Sterhl ratio of vortex beams propagating through atmospheric turbulence

Qiangbo Suo, Jinhong Li ,Linying Chen

Department of Physics, Taiyuan University of Science and Technology, Taiyuan 030024, China

e-mail:690374182@qq.com, jinhongli@live.cn(Corresponding author),cly000818@sohu.com

Keywords: Sterhl ratio; atmospheric turbulence; vortex beam

Abstract. Based on the extended Huygens-Fresnel principle, the analytical expressions for the Strehl ratio(S_R) of Gaussian Schell-model(GSM) vortex beams and Gaussian Schell-model non-vortex beams propagating through atmospheric turbulence and free space are derived. The corresponding results are also calculated. It is shown that the smaller the structure constant C_n^2, the inner scale l_0 and the wave length λ, as well as the larger the outer scale turbulence L_0, the lager the S_R for the GSM vortex beams, and as the spatial correlation length σ_0 increases, the S_R for the GSM vortex beams first increases then decreases. The work will provide theoretical foundation for the practical application of GSM vortex beams.

Introduction

The propagation characteristics of laser beams are widely used in laser communications, remote sensing, aerial mapping and optical radar, etc[1-5]. There has been increasing interest in studying the S_R in valuing the quality of laser beams[6-9]. Aït-Ameur reported the behavior of S_R of Gaussian laser beam in characterising a diffracted laser beam[6]. Zalvida introduced the S_R using the Wigner distribution function[7]. Yang etc. analysed the metrics imprived by the task accounting for the phase, spatial frequency and orientation demands based on the visual S_R [8]. Xi studied the S_R of partially coherent sinh-Gaussian beams in the far field propagating through atmospheric turbulence[9].

Based on the extended Huygens-Fresnel principle, the analytical expressions for the S_R of GSM vortex beams and GSM non-vortex beams propagating through atmospheric turbulence and free space are derived, and used to study the influence of atmospheric turbulence parameters (the structure constant C_n^2, the inner scale l_0, the outer scale turbulence L_0), beam parameters (the spatial correlation length σ_0, the wave length λ).

Theoretical formulation

The field of Gaussian vortex beam at the plane z=0 is written as [10]

$$U(\boldsymbol{s}, z=0) = [s_x + \mathrm{i}\,\mathrm{sgn}(m) s_y]^{|m|} \exp(-\frac{s_x^2 + s_y^2}{w_0^2}), \tag{1}$$

where sgn(·) specifies the sign function, $\boldsymbol{s} \equiv (s_x, s_y)$ denotes the two-dimensional position vector at the z=0 plane, w_0 is the waist width in the Gaussian part, m is the topological charge, set as ±1.

By introducing the Schell correlator, the cross-spectral density function at the source plane z=0 is expressed as[10]

$$W^{(0)}(\boldsymbol{s}_1, \boldsymbol{s}_2, z=0) = [s_{1x}s_{2x} + s_{1y}s_{2y} + \mathrm{i}\,\mathrm{sgn}(m) s_{1x}s_{2y} - \mathrm{i}\,\mathrm{sgn}(m) s_{2x}s_{1y}]^{|m|} \times \exp(-\frac{\boldsymbol{s}_1^2 + \boldsymbol{s}_2^2}{w_0^2}) \exp[-\frac{(\boldsymbol{s}_1 - \boldsymbol{s}_2)^2}{2\sigma_0^2}], \tag{2}$$

where σ_0 is the spatial correlation length, $\boldsymbol{s}_i \equiv (s_{ix}, s_{iy})$ (i=1,2).

