Modeling of Nonlinear Dynamics of Drill Strings in a Supersonic Air Flow

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***Abstract*—In this work we study nonlinear lateral vibrations of a drill string moving in a supersonic air flow. A nonlinear mathematical model of drill string vibrations is created on the basis of the nonlinear theory of elasticity making use of the Hamilton principle. