A Study on Solving the Nonlinear Seepage Flow Model of Three-Region Composite Reservoir

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***Abstract*—This paper builds a seepage model of three-region composite reservoir, in which quadratic-gradient effect, well-bore storage, effective radius and three kinds of outer boundary conditions (constant pressure boundary, closed boundary and infinity boundary) are considered. With Laplace transformation, the seepage flow model is transformed into a boundary value problem of three-region composite zero-order modified Bessel equation by the substitution of variables and introduction of dimensionless variables. Based on the similar structure of the solution of the boundary value problem of differential equation, this paper obtains solutions of dimensionless reservoir pressure and dimensionless bottom-hole pressure of three-region composite reservoir in Laplace space. This research not only contributes to further analyse the inherent law of the solution and compile corresponding well test analysis software, but also solves corresponding mathematical model of reservoir and supplements the study on composite reservoir.**

Keywords-composite reservoir; non-linear seepage; quadratic-gradient term; well-bore storage; effective radius; similar structure of solution; similar kernel function

I. INTRODUCTION

Composite reservoir is the reservoir that involves two regions with different properties (the rock-oriented one and the fluid-oriented one), namely, the inner region and the outer one which are separated by a discontinuous interface. There have been many foreign and domestic researches in the area of composite reservoir.

Reference 1 built seepage flow models of composite reservoir. Also, it studied pressure drop, transient behavior and transient pressure analysis of composite reservoir. Reference 2 proposed a composite reservoir model of rock with fractal characteristics. Numerical Solutions of the dynamic pressure of formation pressure is obtained by using the Laplace Numerical Inversion. Some other literatures [3-5] had studied the well test analysis models of the composite reservoirs, fractal composite reservoirs and multi-bed composite reservoirs which the well-bore storage effect and the skin effect are taken into consideration.

At the beginning of this century, the thought of similar structure of the solution of the boundary value problem of the differential equation began to form in Reference 6. Some gratifying results have been achieved. References [7-9] proposed mathematical model for well test analysis which perfectly described the fractal composite reservoir with the well-bore storage and the skin factor under three kinds of outer boundary conditions (infinite boundary, constant pressure boundary and closed boundary). Solutions of the reservoir pressure and bottom-hole pressure in Laplace space were obtained by using the Laplace transform. A similar structure of the solution was discovered by analyzing the percolation characteristics of fractal composite reservoir under three kinds of outer boundary conditions. Reference s [10, 11] have established the seepage flow model of the nonlinear percolation model of two-region composite reservoir with binary pressure gradient and nonlinear seepage flow. Seepage flow equations were linearized by variable substitution. Solutions in Laplace space were obtained by Laplace transformation and real space solutions obtained by the Stehfest numerical inversion.

However, the above researches mainly studied the mathematical model of the two-region composite reservoir. Base on the above researches, this paper studied the mathematical model of the three-region composite reservoir. With Laplace transformation, the seepage flow model is transformed into a boundary value problem of three-region composite modified zero-order Bessel equation by the substitution of variables and introduction of dimensionless variables. Based on the theory of the similar structure of the solution of the boundary value problem of differential equation, this paper obtains solutions of dimensionless reservoir pressure and dimensionless bottom-hole pressure of three-region composite reservoir in Laplace space. The study simplifies solution procedure and provides a clear idea for compiling corresponding well test analysis software. It can be widely used for well test analysis.

II. THE NONLINEAR SEEPAGE FLOW MODEL OF THREE-REGION COMPOSITE RESERVOIR

To formulate the percolation model, the main assumptions are as follows:

1) The fluid is single-phase, low compressible and follows Darcy's law in two regions.

2) Reservoir has equal thickness, each direction is horizontal and has the same nature; tere exists impermeability around three regions respectively.

3) Neglect the capillary force and gravity effect;

4) The well production quantity is constant.

5) There is no additional pressure drop at the interface of the three seepage regions;

6) Formation pressure is initial reservoir pressure p_0 before producing.

