Convex-Concave Property for Parabola Fitting of Dr. Bridge

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Abstract—**A new method is presented to determine the fitting direction while using Dr. Bridge's linear editor to model various parabolas in this paper. By considering convex-concave property at monotone interval, parabolas' different shapes are determined. Based on the method, two set of symbols are introduced to describe the monotonicity and convex-concave property of parabola, and the judgment of parabola fitting type is determined. With the presented method, a quadratic parabola can be fitted by means of two controlling points as well as three controlling points. The method is illustrated by its simplicity and convenience, but efficient to program realization.**

Keywords-Convex-concave propert; fitting direction

I. INTRODUCTION

Bridge structure usually contains hundreds of or even thousands of units (Yao 2008), it is not advisable to deal with information of units individually because of exhausting input. In Dr. Bridge system, except for some special units, a great deal of input work can actually be reduced by using quick editor. There are ten types of quick editors in Dr. Bridge system such as line, arch, cable, parallel, symmetry, offset, insert, unit, section, and coordinate, which are provided for users to select the most convenient way to model various bridge structures. Among them, linear editor has the capacity of formation of a group of units, whose top edge or midpoint height is located at the same line. When using linear editor, sections at each controlling point are first defined, and then the fitting type of each controlling section is selected. Finally the other sections can be fitted by line or by parabola based on the controlling section which has already been defined.

Dr. Bridge Specifications Instruction indicates when using linear editor to fit a parabola, a parabola must be determined by three points, the parabola fitting type at the first point is not limited, the type at the second point must be 'backward parabola', the type at the third point must be 'forward parabola'. In this way, Dr. Bridge system will regard the three points as controlling points and fits a parabola. However, the case is usually occurred when known two points to determine a parabola (Huang 2012). At this time, only two controlling sections are defined. Moreover, the user's guide of Dr. Bridge does not offer a way how to fit a parabola based on the two controlling points.

By introducing two set of symbols to consider the monotonicity and convex-concave property of a certain parabola in this paper, a new method is presented to determine the fitting direction of parabola when using Dr. Bridge linear editor. Whether two or three controlling points are known, this presented method is suitable for identification of the parabola fitting type. The application of the method is illustrated by the modeling of three-span continuous box girder with variable cross sections, which verified the simplicity and efficiency of the approach in this paper.

II. PARABOLA FITTING METHOD CONSIDERING MONOTONICITY AND CONVEX-CONCAVE PROPERTY

Monotonicity and convex-concave property of function are characteristics of a certain curve (Kohlmann 2001, Chen & Cerone 2003, Tawarmalani et al. 2013). When there is parabola at monotonous interval, we can examine its convex-concave property as well as the parabola fitting direction.

A. Identification of Parabola Fitting Type by Two Controlling Points

When using linear editor to fit parabolas, it will commonly encounter the condition that the controlling sections are only defined at two points. The fitting type at the first point (starting point) is not limited, but it is not mentioned the fitting type at the second point (end point) whether to select 'forward parabola' or to select 'backward parabola' in user's guide of Dr. Bridge, which is always a technical difficulty in modeling process. In fact, by considering monotonicity and convex-concave property of a quadratic curve, the parabola with two controlling points fitted only appears the following four different shapes, as shown in Figure 1.

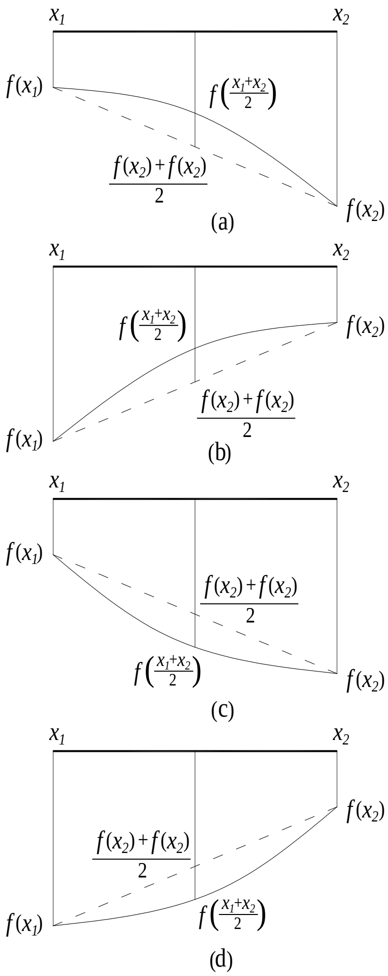

FIGURE I. PARABOLA SHAPES FITTED BY TWO CONTROLLING POINTS.

