Stabilization of complex-valued neural networks with time-varying delays via linear feedback control

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Abstract. In this paper, a class of complex-valued neural networks with time-varying delays is considered without assuming the differentiability of the time-varying delays, and the exponential stabilization for the considered neural networks is investigated. By constructing proper Lyapunov-Krasovskii functional and using the matrix inequality techniques, a delay-dependent criterion for checking the stability of the considered neural networks is presented under designed linear feedback controller. An example with simulations is given to show the effectiveness of the obtained result.

Introduction

In the past decade, delayed neural networks have been successfully applied in many areas such as signal processing, pattern recognition, associative memories, and optimization solvers [1]. Some of these applications require the designed neural network to be stable, and it is therefore important to study the stability of neural networks [2]. A great number of results have been reported on the stability for various neural networks with constant delays or time-varying delays in the literature for example, see [1, 2, 3, 4, 5, 6] and references therein.

As an extension of real-valued neural networks, complex-valued neural networks with complex-valued state, output, connection weight, and activation function become strongly desired because of their practical applications in physical systems dealing with electromagnetic, light, ultrasonic, and quantum waves[7]. In fact, complex-valued neural networks (CVNNs) make it possible to solve some problems which cannot be solved with their real-valued counterparts. For example, the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, but they can be solved with a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potent computational power of complex-valued neurons[8]. Besides, CVNNs has more different and more complicated properties than the real-valued ones [9]. Therefore it is necessary to study the dynamic behaviors of CVNNs deeply [10]. Recently, some stability results of CVNNs have been obtained, for example, see [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] and references therein.

In practice, it may happen that the delayed neural networks are unstable or the convergence rate can not meet the requirements. Under this case, certain controllers may be designed such that the controlled delayed neural networks achieve the desired stability properties. In recent years, many control approaches have been developed to stabilize chaotic complex-valued neural networks such as adaptive control, fuzzy control, sampled-data control, impulsive control and intermittent control and so on. However, To the best of our knowledge, up to now, there are very few results on the stabilization problem of CVNNs.

Motivated by the above discussions, the objective of this paper is to study the exponential stabilization of CVNNs with time-varying delays.

Preliminary

In this paper, we consider the following CVNNs with time-varying delays

$$\dot{x}(t) = -Dx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + u(t) \quad (1)$$

for $t \geq t_0$, where $x(t)$ is the state vector of the network at time t, n corresponds to the number of neurons; D is a positive diagonal matrix, A and B are known constant matrices; $f(x(t))$ denotes the neuron activation at time t; $u(t)$ is a external input vector; $\tau(t)$ is delays and satisfy $0 \leq \tau(t) \leq \tau$. To order to stabilize the origin of neural networks (1) by means of feedback control, we assume that the control exposed on the system is of the form

$$u(t) = Kx(t) \quad (2)$$

where K is the control gain matrix. With control law (2), model (1) can be rewritten as

$$\dot{x}(t) = (K-D)x(t) + Af(x(t)) + Bf(x(t-\tau(t))) \quad (3)$$

Model (1) is supplemented with initial value given by $x(s) = \varphi(s)$, $s \in [-\tau, t_0]$, where is bounded and continuously differential on $s \in [-\tau, t_0]$.

Throughout this paper, we make the following assumption:

Assumption 1. For any $j = 1,2,\cdots,n$, $f_j(0) = 0$ and there exists a positive diagonal matrix $L = diag(l_1, l_2, \cdots, l_n)$ such that for any $x, y \in C$,

$$\left| f_j(x) - f_j(y) \right| \leq l_j \left| x - y \right| \quad (4)$$

Main Results

Theorem 1. Assume that the assumption 1 holds. For viven constant $\alpha > 0$, if there exist three symmetric positive definite Hermitian matrices P_1, P_2 and P_3, two positive diagonal matrices R and S, and four matrices Q_1, Q_2, W and Z such that the following complex-valued linear matrix inequality holds:

