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Competitive Facility Location Problem with Foresight in Discrete Space based on Tabu Search

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***Abstract*—This paper presents the competitive facility location problem with foresight in the discrete space which considers probabilistic customer behavior. A practical bilevel nonlinear integer programming model is formulated to maximize the leader's market share while predicting the follower's response. For solving the model, a two-stage hybrid tabu search algorithm is designed. To illustrate the effectiveness of the proposed algorithm, a set of randomly generated instances are presented and analyzed statistically. The results indicate that the proposed algorithm provides an effective means to solve the problems.**

Keywords—competitive facility location problem; discrete space; gravity-base model; tabu search

I. INTRODUCTION

Competitive facility location problem with foresight is a subclass of the facility location problem which includes two decision makers: the leader and the follower. They compete for the clients and wish to maximize their market share. The decision process usually consists of two stages: first, the leader locates *p* new facilities and the follower, knowing the leader's facility locations, places *r* new facilities later. It's also called discrete (*r*|*p*)-centroid problem which is introduced in competitive location problem by Hakimi [1].

A detailed review about the leader-follower problem till 1996 is studied in Eiselt and Laporte [2]. Plastria provided an overview of optimization approaches in the static competitive facility location [3]. Kress and Pesch reviewed the recent developments of the models in the field of the sequential competitive location problem on networks [4]. Arabani and Farahani studied the dynamics of the facility location problems [5]. Saiz et al. designed a branch-and-bound approach for a Huff-like Stackelberg location problem in which leader and follower located only a single new facility [6]. Ahn et al. considered that two players successively placed one facility in the competitive facility location problem, until each of them has placed *n* facilities [7]. Beresnev proposed a branch-and-bound algorithm for the competitive facility location problem [8]. Ashtiani et al. provided the robust model for determining the optimal locations for the leader's new facilities considering that the number of the follower's new facilities is unknown for the leader [9]. Plastria and Vanhaverbeke solved the competitive location with foresight based on the maximal covering model by considering the competitor will locate a single new facility [10]. Alekseeva are concerned with the discrete (r|p)-centroid problem aiming at maximize their own profits, based on the deterministic customer behavior [11]. Drezner et al. investigated the leader-follower competitive location problem based on the concept of cover [12].

After reviewing the relevant literature, one conclusion can be drawn that most of them are modeled and analyzed in the case of deterministic customer behavior and with respect to the competitive facility location problem with foresight, few researches consider probabilistic customers behavior. In this paper, we consider the probabilistic customer behavior and formulate a bilevel nonlinear integer programming model. In the meanwhile, a two-stage hybrid tabu search algorithm is developed.

II. MODEL DESCRIPTION

In this section, the notations and proposed mathematical model are presented. Given a two dimensional market region, there is a set $J=\{1,\ldots,n\}$ of demand points which are assumed inelastic and are supposed to be aggregated at *n* demand points. In the market two competitors provide same service and have opened some facilities. The set $I=\{1,\ldots,m\}$ of existing facilities is the subset of set *J*, in which the top *t* facilities belong to the leader and the rest of *m-t* facilities belong to the follower. The set $P=\{1,\ldots,n\text{-}m\}$ of potential locations consists of the rest of *n-m* demand points. Now the leader needs to opens *p* new facilities in the set *P* to maximize its market share, given that the follower surely react to his action by launching *r* new facilities in the set *P* with the aim of maximizing his market share. It's assumed that only one new facility can be opened in each potential location. The customer behavior is probabilistic in which each demand point visit any facility with respect to some probability.

A. Notations definition

The notations of our model are as follows:

i	index of existing facility, i=1,2,…,m(the first t facilities belong to the leader, while the rest to the follower)
j	index of demand points, j=1,2,…,n
k	index of leader's new facilities, k=1,2,…,p
h	index of follower's new facilities, h=1,2,…,r
n_{pot}	number of potential locations(n_{pot} =n-m)
s	index of potential locations, s=1,2,…, n_{pot}
b_j	buying power of demand point j
d_{ij}	Euclidean distance between existing facility i and demand point j
d_{sj}	Euclidean distance between the potential location s and demand point j
q_{ij}	quality of existing facility i for demand point j
q_{Lj}	quality of the leader's new facilities for demand point j
q_{Fj}	quality of the follower's new facilities for demand point j
X_s	binary variable which is equal to 1 if leader opens new facility in potential location s, and 0 otherwise
Y_s	binary variable which is equal to 1 if follower opens new facility in potential location s, and 0 otherwise

