

Establish of Electricity Market Model with Variable Coefficient and Analysis of Its Practical Stability

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Abstract—The paper analyzes the dynamic equilibrium of the electricity market by using the principle of economics and establishes the non-autonomous linear discrete periodic mathematic models, the models describe the actions of the electricity market under the assumption that market demand is the linear function with variable coefficient of forecast price and that the market supply is the linear equation with varied coefficient of spot price. The practical stability of the electric market is discussed by using Lyapunov function and the calculus inequality. A number of critical conditions to judge the practical stability of electric market and the existing of practically asymptotically stable periodic solution of electricity market are obtained and these conclusions are explained by example.

Keywords—Electric market; model of deference equation; practical stability; periodic solution; practically quasi-stable

I INTRODUCTION

Electric power system is physical basis of power market; Power market is the operation mode of the electric power system. The economical stability of the power market and the physical stability of Electric Power System are lined and affected each other[1]. The study on stability of economical system has existed for long times, but the study on stability of power market begin just recently. The stability of the electricity market does not only embody physical stability such as the power angle stability, voltage stability and frequency stability of the system, but also needs to consider the stability of economic system related with the electric market. The electric market is a special kind of commodity markets with power transaction. Its market actions not only obey the general economic law [1- 2], but also has its own characteristics, such as difficult storage and balance between the electricity production, the consumption and the demand of the power with periodicity. The making the law of the power market with stable operation has new features. The price and volume of transaction is the core of the market and each participant is acceptor with the same price in the electricity market and the market for sale is accorded to the period of the production. In the following time, the producers establish the forecast of the market price in according to the present price, and put forward the price quoted. Producers will continue to repeat this process when the market price is observed. The demand curves based on needs of the consumers and the demand based on the current

market price. Literature [3-6] studied dynamic changes of the market price and the output of electric power, and strategy of bid, but did not involve stability of electricity market. The electric power market crisis happened in the world shows that the stable operation of power market is very important. Alvarado established a general model of differential equations describing the marketplace in paper [7-8] firstly and he studied the lyapunov stability of electricity market by theory of characteristic value first. Their works are based on the continuous market model, but discrete model is more practical in actual. In addition, the paper [7-8] did not consider the periodicity of actual market. Lyapunov stability is sometimes difficult to materialize in practical engineering. On the basis of the paper [7-8], the paper [9] studied practical stability of electricity market firstly by using theory of characteristic value. This paper will establish a kind of model of difference equation for describing the power market and study the practical stability of the power market. Finally, the rationality of the model is explained by a example.

II THE ESTABLISHMENT OF ONE-ORDER DIFFERENCE MODEL OF ELECTRIC MARKET

Certain assumptions are made in the paper according to the characteristics of the electricity market:

- (1) All market transaction prices are unique at a given time;
- (2) There is not energy storage and all production and sales is at the same time;
- (3) The market demand is a linear function with varied coefficient of the price in stock markets;
- (4) The market supply is a linear function with varied coefficient of the price in spot markets;
- (5) Market consume is determined by predicting the price and the supply is determined by the demand in some time.

We establish the model to describe the electricity market:

$$q^d(t) = a(t) - b(t)P(t) \quad (1)$$

$$P(t) = c(t) + e(t)q^s(t) \quad (2)$$

$$q^d(t) = q^s(t) \quad (3)$$

where $q^d(t)$ is the demand in moment t , $q^s(t)$ is the supply in moment t , $P(t)$ is the price in moment t , $\hat{P}(t)$ is

the predicting price in moment t for user; $a(t)$, $b(t)$, $c(t)$, $e(t)$ are periodic functions with respect to T . Equation (1) and (2) describe the actions of the demand and the supply in the market, and balance between the demand and the supply market is described by equation (3).

Let predict the price be $\hat{P}(t) = f(t)P(t-1)$. The above yields the following difference equation model of electric market

$$P(t) = -e(t)b(t)f(t)P(t-1) + [a(t)e(t) + c(t)], t \geq 0 \quad (4)$$

The equation (4) describes changes of the price in the power market. This model is appropriate when the quantities supplied and consumed are adjusted comparatively slowly relative to the price. The dynamics of the model presented here include the further requirement that supply and demand be in precise balance at all times.