$$\Omega = \left(\Omega_{ij}\right)_{8\times 8} < 0 \quad (5)$$

Where

$\Omega_{ij} = \Omega_{ji}$, and

$\Omega_{11} = \alpha P_1 + P_2 - Q_1 - Q_1^* + LRL + Z - WD + Z^* - DW^*$, $\Omega_{12} = P_1 + Z^* - DW^* - W$, $\Omega_{13} = Q_1$, $\Omega_{15} = WA$, $\Omega_{16} = WB$, $\Omega_{17} = Q_1$, $\Omega_{22} = \tau e^{\alpha\tau} P_2 - W - W^*$, $\Omega_{25} = WA$, $\Omega_{25} = WA$, $\Omega_{28} = Q_2$, $\Omega_{33} = -Q_2 - Q_2^* + LSL$, $\Omega_{34} = Q_2$, $\Omega_{38} = Q_2$, $\Omega_{44} = -\tau e^{-\alpha\tau} P_2$, $\Omega_{55} = -R$, $\Omega_{66} = -S$, $\Omega_{77} = -P_3$, $\Omega_{88} = -P_3$, and the rest of Ω_{ij} are zero, then the origin of system (3) is globally exponerntially stable, and the gain matrix of control law (2) is

$$K = W^{-1} Z \quad (6)$$

Proof. Consider the following Lyapunov-Krasovskii functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

where

$$V_1(t) = x^*(t) P_1 x(t) \quad (7)$$

$$V_2(t) = \int_{t-\tau}^{t} e^{\alpha(s-t)} x^*(s) P_2 x(s) ds \quad (8)$$

$$V_3(t) = \int_{-\tau}^{0} d\xi \int_{t+\xi}^{t} e^{\alpha(s-t)} \dot{x}^*(s) P_3 \dot{x}(s) ds \quad (9)$$

Calculating the time derivative of $V(t)$, we obtain

$$\begin{aligned} \dot{V}_1(t) &= x^*(t) P \dot{x}(t) + \dot{x}^*(t) P_1 x(t) \\ &= -\alpha V_1(t) + \alpha x^*(t) P_1 x(t) + x^*(t) P_1 \dot{x}(t) + \dot{x}^*(t) P_1 x(t) \end{aligned} \quad (10)$$

$$\dot{V}_2(t) = -\alpha V_2(t) + x^*(t)P_2x(t) - e^{-\alpha\tau}x^*(t-\tau)P_2x(t-\tau) \tag{11}$$

$$\begin{aligned}\dot{V}_3(t) &= -\alpha V_3(t) + \tau e^{\alpha\tau}\dot{x}^*(t)P_3\dot{x}(t) - \int_{-\tau}^{0} e^{\alpha\xi}x^*(t+\xi)P_3x(t+\xi)d\xi \\ &\le -\alpha V_3(t) + \tau e^{\alpha\tau}\dot{x}^*(t)P_3\dot{x}(t) - \int_{t-\tau}^{t} x^*(s)P_3x(s)ds\end{aligned} \tag{12}$$

It follows from (10)-(12) that

$$\begin{aligned}\dot{V}(t) \le& -\alpha V(t) + x^*(t)(\alpha P_1 + P_2)x(t) + x^*(t)P_1\dot{x}(t) + \dot{x}^*(t)P_1x(t) \\ &- e^{-\alpha\tau}x^*(t-\tau)P_2x(t-\tau) + \tau e^{\alpha\tau}\dot{x}^*(t)P_3\dot{x}(t) - \int_{t-\tau}^{t} x^*(s)P_3x(s)ds\end{aligned} \tag{13}$$

