A Novel Positioning Method Based on Swedish Wheel and Gyroscope for Mobile Robot

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Abstract. In this paper a general positioning subsystem that used for several kinds of mobile robot is presented. The subsystem consist of a Optical Fiber Gyroscope(OFG) with high precision and a Orthogonal Passive Wheel System(OPWS) which is combination of Swedish Wheels and Optical Encoders. The OPWS is an independent planar odometer which is regarded as the local reference of the kinematic model, while the OFG provides the instant angle between global and local reference frame of the robot. A dead recking model is developed regardless of the specific robot base structure. The error sources of the proposed positioning method is analyzed in allusion to an omnidirectional robot configuration. Calibration method based on geometric trajectory tracking is introduced according to the error sources, followed by the experimental results that confirm both feasibility of the positioning method and effectiveness of the calibration method.

Introduction

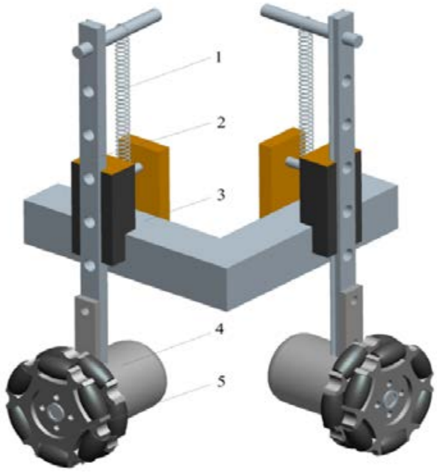
There are many kinds of position estimation techniques for mobile robots. GPS is a well-known method for outdoor applications[1,2]; as for indoor applications image processing and multi-sensor fusion techniques are often used[3,4]. All the methods above have both advantages and disadvantages; for example, robots with image processing techniques are relatively robust and usually have high precision, but the algorithms are usually hard to implement and time consuming.

Odometry is another widely used method for positioning of wheeled mobile robot[5]. Odometry is simple, inexpensive and easy to implement in real time, but it has unbounded accumulation of errors. In most mobile robots, odometer is implemented by reusing the active wheel of the robot base[6-10]. This kind of positioning usually causes coupling of different errors, for example, the well-known calibration method for differential drive mobile robot UMBmark uses clockwise and counter clockwise square path to decoupling the two errors caused by unequal wheel diameters and uncertainty of the wheelbase[6-10]. Maddahi developed an improved UMBmark method by classifying the error by lateral and longitudinal position error, which is applicable to both differential drive and omnidirectional wheeled robots[11,12]. The procedure above is relatively complex and blind to artificial disturbs; furthermore, all the experiments above are conducted at relatively low speed which is lack of persuasion.

Swedish wheel has omnidirectional kinematic performance because of its special mechanism[2]. In most applications, Swedish wheel is used to compose the omnidirectional robot base wheels, moving actively from the perspective of overall robot. On the other hand, when the Swedish wheel with a 90^{o} roller offset angle moves passively, the wheel becomes unidirectional, which means it can only record the motion strictly along the direction perpendicular to the pivot axis regardless of direction of robot's motion. This feature is applied to a new method for mobile robot positioning, in which two Swedish Wheels with orthogonal layout are used to measure the length of X-axis and Y-axis respectively. As for measuring the length quantitively, the commonly used optical encoder is applied by installing the encoder shaft be coincide with that of Swedish wheel.

Structure of the positioning subsystem

As described above, the positioning subsystem consist of two parts: OPWS and OFG. Fig.1 shows the overall mechanical structure of OPWS. There are two special odometers, each of which includes a relatively small Swedish wheel, an optical encoder and a linear slide rail. The odometers are connected by a L-bracket as shown in Fig.1 which is regarded as the Cartesian base. As for the slide rail, it is designed to guarantee the metrical accuracy by making Swedish wheel have stable contact with floor while moving at high speed or moving on uneven surface.



1.Holddown spring. 2.Linear slide rail. 3.L-like support.
4.Optical encoder. 5. Swedish wheel with 90^{o} roller offset angle

Fig.1 3D model of OPWS.

Gyroscope can measure instant orientation angle of a mobile robot, applications that use OFG in conjunction with odometry information obtain relative high positioning accuracy[10,13]. However, the method above always require complex algorithm to fuse information from different sensors, which is time consuming. In the new method for robot positioning, a OFG with high precision is used for measuring the orientation angle instantly.

Dead recking method based on OPWS and OFG

Generally, a mobile robot that operates on horizontal plane can be modeled as a rigid body on wheels. The robot has three dimensionality, two for position in plane and one for orientation. Before introducing dead recking formula, two robot reference frames are established as shown in Fig.2, naming global reference frame $X_I O_I Y_I$ and local reference frame $X_R O_R Y_R$ respectively. Global reference frame stands for an arbitrary inertial basis on the plane, in which the global pose vector P_I is defined as follows:

$$P_I = \begin{bmatrix} X_I & Y_I & \theta \end{bmatrix}^T \quad (1)$$

The planar odometer described above represents the local reference frame which means the frame is connected to the robot base rigidly and moves synchronously with robot. The local pose vector P_R is defined as follows:

$$P_R = \begin{bmatrix} X_R & Y_R & \theta \end{bmatrix}^T \quad (2)$$

where θ is the angle between global and local reference frame which is provided by OFG. While X_R and Y_R are calculated from the information of OPWS. Note that the subscript(superscript in the following sections) $"R"$ and $"I"$ stands for local and global reference frame respectively.

Practically in discrete system, time is divided into fixed intervals, ΔT, in which the wheel velocities of odometer and the orientation angle are assumed to be constant. So, the incremental poses ΔP_I and ΔP_R of the robot within ΔT satisfy the following formula:

$$\Delta P_I = R(\theta)\Delta P_R \quad (3)$$

where $R(\theta)$ is orthogonal rotation matrix which is used to transform position information fro m local reference frame to global reference frame. $R(\theta)$ is defined as follows:

$$R(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \tag{4}$$

Assume that the robot pose in global reference frame at time t is $\Delta P_{I(t)}$, the pose at time $t+\Delta T$ can be estimated as:

$$P_I(t+\Delta t)=\begin{bmatrix}X_{I(t)}\\ Y_{I(t)}\\ \theta_t\end{bmatrix}+\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta X_R\\ \Delta Y_R\\ \Delta\theta\end{bmatrix} \tag{5}$$

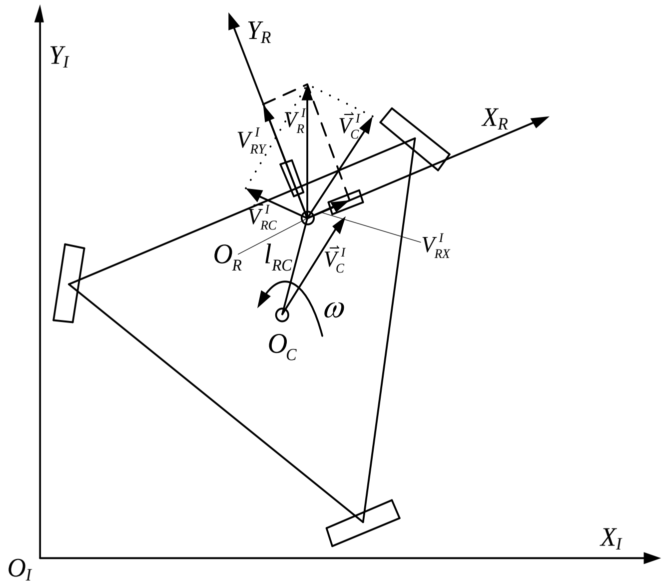


Fig.2 Coordinate frames of three-wheeled omnidirectional mobile robot in 2D workspace

The position of the origin of OPWS O_R can be calculated via the formula above, while the robot center O_c which is the reference for motion control is not coincide with O_R in general . As shown in Fig.2, assume that the translational velocity of O_R is $\vec{V}_R^I(V_{RX}^I, V_{RY}^I)$, and that of O_c is $\vec{V}_C^I(V_{CX}^I, V_{CY}^I)$, then the relation of the two velocities can be described as follows:

$$\vec{V}_R^I=\vec{V}_C^I+\vec{V}_{RC}^I \tag{6}$$

where $\vec{V}_{RC}^I$ is the relative speed of O_R using O_C as reference point. Then the following formula can be derived:

$$\begin{cases}V_{CX}^I=V_{RX}^I+\omega l_{RC}\sin\theta\\ V_{CY}^I=V_{RY}^I-\omega l_{RC}\cos\theta\end{cases} \tag{7}$$

where ω is the rotational velocity, l_{RC} is the length between O_R and O_C.

The translational velocity of O_R can be replaced by ΔP_I within fixed control intervals ΔT . Then the pose of O_C can be estimated as follows:

$$P_{I(t+\Delta t)}=\begin{bmatrix}X_{I(t)}\\ Y_{I(t)}\\ \theta_t\end{bmatrix}+\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta X_R\\ \Delta Y_R\\ \Delta\theta\end{bmatrix}+\begin{bmatrix}\Delta\theta * l_{RC}\sin\theta\\ -\Delta\theta * l_{RC}\cos\theta\\ 0\end{bmatrix} \tag{8}$$

Kinematic model and control of swedish wheeled robot

In the following part, geometric trajectory tracking and orientation control based on omnidirectional robot base is introduced which is used in calibration of positioning error. As for the kinematic analysis of the omnidirectional mobile robot, many literatures have done in detail[2,14].

The offline planned path can be divided into combination of straight line and circle. For straight line, apply the robot translational velocity V_I with the vector from current positioning point to the target point; while apply the translational velocity along with tangent direction for circle path which is relatively complex.

As for orientation, apply the robot rotational velocity with the following formulation：

$$\omega_t = \Delta\theta_t \times V_I \div \Delta S_t \tag{9}$$

where $\Delta\theta_t$ and ΔS_t stands for remaining orientation angle and remaining length of current path specifically at time t.

Theoretically, the robot can move along the ideal path by using the method above. However, the robot will deviate from the path due to imperfections in the design and mechanical implementation. A PID controller is designed to calibrate the trajectory tracking error, Fig.3 illustrates the schematic diagram of the controller.

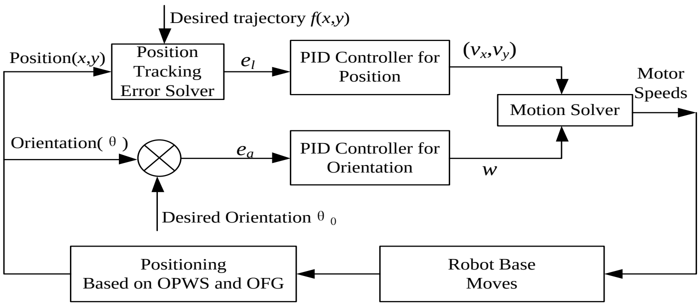
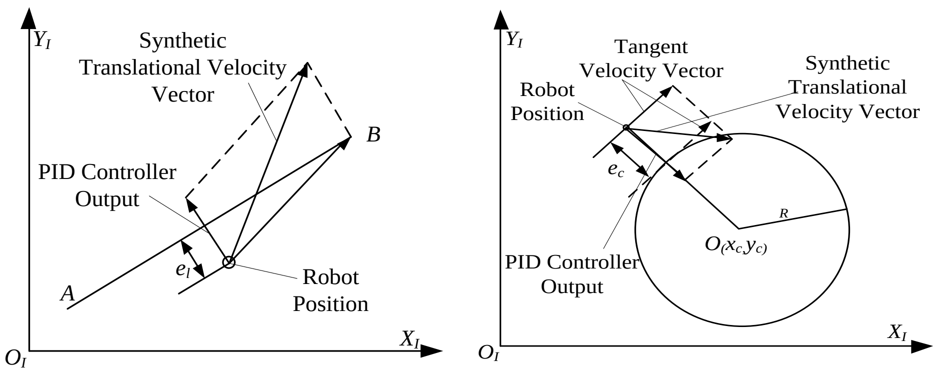


Fig.3 Robot control structure diagram

The position tracking error is calculated as follows:

1)for line path, the the tracking error is defined as the length from robot to the line as shown in Fig.4(a).

2)for circle path, the the tracking error is defined as the difference between the length from robot to circle center and the circle radius as shown in Fig.4(b).



a. Straight line path b. Circle path

Fig.4 Principle for geometric trajectory tracking

Calibration of positioning error

Like any other positioning system, there must be accumulative error during robot moving[9]. However, because of the passive use of Swedish wheel, the position and orientation error are independent, which is different from the traditional positioning method as mentioned above[5,13].

Initial Pose Error in Global Reference Frame. Generally, the origins of local and global reference frame don't coincide as the global reference are usually expressed by L-like reference wall. So The initial robot pose especially the initial orientation angle in the global reference should be accurately measured, otherwise there will be a lot of accumulation error. The influence of the initial pose error can be described as follow:

$$\begin{bmatrix} e_X \\ e_Y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos e_\theta & -\sin e_\theta & 0 \\ \sin e_\theta & \cos e_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta X_I \\ \Delta Y_I \\ e_\theta \end{bmatrix} \tag{10}$$

ΔX_t、ΔY_t stands for the incremental length of X-axis and Y-axis from start to end point in global reference frame respectively.

Error from Swedish Wheel and Interaction With Moving Surface. This kind of error is a combination of systematic and nonsystematic error, which is assumed belongs only to the position error because of the use of OFG for orientation measurement. The sources are summarized as follows:

- Average effective diameters differ from nominal diameter
- Travel over uneven floors
- Velocity of the Swedish wheel
- Discontinuous contact between Swedish wheel and floor

The first source is systematic error while the other three are nonsystematic error. In order to simplify the procedure of calibration, two nondeterministic parameters named k_X、k_Y, are introduced for robot local reference frame. The following experiment is designed to determine the parameters.

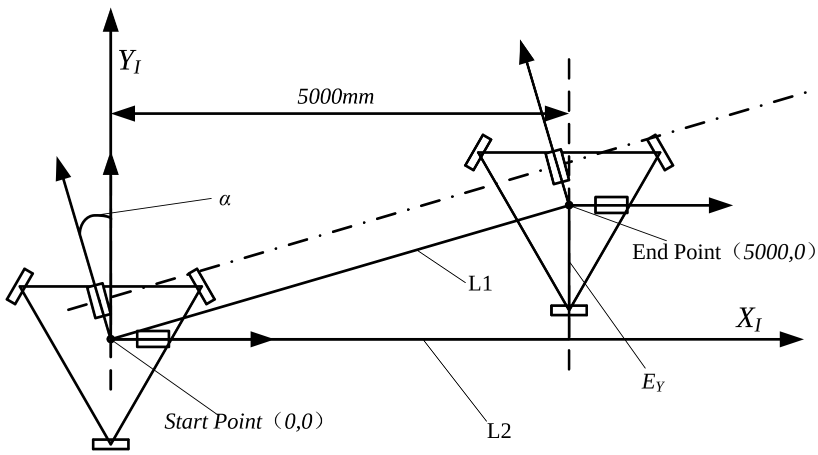


Fig.5 Motion for straight line, angular error of OPWS(larger than 90 degrees) is assumed

As shown in Fig.5, the robot is programmed to travel along the X-axis of local reference frame. because actual angle between axises of OPWS is unequal to 90 degrees due to imperfections of mechanical implementation, and the PID controller will apply additional velocity to maintain the result of Y-axis zero. So, the robot will not travel strictly along the expected path. However, the angular error named α can be estimated within ±1degree. Assume that actually traveled length of the Swedish wheel on X-axis is L_1, while the length from start to end point is L_2 , of which the center point of the robot base is used as the measuring basis and do not need any other references. the following relation can be derived:

$$\cos\alpha = \frac{L_1}{L_2} \geq 0.9998 \tag{11}$$

So, L_2 can replace L_1 in the condition that the test path is long enough. k_x can be calculated as follows:

$$k_X = \frac{\sum_1^N L_{2(i)}}{N\left(X_1^I - X_0^I\right)} \quad (12)$$

where N is the number of test, X_0^I and X_1^I stand for the X coordinate of start and end point specifically, $L_{2(i)}$ means the i th measuring result of L_2 .k_Y is determined through similar procedure above. Calibration can be done by replacing $[\Delta X_R, \Delta Y_R, \Delta\theta]^T$ with $[\Delta X_R * k_X, \Delta Y_R * k_Y, \Delta\theta]^T$ in formula (8).

Specifically, the measuring work of the special method above does not depend on the robot initial pose, while most exited calibration methods[5,14] that the totally same initial pose is required during the repetitive tests, which is impossible. As the former part has shown that the initial pose error will produce obvious positioning error, so this method is better theoretically.

Angular error between Axises of OPWS. After the calibration described in Section 5.2, the two odometers of OPWS are believed to be accurate. As shown in Fig.5 , the angular error between axises of OPWS has little influence on calibration of error type A, but it is essential to correct it because unignorable length error will be caused on Y-axis of global reference frame during the test. Assume that the length of the straight line is 5000mm, the error can be estimated as follows:

$$E_Y \approx L_1 * \alpha = 87.27mm \quad (13)$$

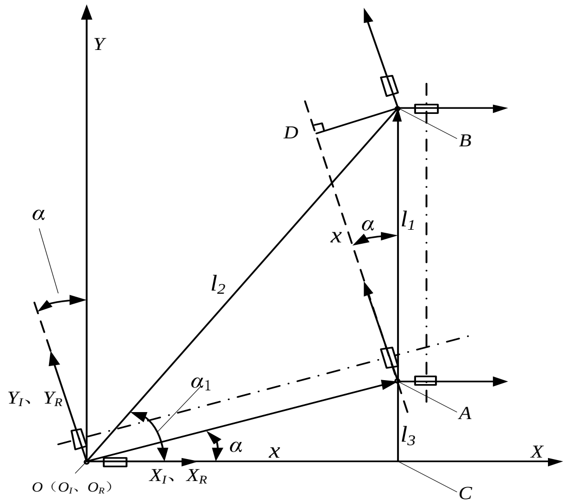


Fig.6 Analysis of straight line motion for calibration of angular error of OPWS

Fig.6 illustrates the schematic diagram of the calibration. The angular error can be measured through the following procedure:

- Given a straight path *OB* of which the start and end pose are $O(0,0,0)$ and $B(x, x, 0)$. Assume that the X-axis of local reference frame is coincident with that of global reference frame.
- To analyze the geometric relation in Fig.6 , the test path is divided into two procedures. First, the robot moves to point A which means complementing the motion along the X-axis. Point C is the projection of A on the X-axis of local reference frame, so the length of *OC* is *x*
- The robot moves from point A to Point B, point D is the projection of B along the Y-axis of the actual local reference frame, so the length of *AD* is *x* as the odometers have been calibrated.

According to the geometric relations in Fig.6, the following formulas are deduced:

$$\begin{cases} l_3 = x * \tan\alpha \\ l_1 = x/\cos\alpha \\ l_2 = \sqrt{x^2 + (l_1 + l_3)^2} = x\sqrt{1 + \left(\frac{1+\sin\alpha}{\cos\alpha}\right)^2} \end{cases} \tag{14}$$

Simplify the formula above:

$$\cos 2\alpha_1 = \cos\left(\frac{\pi}{2} + \alpha\right) \tag{15}$$

In the case that the actual angle between axis of OPWS is larger than 90 degrees as shown in Fig.6:

$$\alpha_1 = \left(\frac{\pi}{2} + \alpha\right) \Big/ 2 \tag{16}$$

While if the angle between axises of OPWS is smaller than 90 degrees:

$$\alpha_1 = \left(\frac{\pi}{2} - \alpha\right) \Big/ 2 \tag{17}$$

It is concluded that α_1 is half of the actual angle between axises of OPWS. α_1 can be calculated by

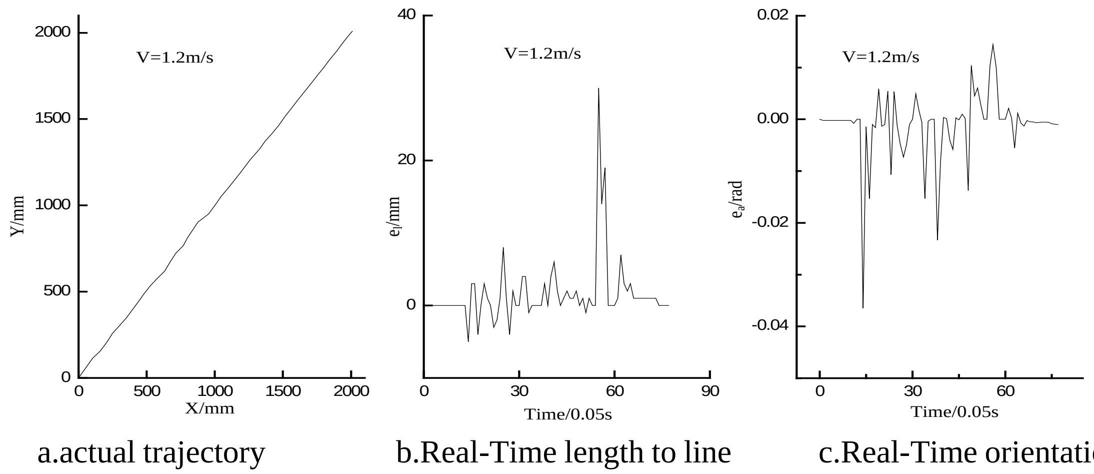
$$\alpha_1 = \arccos\frac{x}{l_{OB}} \tag{18}$$

Where l_{OB} is the metrical length of the experiment. Then α can be calculated through (16) or (17). Calibration can be done by replacing $[\Delta X_R, \Delta Y_R, \Delta\theta]^T$ in formula (8) With $[\Delta X_R, \Delta Y_R/\cos\alpha, \Delta\theta]^T$. Like the calibration above, this method is also has nothing to do with other references.