Nonlinear seepage flow basic equations of three-region composite reservoir which consider the influence of well-bore storage and the effective radius $r_{we} = r_w e^{-S}$ are as follows:

Inter region:

$$\frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r}\frac{\partial p_1}{\partial r} + C_{1L}\left(\frac{\partial p_1}{\partial r}\right)^2 = \frac{\phi_1 \mu_1 C_{1t}}{3.6k_1}\frac{\partial p_1}{\partial t},$$

$$r_{we} \le r < \alpha r_{we}, t > 0, \quad (1)$$

Middle region:

$$\frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r}\frac{\partial p_2}{\partial r} + C_{2L}\left(\frac{\partial p_2}{\partial r}\right)^2 = \frac{\phi_2 \mu_2 C_{2t}}{3.6k_2}\frac{\partial p_2}{\partial t},$$

$$\alpha r_{we} \le r \le \beta r_{we}, t > 0, \quad (2)$$

Outer region:

$$\frac{\partial^2 p_3}{\partial r^2} + \frac{1}{r}\frac{\partial p_3}{\partial r} + C_{3L}\left(\frac{\partial p_3}{\partial r}\right)^2 = \frac{\phi_3 \mu_3 C_{3t}}{3.6k_3}\frac{\partial p_3}{\partial t},$$

$$\beta r_{we} \le r \le R, t > 0, \quad (3)$$

Initial condition: $p_1(r,0) = p_2(r,0) = p_3(r,0) = p_0$, (4)

Inter condition:

$$\begin{cases} p_w(t) = p_1\big|_{r=r_{we}} \\ \left(r\frac{\partial p_1}{\partial r}\right)\Big|_{r=r_{we}} = \frac{\mu_1}{2\pi k_1 h}\left[1.842\times10^{-3}Bq + 4.421\times10^{-6}C\frac{dp_w}{dt}\right] \end{cases}, \quad (5)$$

Convergence condition:

(αr_{we} is the radius of initial region) ,

$$\begin{cases} p_1(\alpha r_\omega, t) = p_2(\alpha r_\omega, t) \\ \frac{k_1}{\mu_1}\frac{\partial p_1}{\partial r}\Big|_{r=\alpha r_\omega} = \frac{k_2}{\mu_2}\frac{\partial p_2}{\partial r}\Big|_{r=\alpha r_\omega} \\ p_2(\beta r_\omega, t) = p_3(\beta r_\omega, t) \\ \frac{k_2}{\mu_2}\frac{\partial p_2}{\partial r}\Big|_{r=\beta r_\omega} = \frac{k_3}{\mu_3}\frac{\partial p_3}{\partial r}\Big|_{r=\beta r_\omega} \end{cases} \quad (6)$$

Outer condition:

$$p_3(R,t) = p_0, or \quad \frac{\partial p_3}{\partial r}\Big|_{r=R} = 0, \ or \quad p_3(r,t)\big|_{r=R\to\infty} = p_0 \quad (7)$$

Firstly, in order to facilitate the research and description, dimensionless variables are introduced:

$$p_{jD}(r_D, T_D) = \frac{542.867k_1 h}{Bq\mu_1}[p_0 - p_j(r,t)],$$

$$C_{jLD} = C_{jL}\frac{Bq\mu_j}{542.867k_j h}, (j=1,2,3), r_D = \frac{r}{r_w e^{-S}}, R_D = \frac{R}{r_w e^{-S}},$$

$$\lambda_1 = \frac{k_2\mu_1}{k_1\mu_2},$$

$$\lambda_2 = \frac{k_3\mu_2}{k_2\mu_3}, C_D = \frac{C}{6.283\times10^6 \phi_1 C_{1t} h r_w^2}, T_D = \frac{3.600k_1 t}{C_D \phi_1 \mu_1 C_{1t} r_w^2},$$

$$\sigma_1 = \frac{\eta_1}{\eta_2} = \frac{\phi_2\mu_2 C_{t_2}}{\phi_1\mu_1 C_{t_1}}\cdot\frac{k_1}{k_2}, \sigma_2 = \frac{\eta_2}{\eta_3} = \frac{\phi_3\mu_3 C_{t_3}}{\phi_2\mu_2 C_{t_2}}\cdot\frac{k_2}{k_3}.$$

Secondly, substituting variable are as follows:

$$p_{jD}(r_D, T_D) = -C_{jLD}\ln\left[u_j(r_D, T_D) + 1\right],$$

$$p_{wD}(T_D) = -C_{1LD}\ln\left[u_w(T_D) + 1\right].$$

Thirdly, the Laplace transform is taken to the seepage flow model of three-region composite reservoir with dimensionless variable t_D, i.e.