Definition 1 Suppose a parabola *f(x)* is fitted by two controlling points, x_1 and x_2 at its monotone interval. If the relation (1) is satisfied at interval [x_1, x_2], as shown in Figure 1 (a) and (b), thus we define the parabola *f(x)* is convex. If the relation (2) is satisfied at interval [x_1, x_2], as shown in Figure 1 (c) and (d), thus we define the parabola *f(x)* is concave.

$$f\left(\frac{x_1+x_2}{2}\right)>\frac{f(x_1)+f(x_2)}{2} \tag{1}$$

$$f\left(\frac{x_1+x_2}{2}\right)<\frac{f(x_1)+f(x_2)}{2} \tag{2}$$

Theorem 1 Two set of symbols are introduced. The first set of symbols marks parabolic monotonicity, and the second set of symbols marks parabolic concave-convex property. In the first set of symbols, it is marked ＋ if the parabola is decreasing function from the first point (starting point) to the second point (end point), and it is marked － if the parabola is increasing function from the first point (starting point) to the second point (end point). Meanwhile, in the second set of symbols, it is marked ＋ if the parabola is convex, otherwise it is marked －.Thus, if the two set of symbols are the same signs, the parabola fitting type at the second point (end point) is 'forward parabola', and if the two set of symbols are opposite signs, the parabola fitting type at the second point (end point) is 'backward parabola'.

According to definition 1 and theorem 1, with regard to four shapes shown in Figure 1, the procedure of determination of parabola fitting direction by two controlling points is given in Table 1.

TABLE I. IDENTIFICATION OF PARABOLA FITTING TYPE BY TWO CONTROLLING POINTS.

Parabola shape	**First set of symbols**	**Second set of symbols**	**Parabola fitting type at end point**
(a)	＋	＋	‘Forward parabola’
(b)	－	＋	‘Backward parabola’
(c)	＋	－	‘Backward parabola’
(d)	－	－	‘Forward parabola’

B. Identification of Parabola Fitting Type by Three Controlling Points

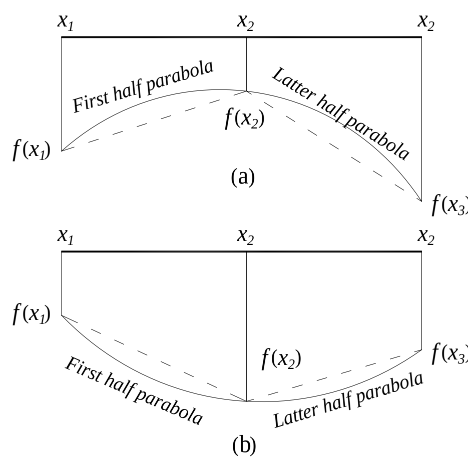

FIGURE II. PARABOLA SHAPES FITTED BY THREE CONTROLLING POINTS.

On the other hand, if a parabola is determined by three controlling points, the proposed parabola fitting method by considering monotonicity and convex-concave property is also suitable for the determination of the parabola fitting direction. At this time, the whole parabola is divided into the first half parabola (from the first point to the second point) and the latter half parabola (from the second point to the third point), as shown in Figure 2. Thus, the midpoint (the second point) is regarded as not only end point of the first half parabola but also starting point of the latter half parabola. Then let theorem 1 apply respectively at the first half parabola and the latter half parabola to determine their fitting direction. The procedure of determination of parabola fitting direction by three controlling points is given in Table 2.

TABLE II. IDENTIFICATION OF PARABOLA FITTING TYPE BY THREE CONTROLLING POINTS.