Experiments and results

The experiments include feasibility and calibration of positioning error. All of the experiments was conducted with a three wheeled robot prototype as shown in Fig.9.

Feasibility for line path. The robot run a line from point (0mm,0mm,0rad) to (2000mm,2000mm,0rad) at the speed of 0.8m/s and 1.5m/s respectively. Fig.7(a) shows the X and Y position that calculated by the robot,Fig.7(b) Shows the tracking error during moving, and Fig.7(c) is the real-time orientation angle of the robot. The result validates the feasibility of the positioning method, while the accuracy is not measured.



a.actual trajectory b.Real-Time length to line c.Real-Time orientation error

Fig.7 Result for straight line motion test

Performance of OFG. The test was conducted to measure the uncertainty of OFG. The error between actual value and integral value at different orientation angle is shown in Fig.8. The result shows that the integral error is within 1 degree, which meets the requirement of the robot.

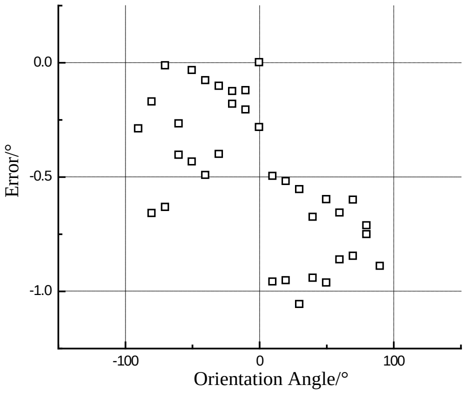


Fig.8 Result of integral effect test of OFG

Error of OPWS

- **Odometer calibration.** The robot was controlled as described above, three different traveling velocities, 0.4 m/s, 0.8 m/s and 1.2 m/s, are tested 10 times for each. The result shows in Table 1. the result shows that the error of single axis of OPWS is between 0.6％ and 0.8％.

TABLE 1: Result of odometer calibration test.

	1	2	3	4	5	6	7	8	9	10	avg.
X-aixs	5036	5032	5038	5033	5031	5033	3030	5031	5030	5033	5032.7
Y-aixs	5035	5038	5038	5039	5034	5036	5038	5033	5035	5036	5036.2

- **Angular error of OPWS.** The calibration the angular error of OPWS was conducted above, in which the value of x is chosen to be 2000mm. The measuring value of l_{OB} is shown in Table 2. The angular error is calculated to be 0.62 degree according to formula(17) .

TABLE 2: Result of angular error of OPWS calibration test

	1	2	3	4	5	6	7	8	9	10	avg.
l_{OB}	2851	2855	2860	2861	2858	2850	2853	2859	2859	2863	2857

- **Overall Positioning Test.** The overall positioning test is based on geometric trajectory tracking and orientation control described above. The test is conducted by controlling the robot to track a straight line path from (0mm,0mm,0rad) to (2000mm,2000mm,0rad). As the test needs to measure the position of robot in real global reference frame, a mechanical bracket was used to make the robot initial pose constant as shown in Fig.9, Which reduce the influence of initial pose error to a great extent. Table 3 is the result before calibration, and Table 4 is the result after calibration. The result in Table3 shows that the measuring value on X-axis is bigger than that of Y-axis, which means that the value of α is less than 90 degree according to description .Results shows that the error changed from 1.5％ to 0.3％, which proves that calibration method is workable.

Fig.9 Robot prototype and its initial state

TABLE 3: Measuring values of robot position before calibration

	1	2	3	4	5	avg.
X	2030	2028	2035	2033	2030	2031.2
Y	2010	2015	2008	2010	2010	2010.6

TABLE 4: Measuring values of robot position after calibration

	1	2	3	4	5	avg.
X	2005	2007	2004	2006	2007	2006
Y	2000	2006	2008	2010	2002	2005

Conclusions

In this paper, a novel positioning method based on OFG and OPWS has been presented. The overview of the OPWS has been introduced, along with the kinematic model of the dead recking method. In order to validate the feasibility of the positioning method, geometric trajectory tracking and orientation control has been introduced. This paper has analyzed the main error sources, and given detailed calibration model and procedure. Several tests in allusion to the error source have been conducted, the results shows that the precision of OFG is within 1 degree, and that of OPWS is between 0.6% and 0.8%. The result of overall positioning test proves that the accuracy has improved from 1.5% to 0.3%, which confirm the feasibility of both positioning and calibration method. Further improvement will include research on multisensor fusion technique.

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