B. Model formulation

In the gravity-based model, the facility attractiveness level for a customer is proportional to the quality of the facility and inversely proportional to the squared distance between the customer and the facility. Now the quality levels of all facilities are supposed to be predetermined. According to this, the attractiveness level of facility i for point j is given by:

$$A_{ij} = \frac{q_{ij}}{\left(\varepsilon + d_{ij}^2\right)} \tag{1}$$

In the equation, ε is added to keep denominator from being 0 when d_{ij}^2 becomes 0. Then, the total attractiveness of point j is:

$$A_j = \sum_{i=1}^{m} A_{ij} + \sum_{s=1}^{n_{pot}} A_{sj} X_s + \sum_{s=1}^{n_{pot}} A_{sj} Y_s \tag{2}$$

The probability that point j patronizes facility i is:

$$P_{ij} = \frac{A_{ij}}{A_j} \quad i = 1, 2, \dots, m \tag{3}$$

Consequently, the probability that point j chooses leader's facilities is:

$$P_{Lj} = \frac{\sum_{i=1}^{t} A_{ij} + \sum_{s=1}^{n_{pot}} A_{sj} X_s}{A_j} \tag{4}$$

Accordingly, the total market share captured by the leader is calculated by adding all customers buying power multiplying the probability and is illustrated as:

$$M_L = \sum_{j=1}^{n} b_j P_{Lj} \tag{5}$$

Similarly, the probability that point j patronizes follower's facilities is:

$$P_{Fj} = \frac{\sum_{i=t+1}^{m} A_{ij} + \sum_{s=1}^{n_{pot}} A_{sj} Y_s}{A_j} \tag{6}$$

As a result, the total market share captured by the follower is:

$$M_F = \sum_{j=1}^{n} b_j P_{Fj} \tag{7}$$

Denote X as the optimal locations of leader's facilities and Y as the optimal locations of follower's, consequently the model of the problem is as follows:

$$Max\ M_L = \sum_{j=1}^{n} b_j P_{Lj} \tag{8}$$

s.t.

$$\sum_{s=1}^{n_{pot}} X_s = p \tag{9}$$

$$X_s \in \{0,1\} \tag{10}$$

Where Y solves

$$Max\, M_F = \sum_{j=1}^{n} b_j P_{Fj} \tag{11}$$

s.t.

$$\sum_{s=1}^{n_{pot}} Y_s = r \tag{12}$$

$$X_s + Y_s \le 1 \qquad s = 1, 2, \dots, n_{pot} \tag{13}$$

$$Y_s \in \{0,1\} \tag{14}$$

The formulated model is a bilevel nonlinear integer programming model in which the upper level represents the model of leader's market share (Leader-Model) and the lower level is the model of follower's market share (Follower-Model). In the upper level, constraint (8) is the objective function aiming to maximize the leader's market share. Constraint (9) ensures that each leader's new facility is located in only one of the predefined potential locations. In the lower level, constraint (11) is the follower's objective function in order to maximize follower's market share. The constraint (12) can be explained as constraint(9). Constraint (13) shows that only one new facility, both the leader and follower, can be opened in each potential location. Given X, the lower level problem is the so-called (r|X_p)-medianoid problem which is NP hard problem[1]. The upper level leader problem is much more difficult to solve, since the leader has to decide on location with the knowledge of the optimal action of the follower.

III. HYBRID TABU SEARCH HEURISTIC (HTS)

In this section, a hybrid tabu search heuristic is proposed to solve the problem. Since the problem is NP hard, exact methods are inapplicable for solving large scale instances that is often encountered in practice. Tabu search is a local search heuristics which was first proposed by Glover [13]. According to recent literature [14], tabu search is widely used for solving competitive facility location problem and proved to be effective. Next, the hybrid tabu search heuristic algorithm and its ingredients are introduced.

A. *The algorithm*

Notation:	**Meaning**
X,Y	leader/follower solution
X^*,Y^*	The global optimal leader/follower solution
f_L^*, f_F^*	Leader's/Follower's market share of X^* / Y^*
N(X),N(Y)	The neighborhood of *X,Y*
$\tilde{N}(X),\tilde{N}(Y)$	The admissible subset of *N(X),N(Y)* (i.e. non-tabu or allowed by aspiration)
T_L,T_F	Leader's/Follower's Tabu list

The pseudocode of the algorithm is described in Algorithm 1 which consists of two phases, leader phase and follower phase. Its output is the optimal locations of the leader found during the process and its corresponding leader's market share.