III STUDY ON PRACTICAL STABILITY OF ELECTRIC MARKET

Let $-e(t)b(t)f(t) = A(t)$, $a(t)e(t) + c(t) = B(t)$,

$P(t) = x(t+1)$, then the following equation can be derived

from equation (4)

$$x(t+1) = A(t)x(t) + B(t) \quad (5)$$

The corresponding homogeneous equation

$$x(t+1) = A(t)x(t) \quad (6)$$

Dynamical behaviors of non-autonomous system (5) and (6) are defined over time interval $J = [0, K)$, $K \in [0, +\infty)$, so finite time stability and practical stability can be treated simultaneously. It is obvious that $J \in [0, +\infty)$. Next, we will discuss the case $K = +\infty$.

Theorem 1 Assume that

- 1) $|A(t)| \leq 1 + c_t$ ($c_t \geq 0$), and series $\sum_{t=1}^{\infty} c_t$ converges;
- 2) There are constants $\lambda > 0$, $A > 0$ such that $\lambda e^{\sum_{t=1}^{\infty} c_t} < A$

Then the solution of system (6) is practical stable with respect to (λ, A) .

Proof The solution of (6) with the initial value $(0, x_0)$ can be expressed in terms of fundamental matrix as it is written below

$$x(t+1) = x_0 \prod_{i=0}^t A(i)$$

Evaluating the norm $|x(t)|_2$ of both sides of this equations and using the known inequality $e^x \geq 1 + x$, $x \in (-\infty, +\infty)$ yields

$$|x(t+1)| \leq |x_0| \prod_{i=0}^t |1 + c_i| \leq |x_0| e^{\sum_{i=0}^t c_i} \leq \lambda e^{\sum_{i=0}^{\infty} c_i}$$

Applying the conditions 2) of Theorem, one can get $|x(t+1)| \leq A$. So the solution of system (6) is practical stable with respect to (λ, A) .

Theorem 2 Assume that

- 1) the matrix $A(t)$ is periodic functions with respect to T , and such that $\prod_{i=0}^T |A(t)| < 1$, let $M = \max_{0 \leq t \leq T} \{ |A(t)| \}$;
- 2) There are constants $\lambda > 0$, $A > 0$, $N > 0$, such that

$$\left(\prod_{i=0}^T |A(i)| \right)^k < \frac{A}{\lambda M^T}, k > N$$

Then the solution of system (6) is practically asymptotically stable respect to (λ, A, N) .

Proof Since the matrix $A(t)$ are periodic functions with respect to T . For any t , there is a constant k such that $kT \leq t \leq (k+1)T$. So we have

$$|x(t+1)| = |x_0| \prod_{i=0}^t |A(i)| = |x_0| \left(\prod_{i=0}^T |A(i)| \right)^k \prod_{i=kT+1}^t |A(i)|$$

$$\leq |x_0| \left(\prod_{i=0}^T |A(i)| \right)^k M^T$$

Since in view of 2), we have

$$|x(t+1)| \leq |x_0| \left(\prod_{i=0}^T |A(i)| \right)^k M^T < \lambda M^T \frac{A}{\lambda M^T} = A$$

So the solution of system (6) is practical quasi stable with respect to (λ, A, N) .

Theorem 3 Assume that the condition 1) of theorem 2 and $M^T \lambda < A$ hold, then the solution of system (6) is practically asymptotically stable with respect to (λ, A) .

Proof Since the matrix $A(t)$ are periodic functions with respect to T , so $|A(t)| \leq M$. For any t , there is a constant k such that $kT \leq t \leq (k+1)T$. So we have

$$|x(t+1)| < |x_0| \left(\prod_{i=0}^T |A(i)| \right)^k M^T < \lambda M^T < A$$

So the solution of system (6) is practical stable with respect to (λ, A) .