$$\bar{P}_{iD}(r_D, z) = \int_0^\infty e^{-zt_D} P_{iD}(r_D, t_D)\, dt_D \quad (i=1,2,3),$$

$$\bar{P}_{\omega D}(z) = \int_0^\infty e^{-zt_D} P_{\omega D}(t_D)\, dt_D$$

Finally, the boundary value problem of three-region composite modified Bessel equation with parameter z (where z is Laplace space variable) is obtained as below:

$$\begin{cases} \frac{d^2\bar{P}_1}{dr_D^2} + \frac{1}{r_D}\frac{d\bar{P}_1}{\partial r_D} = \frac{z}{C_D e^{2S}}\bar{P}_1, 1 \le r_D < \alpha, \\ \frac{d^2\bar{P}_2}{dr_D^2} + \frac{1}{r_D}\frac{d\bar{P}_2}{dr_D} = \frac{\sigma_1 z}{C_D e^{2S}}\bar{P}_2, \alpha \le r_D \le \beta, \\ \frac{d^2\bar{P}_3}{dr_D^2} + \frac{1}{r_D}\frac{d\bar{P}_3}{dr_D} = \frac{\sigma_2 z}{C_D e^{2S}}\bar{P}_3, r_D \ge \beta, \\ \bar{P}_w(z) = \bar{P}_1(1,z), \left[(C_{1LD} + z)\bar{P}_w - \frac{d\bar{P}_1}{dr_D}\right]\Big|_{r_D=1} = \frac{C_{1LD}}{z} \\ \bar{P}_{1D}(\alpha, z) = \bar{P}_{2D}(\alpha, z), \frac{d\bar{P}_{1D}}{dr_D}\Big|_{r_D=\alpha} = \lambda_1\frac{d\bar{P}_{2D}}{dr_D}\Big|_{r_D=\alpha} \\ \bar{P}_{2D}(\beta, z) = \bar{P}_{3D}(\beta, z), \frac{d\bar{P}_{2D}}{dr_D}\Big|_{r_D=\beta} = \lambda_2\frac{d\bar{P}_{3D}}{dr_D}\Big|_{r_D=\beta} \\ \bar{P}_{3D}(\infty, z) = 0 \text{ or } \bar{P}_{3D}(R_D, z) = 0 \text{ or } \frac{d\bar{P}_{3D}}{dr_D}\Big|_{r_D=R_D} = 0 \end{cases}. \quad (8)$$

III. SOLVING THE NONLINEAR SEEPAGE FLOW MODEL OF THREE-REGION COMPOSITE RESERVOIR

Solutions of three regions of the BVP (8) which have the form of the product of continued fraction (the similar structure) are obtained as follows:

$$\bar{P}_1(r_D,z)=\frac{C_{1LD}}{z}\cdot\frac{1}{(C_{1LD}+z)+\frac{1}{\Phi_1(1,z)-1}}\cdot\frac{1}{\Phi_1(1,z)-1}\cdot\Phi_1(r_D,z),\quad(1\le r_D<\alpha)\quad,\qquad(9)$$

$$\bar{P}_2(r_D,z)=\frac{C_{1LD}}{z}\cdot\frac{1}{(C_{1LD}+z)+\frac{1}{\Phi_1(1,z)-1}}\cdot\frac{1}{\Phi_1(1,z)-1}\cdot\frac{\Psi_{0,1}\left(\alpha,\alpha,\sqrt{z}\right)}{\Phi_2(\alpha,z)\sqrt{z}\Psi_{1,1}\left(1,\alpha,\sqrt{z}\right)+\lambda_1\Psi_{1,0}\left(1,\alpha,\sqrt{z}\right)}\cdot\Phi_2(r_D,z),\quad(\alpha\le r_D\le R_D),\qquad(10)$$

$$\bar{P}_3(r_D,z)=\frac{C_{1LD}}{z}\cdot\frac{1}{(C_{1LD}+z)+\frac{1}{\Phi_1(1,z)-1}}\cdot\frac{1}{\Phi_1(1,z)-1}\cdot\frac{\Psi_{0,1}\left(\alpha,\alpha,\sqrt{z}\right)}{\left[\Phi_2(\alpha,z)\sqrt{z}\Psi_{1,1}\left(1,\alpha,\sqrt{z}\right)+\lambda_1\Psi_{1,0}\left(1,\alpha,\sqrt{z}\right)\right]}\cdot\frac{\Psi_{0,1}\left(\beta,\beta,\sqrt{\sigma_1 z}\right)}{\left[\Phi_3(\beta,z)\sqrt{\sigma_1 z}\Psi_{1,1}\left(\alpha,\beta,\sqrt{\sigma_1 z}\right)+\lambda_2\Psi_{1,0}\left(\alpha,\beta,\sqrt{\sigma_1 z}\right)\right]}\cdot\Phi_3(r_D,z),\quad(\beta\le r_D\le R_D)\qquad(11)$$

where the similar kernel function of outer region is delimited as below:

$$\Phi_3\left(r_D,z\right)=\Phi_{3j}\left(r_D,z\right)\quad\left(j=1,2,3\right).\qquad(12)$$