By Newton-Leibniz formulation, we have

$$\begin{aligned}0 =& x^*(t)Q_1[-x(t) + x(t-\tau(t)) + \int_{t-\tau(t)}^{t}\dot{x}(s)ds] \\ &+ [-x(t) + x(t-\tau(t)) + \int_{t-\tau(t)}^{t}\dot{x}(s)ds]^*Q_1x(t) \\ \le& x^*(t)[-Q_1 - Q_1^* + Q_1P_3^{-1}Q_1^*]x(t) + x^*(t)Q_1x(t-\tau(t)) \\ &+ x^*(t-\tau(t))Q_1^*x(t) + \int_{t-\tau(t)}^{t}\dot{x}^*(s)P_3\dot{x}(s)ds\end{aligned} \tag{14}$$

$$\begin{aligned}0 =& x^*(t-\tau(t))Q_2[-x(t-\tau(t)) + x(t-\tau) + \int_{t-\tau}^{t-\tau(t)}\dot{x}(s)ds] \\ &+ [x(t-\tau) - x(t-\tau(t)) - \int_{t-\tau}^{t-\tau(t)}\dot{x}(s)ds]^*Q_2x(t-\tau(t)) \\ \le& x^*(t-\tau(t))[-Q_2 - Q_2^* + Q_2P_3^{-1}Q_2^*]x(t-\tau(t)) \\ &+ x^*(t-\tau(t))Q_2x(t-\tau) + x^*(t-\tau)Q_2^*x(t-\tau(t)) \\ &+ \int_{t-\tau}^{t-\tau(t)}\dot{x}^*(s)P_3\dot{x}(s)ds\end{aligned} \tag{15}$$

In addition, we can obtain from *Assumption 1* that

$$f^*(x(t))Rf(x(t)) \le x(t)LRLx(t) \tag{16}$$

$$f^*(x(t-\tau))Sf(x(t-\tau)) \le x(t-\tau)LSLx(t-\tau) \tag{17}$$

From model (3), we have that

$$\begin{aligned}0 =& \dot{x}^*(t)W[-\dot{x}(t) + (K-D)x(t) + Af(x(t)) + Bf(x(t-\tau(t)))] \\ &+ [-\dot{x}(t) + (K-D)x(t) + Af(x(t)) + Bf(x(t-\tau(t)))]^*W^*\dot{x}(t)\end{aligned} \tag{18}$$

It follows from inequality (13)-(18) that

$$\dot{V}(t) \le -\alpha V(t) + \eta^*(t)\Theta\eta(t) \tag{19}$$

where $\eta(t) = (x^*(t), \dot{x}^*(t), x^*(t-\tau(t)), x^*(t-\tau), f^*(x(t)), f^*(x(t-\tau)))^*$, and $\Theta = (\Theta_{ij})_{6\times 6}$ with $\Theta_{11} = \Omega_{11} + Q_1P_3^{-1}Q_1^*$, $\Theta_{33} = \Omega_{33} + Q_2P_3^{-1}Q_2^*$, the rest of Θ_{ij} are Ω_{ij}.

From Schur theorem, we know that $\Omega < 0$ is equivalent to $\Theta < 0$. Therefore, we have from inequality (5) and (20) that

$$\dot{V}(t) \le -\alpha V(t) \tag{20}$$

Thus, $V(t) \le V(t_0)e^{-\alpha(t-t_0)}$ for any $t \ge t_0$. From the definition of $V(t)$, we know that $V(t) \ge \lambda_{\min}(P_1)\|x(t)\|^2$. Let $M = (V(t_0)/\lambda_{\min}(P_1))^{\frac{1}{2}}$, then

$$\|x(t)\| \le Me^{-\frac{1}{2}\alpha(t-t_0)}, \quad t \ge t_0 \tag{21}$$

Inequality (21) implies model (3) is globally exponentially stable. The proof is complete.

Example

In this section, we will provide an example to illustrate the effectiveness of the obtained result. Consider a 2-dimensional neural networks (1) , where

$$D=\begin{pmatrix}0.8 & 0\\ 0 & 0.9\end{pmatrix},\quad A=\begin{pmatrix}0.1+0.3i & -0.4-0.6i\\ -0.2-0.6i & -1+i\end{pmatrix},\ B=\begin{pmatrix}0.7+0.2i & 0.4-i\\ -0.1+0.i & 0.5-i\end{pmatrix},$$

$$f_1(x)=f_2(x)=\frac{1}{20}(|x-1|-|x+1|),\ \tau(t)=8|\sin(t)|\ .$$

It is easy to see that $\tau(t)=8$, and *Assumption 1* is satisfied with $L=\begin{pmatrix}0.1 & 0\\ 0 & 0.1\end{pmatrix}$. Taking $\alpha=0.1$, by the YALMIP toolbox in MATLAB , we can find a solution to the complex-valued linear matrix inequality (5) as follows:

$$P_1=\begin{pmatrix}77.1925 & -3.9825+0.1936i\\ -3.9825-0.1936i & 68.9255\end{pmatrix},\quad P_2=\begin{pmatrix}57.4822 & 0.5662-0.1293i\\ 0.5662+0.1293i & 57.9773\end{pmatrix},$$

$$P_3=\begin{pmatrix}1.8901 & -0.4011+0.0295i\\ -0.4011-0.0295i & 1.0653\end{pmatrix},\quad R=\begin{pmatrix}77.4715 & 0\\ 0 & 112.5789\end{pmatrix},$$

$$S=\begin{pmatrix}84.0433 & 0\\ 0 & 81.2348\end{pmatrix},\quad Q_1=\begin{pmatrix}0.2872-0.0013i & -0.0712+0.0058i\\ -0.0827-0.0089i & 0.1430+0.0013i\end{pmatrix},$$

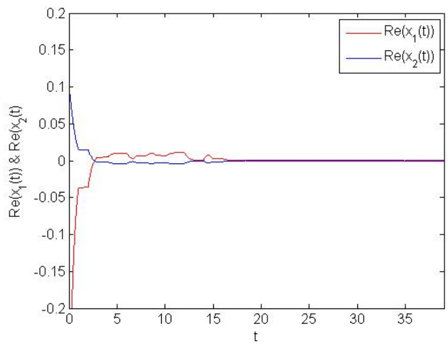
$$Q_2=\begin{pmatrix}1.8237-0.0008i & -0.3858+0.0237i\\ -0.3896-0.0249i & 1.0076+0.0005i\end{pmatrix},\quad W=\begin{pmatrix}31.1013-0.0454i & -6.3074+0.2873i\\ -6.2808-0.2961i & 17.8358+0.0307i\end{pmatrix},$$

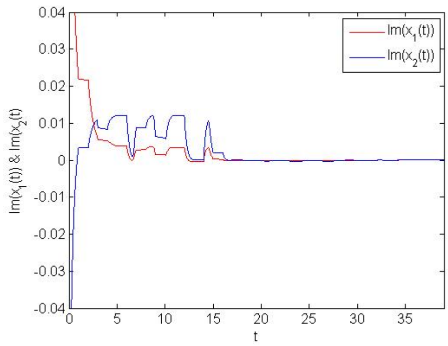
$$Z=\begin{pmatrix}-43.6642+0.0171i & -4.1023+0.2865i\\ -3.4447-0.2847i & -48.6779-0.0075i\end{pmatrix}.$$

Therefore, by *Theorem 1*, we know that the origin of system (3) is globally exponerntially stable, and the gain matrix of control law (2) is

$$K=W^{-1}Z=\begin{pmatrix}-1.5545-0.0033i & -0.7382+0.0369i\\ -0.7406-0.0417i & -2.9898+0.0054i\end{pmatrix}.$$

The numerical simulation is shown in figure 1 and figure 2.





Conclusions

In this paper, the stabilization problem for a class of complex-valued neural networks with time-varying delay has been investigated. A delay-dependent criterion for checking the stability of the considered neural networks has been obtained by constructing proper Lyapunov-Krasovskii functional and using the matrix inequality techniques. An example with simulations is also given to show the effectiveness of the obtained result.

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