Since $\prod_{i=0}^T |A(i)| < 1$, for any $\varepsilon > 0$, there a positive integer $N > 0$, such that

$$\left(\prod_{i=0}^T |A(i)| \right)^k < \frac{\varepsilon}{\lambda M^T}, \text{ when } k > N$$

$$|x(t+1)| \leq |x_0| \left(\prod_{i=0}^T |A(i)| \right)^k M^T < \lambda M^T \frac{\varepsilon}{\lambda M^T} = \varepsilon$$

So Then the solution of system (6) is practically asymptotically stable with respect to (λ, A) .

Theorem 4 Assume that the conditions of theorem 3 holds and $B(t)$ is a periodic matrixes functions with respect to T , Then system (5) has only one periodic solution with respect to T and this periodic solution is practically asymptotically stable with respect to (λ, A) .

Proof The solution of equation (5) with the initial value $(0, x_0)$ can be expressed in terms of fundamental matrix as it is written below

$$x(t) = x_0 \prod_{i=0}^t A(i) + \sum_{r=1}^t \prod_{i=1}^{r-1} A(t-i) B(t-r)$$

$$\text{where } \prod_{i=0}^t A(i) = 1 \text{ when } t < 0.$$

$$x_0 = \frac{\sum_{r=1}^T \prod_{i=1}^{r-1} A(t-i) B(t-r)}{1 - \prod_{i=0}^T A(i)}$$

Let us take , the solution of equation (5) with the initial value $(0, x_0)$ such that

$$x(T) - x(0) = x_0 \left\{ \prod_{i=0}^T A(i) - 1 \right\} + \sum_{r=1}^T \prod_{i=1}^{r-1} A(t-i) B(t-r) = 0$$

So the solution $x(t, 0, x_0)$ is periodic solution with respect to T . Next, we will proof this periodic solution is only one.

If it is not true, there are two periodic solutions $\varphi_1(t), \varphi_2(t)$ with respect to T . Let $y(t) = \varphi_1(t) - \varphi_2(t)$, then $y(t)$ is a solution of system (6). By Theorem 3, we have

$$\lim_{t \rightarrow +\infty} y(t) = \lim_{t \rightarrow +\infty} [\varphi_1(t) - \varphi_2(t)] = 0$$

So $\varphi_1(t) = \varphi_2(t)$, That is system (6) has only one periodic solution with respect to T .

Because the zero solution of (6) is practical asymptotically stable, so this periodic solution is asymptotically stable, the proof is complete.

IV APPLICATIONS

Next we will analysis the data of market prices and demand in April 15, 1998 in PX market of California [10] by using the above model. The coefficient $b(t), d(t), f(t)$ of equation (1)–(4) can be derived from economics meaning by using fitting of historical data. The Consumers present linear forecast function $\hat{P}(t)$ of price by analyzing the price before April 15, 1998. From equation (2), we can calculate the coefficient $c(t)$ by using the demand $q^d(t)$ and market price $P(t)$ and fitting coefficient $e(t)$. We Simulate system (4) by using these coefficient, we can get only one period solution. Figure 1 is used to illustrate this last point. This figure shows that system (4) has only one periodic solution and this periodic solution is asymptotically stable.

These coefficients satisfy the conditions of Theorem 4, system (6) has only one periodic solution with respect to T and this periodic solution is practically asymptotically stable with respect to (λ, A) by Theorem 4. This result is the same with numerical simulations. These shows that established mathematical model in this paper is the same with the actual situation. These theoretical results offer theoretical basis to ensure stability of electricity market.

V CONCLUSION

The paper establishes a difference equation model of electricity market and discusses its practical stability. A series of sufficient conditions of practical stability of electricity market are obtained. These conditions are expressed by coefficients of the model. It is simple and convenient to judge whether the electricity market is stable. The last example shows that this model is very useful.

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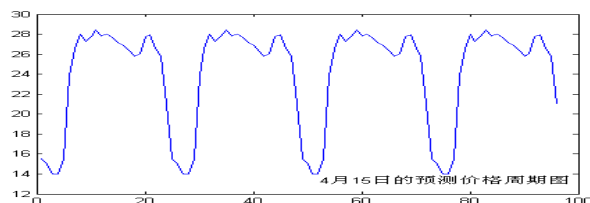


Figure 1. Simulation figure of PX from California