Algorithm 1. Hybrid tabu Algorithm

```
1:  Construct an initial solution $X_0$
2:  Set $X \leftarrow X_0$, $f_L^* \leftarrow 0$, $X^* \leftarrow X_0$, $T \leftarrow \varnothing$
3:  While the termination criterion not satisfied do
4:    Set $f_{Lt}=0$,
5:    For each $X' \in \tilde{N}(X)$, $X \leftarrow X'$
6:      Construct an initial solution $Y_0$
7:      Set $Y \leftarrow Y_0$, $f_F^* \leftarrow f_F(Y_0)$, $Y^* \leftarrow Y_0$, $T_F \leftarrow \varnothing$
8:      While termination criterion not met do
9:        Select $Y$ in $\mathrm{argmax}_{Y' \in \tilde{N}(Y)}\left[f_F(Y')\right]$
10:       If $f_F(Y) > f_F^*$, then set $f_F^* \leftarrow f_F(Y)$, $Y^* \leftarrow Y$
11:       Record tabu for the current move in $T_F$ (delete
          oldest entry if necessary)
12:     End while
13:     If $f_L(X) > f_{Lt}$, then set $f_{Lt} \leftarrow f_L(X)$, $X_t \leftarrow X$
14:   End for
15:   $X \leftarrow X_t$
16:   If $f_L(X) > f_L^*$, then set $f_L^* \leftarrow f_L(X)$, $X^* \leftarrow X$
17:   Record tabu for the current move in $T_L$ (delete oldest
      entry if necessary)
18: End while
```

In this algorithm, *argmax* returns the subset of solutions in $\tilde{N}(Y)$ that maximize f_F

B. *Initialize methods*

An initial solution of the leader is chosen at first (line 1). 4 ways are designed to create leader's initial solution X_0:(1) random method, that is, select *p* locations from potential locations set *P* randomly; (2) demand method, select top *p* points from the potential locations set *P* according to buying power; (3) cover-demand method, select top *p* points by the total buying power of the covered demand points which are within a distance; (4) greedy method, solve the leader problem without considering the follower's reaction and initialize the leader's solution with the optimal result.

C. *Neighborhood structure*

In the algorithm, the neighborhood structure involves the 1-Swap move that moves an open facility from one point to another candidate point. For the leader, neighborhoods are constructed by swapping each component X_{s_k} of leader's current solution $X=(X_{s_1},X_{s_2},\ldots,X_{s_p})$ with each point in leader's candidate swap list *LC*, represented by the point pair (X_{s_k},LC_j). And the follower's neighborhoods are constructed by swapping each member Y_{s_h} of the follower's current solution $Y=(Y_{s_1},Y_{s_2},\ldots,Y_{s_r})$ with each point in follower's candidate swap list *FC,* represented by point pair (Y_{s_h}, FC_j). Given *X* and *Y*, the list *FC* is created by selecting *a* points from the rest $n_{pot}-p-r$ potential locations, and 4 methods are designed to generate the list: (1) random method, select *a* points randomly; (2) cover-demand method, select *a* points which have most cover-demand; (3)uncover-demand method, uncover-demand means the total buying power of locations which are within the service area of the facility under consideration while beyond that of any other facilities; (4) hybrid method, divide the list into quarters and each part is generated by one of above methods. Similarly, given *X*, the list *LC* is generated by first solving a follower's solution *Y*, taking *Y* as subset of *LC*, and then selecting *a* points from the rest $n_{pot}-p-r$ potential locations and adding them to *LC.*

In the research, the tabu list *T* recording the last few transformations performed on the current solution and prohibit reverse transformations. The aspiration criteria allows a tabu move when it results in a solution with better objective value than the current best-know solution.

IV. RESULTS AND COMPUTATIONAL ANALYSIS

In order to evaluate the effectiveness and efficiency of the algorithms, we randomly generate 6 small instances and 4 large instances. The buying power of each demand point is randomly generated in the interval [1, 10]. Quality values for all facilities are randomly generated in [1, 5].