The similar kernel function of middle region is delimited as below:

$$\Phi_2\left(r_D,z\right)=\Phi_{2j}\left(r_D,z\right)\quad\left(j=1,2,3\right).\qquad(13)$$

The similar kernel function of inter region is delimited as below:

$$\Phi_1\left(r_D,z\right)=\Phi_{1j}\left(r_D,z\right)\quad\left(j=1,2,3\right).\qquad(14)$$

where $j=1$ denotes that the outer boundary condition is infinite, $j=2$ denotes that the outer boundary condition is constant pressure, and $j=3$ denotes that the outer boundary condition is closed.

Case 1. When the outer boundary condition is infinite $\bar{P}_{3D}(\infty,z)=0$ (i.e. $j=1$),

$$\Phi_{31}\left(r_D,z\right)=-\frac{K_0\left(\sqrt{\sigma_2 z}r_D\right)}{\sqrt{\sigma_2 z}K_1\left(\beta\sqrt{\sigma_2 z}\right)}\quad\left(\beta\le r_D\le\infty\right)\qquad(15)$$

$$\Phi_{21}\left(r_D,z\right)=\frac{\lambda_2\Psi_{0,0}\left(r_D,\beta,\sqrt{\sigma_1 z}\right)+\sqrt{\sigma_1 z}\Psi_{0,1}\left(r_D,\beta,\sqrt{\sigma_1 z}\right)\Phi_{31}\left(\beta,z\right)}{\lambda_2\sqrt{\sigma_1 z}\Psi_{1,0}\left(\alpha,\beta,\sqrt{\sigma_1 z}\right)+\sigma_1 z\Psi_{1,1}\left(\alpha,\beta,\sqrt{\sigma_1 z}\right)\Phi_{31}\left(\beta,z\right)}\quad\left(\alpha\le r_D\le\beta\right),\qquad(16)$$

$$\Phi_{11}\left(r_D,z\right)=\frac{\lambda_1\Psi_{0,0}\left(r_D,\alpha,\sqrt{z}\right)+\sqrt{z}\Psi_{0,1}\left(r_D,\alpha,\sqrt{z}\right)\Phi_{21}\left(\alpha,z\right)}{\lambda_1\sqrt{z}\Psi_{1,0}\left(1,\alpha,\sqrt{z}\right)+z\Psi_{1,1}\left(1,\alpha,\sqrt{z}\right)\Phi_{21}\left(\alpha,z\right)}\quad\left(1\le r_D\le\alpha\right).\qquad(17)$$

Case 2. When the outer boundary condition is constant pressure $\bar{P}_{3D}(R_D,z)=0$ (i.e. $j=2$),

$$\Phi_{32}\left(r_D,z\right)=\frac{\Psi_{0,0}\left(r_D,R_D,\sqrt{\sigma_2 z}\right)}{\sqrt{\sigma_2 z}\Psi_{1,0}\left(\beta,R_D,\sqrt{\sigma_2 z}\right)}\quad\left(\beta\le r_D\le R_D\right),\qquad(18)$$

$$\Phi_{22}\left(r_D,z\right)=\frac{\lambda_2\Psi_{0,0}\left(r_D,\beta,\sqrt{\sigma_1 z}\right)+\sqrt{\sigma_1 z}\Psi_{0,1}\left(r_D,\beta,\sqrt{\sigma_1 z}\right)\Phi_{32}\left(\beta,z\right)}{\lambda_2\sqrt{\sigma_1 z}\Psi_{1,0}\left(\alpha,\beta,\sqrt{\sigma_1 z}\right)+\sigma_1 z\Psi_{1,1}\left(\alpha,\beta,\sqrt{\sigma_1 z}\right)\Phi_{32}\left(\beta,z\right)}\quad\left(\alpha\le r_D\le\beta\right),\qquad(19)$$

$$\Phi_{12}\left(r_D,z\right)=\frac{\lambda_1\Psi_{0,0}\left(r_D,\alpha,\sqrt{z}\right)+\sqrt{z}\Psi_{0,1}\left(r_D,\alpha,\sqrt{z}\right)\Phi_{22}\left(\alpha,z\right)}{\lambda_1\sqrt{z}\Psi_{1,0}\left(1,\alpha,\sqrt{z}\right)+z\Psi_{1,1}\left(1,\alpha,\sqrt{z}\right)\Phi_{22}\left(\alpha,z\right)}\quad\left(1\le r_D\le\alpha\right).\qquad(20)$$

