Sensor and Actuator Fault Reconstruction for Nonlinear System

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Abstract. This paper presents a fault reconstruction scheme based on sliding mode observers for sensor and actuator faults detection and isolation for a class of uncertain nonlinear systems. A coordinate transformation is first designed for the output equation of the system followed by a first-order low-pass filter in order to convert the sensor faults into equivalent actuator faults. The original system then is transformed into three subsystems through linear transformation.

Introduction

System faults and unknown input disturbances are inevitable during operation of a complicated power system. With regard to physical position, faults can be classified into actuator and sensor faults. Faults can damage the normal system operation and make the system unstable, therefore, the fault detection and isolation (FDI) technique plays an important role in system operation. In the past decades, the study on FDI made great progress, especially for the model-based fault detection [1–3]. Various approaches have been proposed to solve FDI problems, such as differential geometry method, self-adaptive control method, and sliding mode observer technique. Moreover, research on fault tolerance control and stability analysis considerably promoted the FDI [4–6]. The power system used in this study inevitably suffers from unknown input disturbances and potential system faults during operation. Therefore, the sliding mode control technique, an effective method with has high robustness and insensitivity to system uncertainty, has been extensively employed in the study for FDI .

Problem description

A nonlinear system with actuator and sensor faults is given by

$$\begin{cases} \dot{x}(t) = Ax + \Phi(x,u) + Ef_a(t) + Dd(t) + Bu(t) \\ \quad y(t) = Cx(t) + F_s f_s(t) \end{cases} \quad (1)$$

where $x \in R^n$, $u \in R^r$ and $y \in R^p$ denote, respectively, the state variables, inputs and outputs. The nonlinear continuous term $\Phi(x,u) \in R^n$ is assumed to be known. The unknown nonlinear term $d(t) \in R^q$ models the lumped uncertainties and disturbances experienced by the system, which is assumed to be bounded, i.e., a positive constant γ_1 exists such that $\|d(t)\| \leq \gamma_1$. The unknown nonlinear terms $f_a(t) \in R^q$ and $f_s(t) \in R^m$ denote, respectively, actuator faults and sensor faults, which are also bounded, i.e., two constant γ_2 and γ_3 exist such that $\|f_a(t)\| \leq \gamma_2$ and $\|f_s(t)\| \leq \gamma_3$. $A \in R^{n \times n}$, $B \in R^{n \times r}$, $C \in R^{p \times n}$, $D \in R^{n \times q}$, $E \in R^{n \times q}$ and $F_s \in R^{p \times m}$ are known constant matrices with $n > p > q + m$.

For the output signal $y \in R^p$ of system (1), a transformation matrix $S_0 \in R^{p \times p}$ exists such that the following equation could be obtained:

$$S_0 \begin{bmatrix} C & F_s \end{bmatrix} = \begin{bmatrix} C_1 & 0 \\ C_2 & F \end{bmatrix} \quad (2)$$

we obtain

$$S_0 y = \begin{cases} y_1 = C_1 x \\ y_2 = C_2 x + F f_s \end{cases} \quad (3)$$

where $y_1 \in R^{p-m}$, $y_2 \in R^m$, $C_1 \in R^{(p-m)\times n}$, $C_2 \in R^{m\times n}$, and $F \in R^{m\times m}$ is a nonsingular constant matrix. Partition of the outputs y_1 and y_2 represents the fault free sensor signals and the potentially faulty sensor signals, respectively.

A new status variable $z_a \in R^m$ is then designed as the first-order low-pass filter of the output signal y_2

$$\dot{z}_a = A_s z_a + B_s y_2 \quad (4)$$

where $A_s \in R^{m\times m}$, and $B_s \in R^{m\times m}$ are filter matrices to be determined and B_s is a constant matrix with full rank.

The following equation could be obtained by transforming the output signal y_2 of Eq. (3) to Eq. (4)

$$\dot{z}_a = A_s z_a + B_s C_2 x + B_s F f_s \quad (5)$$

Eq. (5) and System (1) are then combined into a new system equation.