The optimal solution is derived by first enumerating all possible leader solutions, and then solving a Follower-Model to optimal by Lingo. The parameters of the HTS algorithm are set as follows: the maximal iteration number of leader and follower is 300, the non-updating times when exploring the solution space of leader and follower (termination criterion) are both set to 30, the tabu list length is 7, $\varepsilon=1$, and *a* is set to be n/3. After comparing the 4 ways in leader's initial solution construction, greedy method is adopted as it can generate higher quality solution in most time. As for the candidate swap list generating, the hybrid method outperforms other methods on the objective value and the rate of convergence. The details of the comparison are not illustrated due to the space considerations. The solutions for small instances calculated by HTS are depicted and compared with optimal in TABLE I.

TABLE I. THE RESULTS OF SMALL INSTANCE

NO.	n	m	t	p	r	Optimal				HTS				
						Leader Locations		Leader Market	Time	Leader Locations		Leader Market		Time(s)
						1	2			1	2	Value	Gap	
1	16	5	3	2	1	6	7	40.6357	16s	6	7	40.6357	0	0.041
					2	6	7	47.442	29s	6	7	47.442	0	0.058
					3	6	7	52.9792	2m47s	6	7	52.9792	0	0.121
					4	6	10	57.2598	11m11s	6	10	57.2598	0	0.119
2	20	5	3	2	1	8	10	39.784	49s	8	10	39.784	0	0.206
					2	8	14	47.0662	4m03s	8	14	47.0662	0	0.349
					3	8	14	51.9945	11m24s	8	14	51.9945	0	0.454
					4	8	14	55.8717	1h01m52s	8	14	55.8717	0	0.549
3	25	5	3	2	1	10	17	48.1744	2m36s	10	17	48.1744	0	0.483
					2	7	14	58.5845	21m42s	7	14	58.5845	0	1.014
					3	14	18	65.8858	1h02m17s	14	18	65.8858	0	1.6
4	26	5	3	2	1	10	14	45.7883	2m47s	10	14	45.7883	0	0.569
					2	11	20	54.8851	13m57s	11	20	54.8851	0	1.288
					3	11	19	61.2382	3h15m30s	11	19	61.2382	0	1.929
5	27	5	3	2	1	14	18	60.5603	3m31s	14	18	60.5603	0	0.683
					2	14	17	74.5096	11m33s	14	17	74.5096	0	1.43
					3	14	17	83.831	3h16m21s	14	17	83.831	0	2.233
6	28	5	3	2	1	14	18	61.4782	4m51s	14	18	61.4782	0	0.756
					2	14	17	76.4655	29m31s	14	17	76.4655	0	1.578

The experimental results demonstrate that the algorithm is effective for small instance to produce the optimal results. And the HTS is also very effective for the large instances, the following is the results of 4 large instances.

TABLE II. THE RESULTS OF LARGE INSTANCES

NO.	n	m	t	p	r	HTS	Leader Locations		Average Time/s
							1	2	
1	30	5	3	2	1	52.951	9	13	0.982
					2	65.44	9	13	2.045
					3	74.404	8	13	3.266
					4	81.83	13	14	4.797
2	50	5	3	2	1	85.193	14	19	2.97
					2	107.817	14	19	7.154
					3	122.758	14	19	12.375
					4	134.394	14	19	31.6
3	70	5	3	2	1	124.046	28	59	6.383
					2	158.549	28	49	15.035
					3	181.795	25	29	27.205
					4	198.387	28	49	52.423
4	100	5	3	2	1	191.913	17	73	5.883
					2	232.667	17	73	14.766
					3	263.043	17	74	38.648
					4	289.011	17	74	57.656

TABLE II. focuses on the computational results of the 4 large instances calculated by HTS. From the last column, we observe that the HTS is very efficient and can solve large instances within 60 seconds. Take the 3th instance as example, the leader will open 2 new facilities. When the follower plans to open one new facility, leader's optimal locations are 28 and 59. On the other hand, when the follower determines to launch 2 new facilities, leader's optimal locations are 28 and 49. However, when the follower plans to open 3 new facilities, leader's optimal locations are 25 and 29. This proves that the follower's reaction plays a critical role in the location decision of the leader in the competitive facility location problem with foresight.

V. CONCLUSIONS

In this paper, the competitive facility locations problem with foresight has been studied by considering probabilistic customer behavior. A bilevel nonlinear integer programming model is proposed for the problem. For solving large scale problems, a two-stage hybrid tabu search algorithm is designed. The effectiveness and efficiency of the designed algorithm is first evaluated by comparing with optimal solutions on small instances. Then the results on large scale problems are reported and analyzed. As a future research, we can extend the problem to consider the situation when the facility number that the follower plans to build is unknown.

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