By using the paraxial form of the extended Huygens-Fresnel principle, the cross-spectral density of GSM vortex beam propagating through atmospheric turbulence is expressed as[2]

$$W(\boldsymbol{\rho},\boldsymbol{\rho}_d,z)=\left(\frac{k}{2\pi z}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W^{(0)}(\boldsymbol{s},\boldsymbol{s}_d,z=0)$$
$$\times\exp\left[\frac{ik}{z}(\boldsymbol{\rho}-\boldsymbol{s})(\boldsymbol{\rho}_d-\boldsymbol{s}_d)-H(\boldsymbol{\rho}_d,\boldsymbol{s}_d,z)\right]d^2sd^2s_d \quad (3)$$

where k specifies the wavenumber related to the wave length λ by $k=2\pi/\lambda$. In Eq.(3) we have used the following sum and difference vector notation.

$$\boldsymbol{\rho}=\frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{2}\ ,\boldsymbol{\rho}_d=\boldsymbol{\rho}_1-\boldsymbol{\rho}_2\,,\boldsymbol{s}=\frac{\boldsymbol{s}_1+\boldsymbol{s}_2}{2}\,,\boldsymbol{s}_d=\boldsymbol{s}_1-\boldsymbol{s}_2\,, \quad (4)$$

where the $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are two arbitrary points in the receiver plane, perpendicular to the direction of propagation of the beam. In Eq.(3) the term $H(\boldsymbol{\rho}_d, \boldsymbol{s}_d, z)$ can be written as[2, 11, 12]

$$H(\rho_d,s_d,z)=4\pi^2k^2z\int_0^1 d\xi\int_0^{\infty}[1-J_0(\kappa|s_d\xi+(1-\xi)\rho_d|)]\Phi_n(\kappa)\kappa d\kappa \quad (5)$$

The Sterhl ratio (S_R) is defined as[9]

$$S_R=\frac{I_{max}}{I_{0max}}\,, \quad (6)$$

where I_{max} and I_{0max} are the maximum intensity of a real beam and a diffraction-limited ideal beam, respectively. In this paper, the GSM non-vortex beam in free space is taken as the diffraction-limited ideal beam.

Letting $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}$, the average intensity of GSM vortex beams in atmospheric turbulence obtained from Eq.(3) is written as

$$I(\boldsymbol{\rho},z)=W(\boldsymbol{\rho},\boldsymbol{\rho},z)=\frac{k^2w_0^4}{4(8z^2A+k^2w_0^2)}[2-B(1\text{-}B\boldsymbol{\rho}^2)w_0^2-C(1-B\boldsymbol{\rho}^2)]\exp[-B\boldsymbol{\rho}^2]\,, \quad (7)$$

where

$$A=\frac{1}{2w_0^2}+\frac{1}{2\sigma_0^2}+T\,, \quad (8a)$$

$$B=\frac{2k^2}{8z^2A+k^2w_0^2}\,, \quad (8b)$$

$$C=\frac{8z^2}{(8z^2A+k^2w_0^2)w_0^2}\,, \quad (8c)$$

where

$$T=\frac{\pi^2k^2z}{3}\int_0^{\infty}\kappa^3\Phi_n(\kappa)d\kappa\,, \quad (9)$$

with $\Phi_n(\kappa)$ being the spatial power spectrum of the refractive index fluctuations of the turbulence, and we use the power spectrum[2]

$$\Phi_n(\kappa)=0.033C_n^2\frac{\exp[-(\kappa^2/\kappa_m^2)]}{(\kappa^2+\kappa_0^2)^{11/6}}\quad(0\le\kappa\le\infty)\,, \quad (10)$$

to model the Von Karman atmospheric turbulence, where $\kappa_0=1/L_0$ (L_0-outer scale of the atmospheric turbulence),$\kappa_m=5.92/l_0$ (l_0-inner scale of the atmospheric turbulence), C_n^2 is the structure constant.

Substituting Eqs.(7) and (8) into Eq.(6), we obtain

$$S_R=\frac{(8z^2A_1+k^2w_0^2)[2-Bw_0^2(1-B\boldsymbol{\rho}^2)-C(1-B\boldsymbol{\rho}^2)]\exp[-B\boldsymbol{\rho}^2]}{(8z^2A+k^2w_0^2)[2-B_1w_0^2(1-B_1\boldsymbol{\rho}^2)-C_1(1-B_1\boldsymbol{\rho}^2)]\exp[-B_1\boldsymbol{\rho}^2]}\,, \quad (11)$$

For T=0, we get the expressions of A_1、B_1、C_1 from that of A、B、C. Eq.(11) is the propagation expression of S_R of GSM vortex beam in atmospheric turbulence.

Letting $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2=\boldsymbol{\rho}$, m=0, the average intensity of GSM non-vortex beams in atmospheric turbulence obtained from Eq. (3) is written as

$$S_R = \frac{(8z^2 A_1 + k^2 w_0^2)\exp[-B\boldsymbol{\rho}^2]}{(8z^2 A + k^2 w_0^2)\exp[-B_1\boldsymbol{\rho}^2]} \quad (12)$$

The values of A,B, A_1,B_1 are the same as in Eq. (11).

Numerical calculations and analyses

Figure1 gives the S_R of GSM vortex beams and GSM non-vortex beams propagating through atmospheric turbulence versus (a) structure constant C_n^2, (b) inner scale of the atmospheric turbulence l_0, and (c) outer scale of the atmospheric turbulence L_0. The calculation parameters are (a) σ_0=1cm, w_0=3cm,λ=1064nm,L_0=10m,l_0=1cm, (b) and (c), $C_n^2=10^{-14}\text{m}^{-2/3}$, the other calculation parameters are the same as in Fig.1 (a). From Fig.1 (a),it can be seen that the S_R of GSM vortex beams and GSM non-vortex beams decrease with increasing structure constant C_n^2. From Fig.1 (b), it can be seen that the S_R of GSM vortex beams and GSM non-vortex beams increase with increasing inner scale of the atmospheric turbulence l_0. From Fig.1 (c), it can be seen that the S_R of GSM vortex beams and GSM non-vortex beams decrease with increasing outer scale of the atmospheric turbulence L_0.It can be seen from Figs.1, the S_R of GSM vortex beams propagating through atmospheric turbulence is larger than that of GSM non-vortex beams.

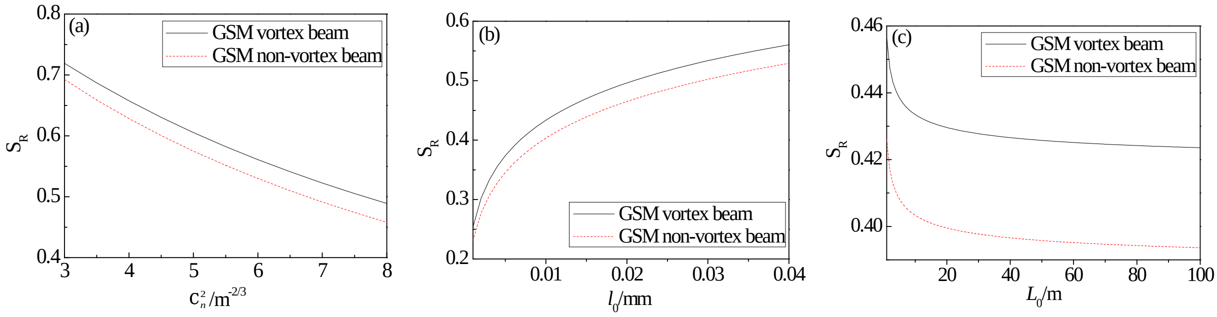

Figure 1 The S_R of GSM vortex beams and GSM non-vortex beams versus structure constant C_n^2 (a), inner scale of the atmospheric turbulence l_0 (b) and outer scale of the atmospheric turbulence L_0 (c)

The S_R of GSM vortex beams and GSM non-vortex beams propagating through atmospheric turbulence versus (a) spatial correlation length σ_0 and (b) wave length λ are depicted in Figure.2(a)-2(b), respectively. The calculation parameters are $C_n^2=10^{-14}\text{m}^{-2/3}$, the other calculation parameters are the same as in Fig1 (a). From Fig.2(a) we see that, when σ_0<0.028 cm, the S_R decrease with increasing spatial correlation length σ_0, when σ_0>0.028cm, the S_R increase with decreasing spatial correlation length σ_0. From Fig.2(b) we see that the S_R increase with increasing wave length λ. It can be seen from Figs.2, the S_R of GSM vortex beams propagating through atmospheric turbulence is larger than that of GSM non-vortex beams.

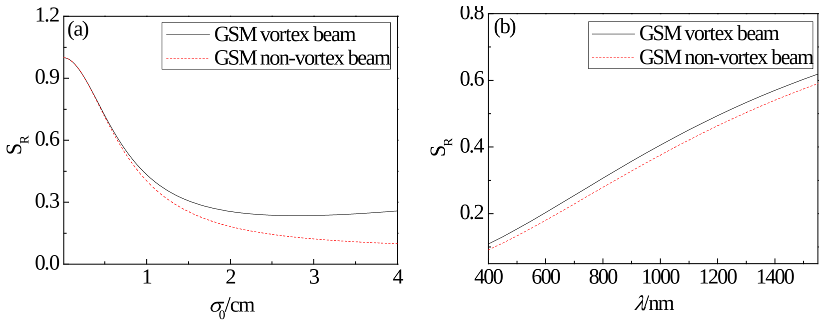

Figure 2 The S_R of GSM vortex beams and GSM non-vortex beams versus spatial correlation length σ_0 (a), wave length λ (b)

Conclusion

In this paper, by using the extended Huygens-Fresnel principle, the analytical expressions for the S_R of GSM vortex beams and GSM non-vortex beams propagating through atmospheric turbulence and free space are derived, and used to study the influence of beam parameters on the S_R. It is shown that with the decrement of structure constant C_n^2, the inner scale l_0 and the wave length λ, the increment of the outer scale turbulence L_0, the S_R for the S_R of GSM vortex beams and Gaussian Schell-model non-vortex beams will increase, with the increment of the spatial correlation length σ_0, the S_R for the S_R of GSM vortex beams and GSM non-vortex beams first decrease then increase. The theoretical analysis presented in this paper can also be used for the practical application of GSM vortex beams.

Acknowledgements

This work was financially supported by the National Natural Science Foundation of China (Nos. 61405136 and 61178067), the Natural Science Foundation for Young Scientists of Shanxi Province (Nos.2012021016 and 2013021010-4).

References

[1] Strohbehn J W: Laser beam propagation in the atmosphere (Springer-Verlag,New York,1978).

[2] Andrews L C and Phillips R L: Laser beam propagation through random media (SPIE press,Bellingham, Washington 2005).

[3] Li Jinhong and Lü Baida Acta Physics Sinica, Vol. 60 (2011) p.074205 (in Chinese)

[4] Zheng Yulong and Ji Xiaoling: High Power Laser and Particle Beams, Vol.24 (2012) p.276-280 (in Chinese)

[5] Duan Meiling,Li Jinhong and Wei Jinlin: High Power Laser and Particle, Vol.25 **(**2010) p.2252-2256 (in Chinese)

[6] Aït-Ameur K: Optics Communications Vol. 285 (2012), p. 699

[7] Zalvidea D, Lehman M and Granieri S,Sicre E E: Optics Communications Vol. 118 (1995), p. 207

[8] Yang Y, Cao G and Zhao H: Optik - International Journal for Light and Electron Optics Vol. 124 (2013), p. 6415

[9] Xiao X, Ji X and Lü B: Optics & Laser Technology Vol. 40 (2008), p. 129

[10] Li J, Zhang H and Lü B: Optics & Laser Technology Vol. 42 (2010), p. 428

[11] Dan Y and Zhang B: Opt. Express Vol. 16 (2008), p. 15563

[12] Yuan Y, Cai Y, Eyyuboğlu H T and Baykal Y,Chen J: Optics and Lasers in Engineering Vol. 50 (2012), p. 752