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{z}_a \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_s C_2 & A_s \end{bmatrix} \begin{bmatrix} x \\ z_a \end{bmatrix} + \begin{bmatrix} \Phi(x,u) \\ 0 \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} f_a(t) + \begin{bmatrix} 0 \\ B_s F \end{bmatrix} f_s(t) + \begin{bmatrix} D \\ 0 \end{bmatrix} d(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\ \begin{bmatrix} y_1 \\ z_a \end{bmatrix} = \begin{bmatrix} C_1 & 0 \\ 0 & I_m \end{bmatrix} \begin{bmatrix} x \\ z_a \end{bmatrix} \end{cases} \quad (6)$$

A set of new status variables and the corresponding matrix are then designed as follows:

$$\bar{x} = \begin{bmatrix} x \\ z_a \end{bmatrix}, \bar{y} = \begin{bmatrix} y_1 \\ z_a \end{bmatrix} = \begin{bmatrix} y_1 \\ y_a \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 \\ B_s C_2 & A_s \end{bmatrix}, \bar{C} = \begin{bmatrix} C_1 & 0 \\ 0 & I_m \end{bmatrix}$$

where $\bar{x} \in R^{n+m}$, $\bar{y} \in R^p$, $\bar{A} \in R^{(n+m)\times(n+m)}$ and $\bar{C} \in R^{p\times n}$.

Eq.(6) could be converted into the following two subsystems

$$\begin{cases} \dot{x}(t) = Ax + \Phi(x,u) + Ef_a(t) + Dd(t) + Bu(t) \\ y_1(t) = C_1 x(t) \end{cases}$$

$$\begin{cases} \dot{z}_a = A_s z_a + B_s C_2 x + B_s F f_s \\ y_a = z_a \end{cases}$$

Fault reconstruction and disturbance estimation

When the system reaches the sliding mode surface, $s_i = \dot{s}_i = 0$ (i=1,2,3) according to the sliding mode equivalent principle . The following equation is obtained:

$$F_1 e_{v1} = F_1 \dot{e}_{v1} = F_1 C_{11} \dot{e}_1 = 0$$
$$F_2 e_{v2} = F_2 \dot{e}_{v2} = F_2 C_{22} \dot{e}_2 = 0$$
$$e_a = \dot{e}_a = 0$$

From lemma 1, e_1 and e_2 will approach zero in some finite time and $\Phi(z,u) = \Phi(\hat{z},u)$.

(1) The reconstruction of actuator fault $f_a(t)$

We obtain the following:

$$F_1 C_{11}((\bar{A}_{11} - L_1 C_{11})e_1(t) + \bar{A}_{12}e_2(t) + \bar{\Phi}_1(z,u) - \bar{\Phi}_1(\hat{z},u) + \bar{E}_1 f_a(t) - \bar{E}_1 r_1(t)) = 0$$

$$f_a(t) \approx r_1(t) = \rho_1 \operatorname{sgn}(F_1 e_{v1})$$

(2) The reconstruction of sensor fault $f_s(t)$

The following equation could be obtained:

$$A_1 e_1(t) + A_2 e_2(t) + A_s e_a(t) + B_s F f_s(t) - r_3(t) = 0$$

$F \in R^{m \times m}$ is a nonsingular matrix and $B_s \in R^{m \times m}$ is a full rank matrix to be designed. Therefore, a matrix B_s exists to make $B_s F$ to be a nonsingular matrix, then

$$f_s(t) \approx (B_s F)^{-1} r_3(t) = (B_s F)^{-1} \rho_3 \, sgn(e_a)$$

(3) The estimation of the unknown input disturbance $d(t)$

we obtain the following:

$$F_2 C_{22}((\bar{A}_{22} - L_2 \bar{C}_{22}) e_2(t) + \bar{A}_{21} e_1(t) + \bar{\Phi}_2(x,u) - \bar{\Phi}_2(\hat{x},u) + \bar{E}_2 f_a(t) - \bar{E}_2 r_2(t) + \bar{D}_2 d(t)) = 0$$