Parabola shape		**First set of symbols**	**Second set of symbols**	**Parabola fitting type at end point**
(a)	First half	−	+	Second point 'backward parabola'
	Latter half	+	+	Third Point 'forward parabola'
(a)	First half	+	−	Second point 'backward parabola'
	Latter half	−	−	Third Point 'forward parabola'

The user's guide of Dr. Bridge indicates that if a parabola is determined by three points, the parabola fitting type at the first point is not limited, the type at the second point must be 'backward parabola', the type at the third point must be 'forward parabola'. In this way, Dr. Bridge will regard the three points as controlling points and fits a parabola. As can be seen in Table 2, the results of the presented method are in agreement with the criterion given in the user's guide of Dr. Bridge, which verified the presented method is applicable to fit parabola by three controlling points.

III. APPLICATION EXAMPLE

A three-span continuous box girder is 100m in length, with span arrangement 30m＋40m＋30m. There are several segments of straight line being located at side spans, midspan join, and pier tables. The length of line segment at side span is 10m, and 4m line segments are located at piers, and 2m line segment is located at midspan join. The element arrangement of three-span continuous box girder is shown in Figure 3. The girder is divided into 100 elements, with each element 1m in length. The configurations of controlling sections are shown in Figure 4, with vertical web, single box and single room. The controlling sections at side spans and midspan are both 2.5m in height, and at piers are 5.0m high.

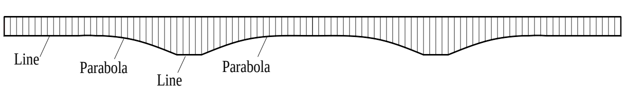

FIGURE III. ELEMENT ARRANGEMENT OF THREE-SPAN CONTINUOUS BOX GIRDER.

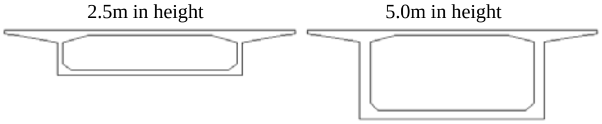

FIGURE IV. CONFIGURATIONS OF CONTROLLING SECTIONS.

We can notice that the top edge of the box girder is all located at the same line, and the bottom edge of the box girder is various from lines to parabolas, as shown in Figure 3. Thus, the linear editor of Dr. Bridge can be used to complete the model of the box girder. Let add ten controlling sections, 0m, 10m, 28m, 32m, 49m, 51m, 68m, 72m, 90m, 100m respectively to the column of distance from controlling point to starting point on the interface. Then input the size of each controlling section, and determine the parabola fitting direction at each controlling section. Based on the presented identification of parabola fitting type by two controlling points in this paper, select at 0m 'linear interpolation', at 10m 'linear interpolation', at 28m 'backward parabola', at 32m 'linear interpolation', at 49m 'backward parabola', at 51m 'linear interpolation', at 68m 'forward parabola', at 72m 'linear interpolation', at 90m 'backward parabola', at 100m 'linear interpolation', respectively. Thereby, the model of three-span continuous box girder with variable cross sections whose span is 100m is established, as shown in Figure 5.

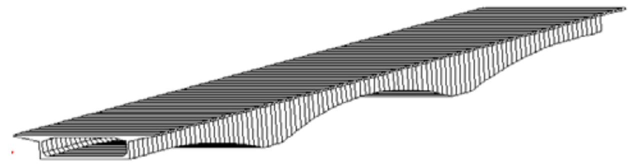

FIGURE V. MODEL OF THREE-SPAN CONTINUOUS BOX GIRDER.

IV. CONCLUSIONS

Based on monotonicity and convex-concave property of a quadratic curve, four different parabola shapes, with two controlling points fitted, are provided in this paper. By introducing two set of symbols to describe parabola characteristics, the method of determination of parabola fitting direction is presented, which explore a new approach to fit a parabola by two controlling points when linear editor of Dr. Bridge is used. The presented method is suitable for determining the parabola fitting direction whether parabola is fitted by two or by three controlling points. The method is illustrated by its simplicity and convenience, but efficient to program realization, which greatly promotes the modeling efficiency.

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