{-\mu t} dt = \frac{1}{\mu} = \tau = 4.1726(h)$$

$$MTBF = MTTF + MTTR = 1558.44(h)$$

Inherent availability $A_i = \frac{\mu}{\lambda + \mu} = 0.9973$

Conclusion

Traditional FTA techniques are tedious for CNC system. Moreover they can only provide an approximate result in same case. To improve the efficiency of the analysis, a novel approach based on PN and fuzzy sets for determining the reliability indices of CNC system is proposed in this paper. The use of PN to perform the CNC system's reliability modeling is a reliable and efficient approach. It is very simply to obtain all the minimal cut sets. The relation between events is more clearly than FTA. Furthermore, the use of fuzzy $\lambda - \tau$ methodology can estimate the reliability indices more accurately with fewer failure data. Finally we obtain three parameters: MTBF, MTTR and Ai. Based on these accurate values, for the purpose of increasing MTBF and reducing MTTR, the CNC system's reliability can be improved step by step.

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