$$d(t) \approx \bar{D}_2^{-1}(\bar{E}_2 r_2(t) - \bar{E}_2 f_a(t)) = \bar{D}_2^{-1} \bar{E}_2(\rho_2 \, sgn(F_2 e_{v2}) - \rho_1 \, sgn(F_1 e_{v1}))$$

$$sgn(F_1 e_{v1}) = \frac{F_1 e_{v1}}{|F_1 e_{v1}| + \sigma_1}, \, sgn(F_2 e_{v2}) = \frac{F_2 e_{v2}}{|F_2 e_{v2}| + \sigma_2}, \, sgn(e_a) = \frac{e_a}{|e_a| + \sigma_3}$$

where σ_1, σ_2, σ_3 are the three minor positive constants.

Simulation example

Consider a single-link robotic arm with a revolute elastic joint rotating in a vertical plane whose motion equations are as follows:

$$\begin{cases} J_1 \ddot{q}_1 + F_n \dot{q}_1 + k(q_1 - q_2) + mgl \sin q_1 = 0 \\ J_m \ddot{q}_2 + F_m \dot{q}_2 - k(q_1 - q_2) = u \end{cases}$$

where q_1 and q_2 are the link displacement and the rotor displacement, respectively. The link intertia J_1, motor rotor inertia J_m, elastic constant k, link mass m, gravity g, connecting rod length l, and viscous friction coefficients F_1, F_m are all positive constant parameters. The control u is the torque delivered by the motor. The robot parameters are include $k=2Nm/rad$, $F_m=1$, $F_1=0.5Nm/(rad/s)$, $J_m=1Nm^2$, $J_1=2Nm^2$, $m=0.15 \, kg$, $g=9.8 \, m/s^2$, and $l=0.3 \, m$. The input $u = \sin(5t) + 4\sin(20t)$.

The state variables are defined as $x_1=q_1$, $x_2 = \dot{q}_1$, $x_3=q_2$, and $x_4 = \dot{q}_2$. The robot parameters in practice vary as time changes with handling different objects. Such changing parameters are defined as unknown input disturbances. Assuming that there exists an actuator fault $f_a(t)$ and a sensor fault $f_s(t)$ on the single-articulation robot, the system equation as shown Eq. (1) can be obtained.

Correspond (1), the parameter matrixes are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k}{J_1} & \dfrac{-F_1}{J_1} & \dfrac{k}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k}{J_m} & 0 & \dfrac{-k}{J_m} & \dfrac{-F_m}{J_m} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -0.25 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & -2 & -1 \end{bmatrix}$$

$$f(x,u,t) = \begin{bmatrix} 0 \\ -0.2205 \sin x_1 \\ 0 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 3 \\ 4 \\ 4 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \dfrac{1}{2} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

The transformational matrix T

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

then

$$TE = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 1 \end{bmatrix}, TD = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The transformational matrices S_0, S

$$S_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Design the matrices as $P_1 = \begin{bmatrix} \dfrac{1}{3} & 0 & 0 \\ 0 & \dfrac{1}{2} & -\dfrac{1}{4} \\ 0 & -\dfrac{1}{4} & \dfrac{3}{8} \end{bmatrix}$, $L_1 = \begin{bmatrix} 4 & 2 \\ -8 & 1 \\ 4 & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 \end{bmatrix}$, $L_2 = \begin{bmatrix} 2 \end{bmatrix}$, $F_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$,

$F_2 = \begin{bmatrix} 1 \end{bmatrix}$, $A_s = \begin{bmatrix} -2 \end{bmatrix}$ and $B_s = \begin{bmatrix} 2 \end{bmatrix}$.