Case 3. When the outer boundary condition is closed $\left.\frac{d\bar{P}_{3D}}{dr_D}\right|_{r_D=R_D}=0$ (i.e. $j=3$),

$$\Phi_{33}\left(r_D,z\right)=\frac{\Psi_{0,1}\left(r_D,R_D,\sqrt{\sigma_2 z}\right)}{\sqrt{\sigma_2 z}\Psi_{1,1}\left(\beta,R_D,\sqrt{\sigma_2 z}\right)}\quad\left(\beta\le r_D\le R_D\right),\qquad(21)$$

$$\Phi_{23}\left(r_D,z\right)=\frac{\lambda_2\Psi_{0,0}\left(r_D,\beta,\sqrt{\sigma_1 z}\right)+\sqrt{\sigma_1 z}\Psi_{0,1}\left(r_D,\beta,\sqrt{\sigma_1 z}\right)\Phi_{33}\left(\beta,z\right)}{\lambda_2\sqrt{\sigma_1 z}\Psi_{1,0}\left(\alpha,\beta,\sqrt{\sigma_1 z}\right)+\sigma_1 z\Psi_{1,1}\left(\alpha,\beta,\sqrt{\sigma_1 z}\right)\Phi_{33}\left(\beta,z\right)}\quad\left(\alpha\le r_D\le\beta\right),\qquad(22)$$

$$\Phi_{13}\left(r_D,z\right)=\frac{\lambda_1\Psi_{0,0}\left(r_D,\alpha,\sqrt{z}\right)+\sqrt{z}\Psi_{0,1}\left(r_D,\alpha,\sqrt{z}\right)\Phi_{23}\left(\alpha,z\right)}{\lambda_1\sqrt{z}\Psi_{1,0}\left(1,\alpha,\sqrt{z}\right)+z\Psi_{1,1}\left(1,\alpha,\sqrt{z}\right)\Phi_{23}\left(\alpha,z\right)}\quad\left(1\le r_D\le\alpha\right).\qquad(23)$$

where $\Psi_{m,n}\left(\alpha,\beta,y\right)=I_m\left(\alpha y\right)K_n\left(\beta y\right)+(-1)^{m-n+1}K_m\left(\alpha y\right)I_n\left(\beta y\right)$ and $I_n\left(\cdot\right)$, $K_n\left(\cdot\right)$ are respectively the first and the second class of modified Bessel functions of order n [12].

IV. SYMBOL DESCRIPTION

The symbol meanings are listed below.

p — Reservoir pressure (MPa);

p_0 — Initial pressure(MPa);

p_w —Bottom-hole pressure (MPa);

q — Well yield (m^3/d);

t —Time (h);

r — The distance from any point in the reservoir to the center of well (m);

R — The outer boundary radius (m);

h —Reservoir thickness(m);

B —Oil volume factor, dimensionless;

k — Reservoir permeability($\mu\mathrm{m}^2$);

μ — The viscosity of fluid in reservoir ($\mathrm{mPa}\cdot\mathrm{s}$);

S —Skin factor, dimensionless;

C — Wellbore storage coefficient (m^3/MPa);

C_t — Total compressibility of reservoir, ($1/\mathrm{MPa}$);

C_L — Fluid compressibility, ($1/\mathrm{MPa}$);

r_w — Wellbore radius(m);

r_{we} — Effective wellbore radius,(m);

ϕ — Porosity, dimensionless;

σ — Elastic storativity ratio, dimensionless;

λ — Interporosity flow coefficient, dimensionless;

w —Well;

D —Dimensionless.

V. CONCLUSIONS

1) Using variable substitution and Laplace transformation, the seepage flow model of three-region composite reservoir is transformed into a boundary value problem of three-region composite zero-order modified Bessel equation. Solutions of dimensionless reservoir pressure and dimensionless bottom-hole pressure can be constructed by two linearly independent solutions of the basic equation and coefficients of boundary conditions. And these solutions have the form of the product of continued fraction, hence the similar structure.

2) The seepage flow model of three-area composite reservoir has the similar structure of solutions under three different outer boundary conditions. The difference is that similar kernel functions are different under three different outer boundary conditions.

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