The actuator fault reconstruction algorithm

$$\hat{f}_a(t) = r_1(t) = \rho_1 \, \mathrm{sgn}(F_1 e_{v1}) = \rho_1 \frac{e_1 + e_3}{|e_1 + e_3| + \sigma_1}$$

The sensor fault reconstruction algorithm

$$\hat{f}_s(t) = (B_s F)^{-1} \rho_3 \, \mathrm{sgn}(e_a) = 0.5(\rho_3 \frac{e_{a1}}{|e_{a1}| + \sigma_3})$$

The estimation of the unknown input disturbance

$$\hat{d}(t) = \bar{D}_2^{-1} \bar{E}_2 (\rho_2 \, \mathrm{sgn}(F_2 e_{v2}) - \rho_1 \, \mathrm{sgn}(F_1 e_{v1})) = \rho_2 \frac{e_4}{|e_4| + \sigma_2} - \rho_1 \frac{e_1 + e_3}{|e_1 + e_3| + \sigma_1}$$

The observer parameters are chosen as $\rho_1 = 32$, $\rho_2 = 60$, $\rho_3 = 20$, $\sigma_1 = 0.1$, $\sigma_2 = 0.02$, and $\sigma_3 = 0.01$. Two overlapped sine signals are used to simulate the incipient fault for actuator fault $f_a(t)$, where $f_a = 2\sin 40t + 2\sin 5t$. The sensor fault $f_s(t)$ is simulated by a white noise with amplitude of 3 and sample period of 0.02 s, which is the combination of abrupt fault and intermittent fault. The unknown input disturbance $d(t)$ is simulated by a sinusoidal, which is $d(t) = 4\sin 20t$. The initial value of state variable x in the simulation example is chosen as 0, 0, −3, and −2. The simulation results are as follows:
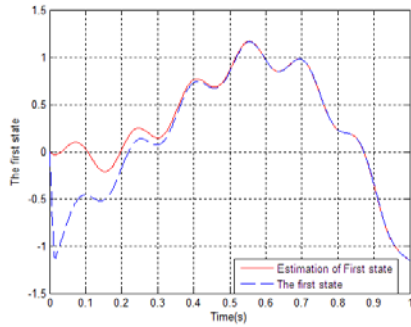
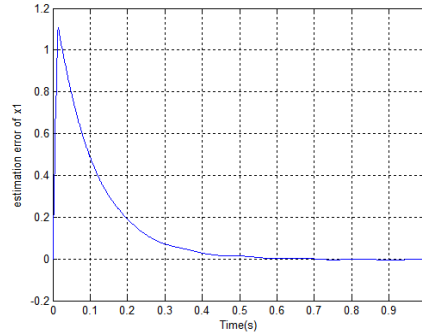Fig. 1. The first state x_1 and its estimation $\hat{x}_1$

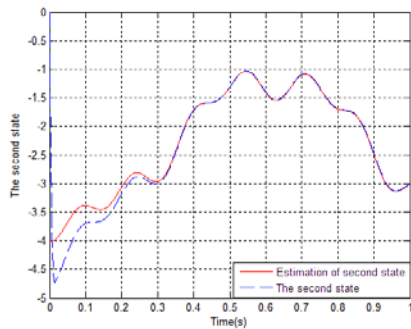Fig. 2. The estimation error e_1

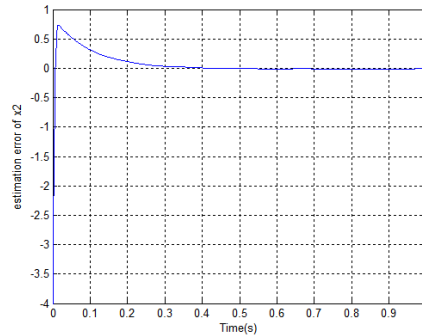Fig. 3. The second state x_2 and its estimation $\hat{x}_2$

Fig. 4. The estimation error e_2

Figures 1, 3, show the estimation results of the four state vectors, the corresponding estimation errors are shown on Figures 2, 4,. The results imply that the observers converge quickly, which lay the foundation for fault reconstruction.

Conclusions

In this paper, a type of nonlinear system with actuator faults, sensor faults, and unknown input disturbances is studied. A fault reconstruction method of nonlinear system is presented.

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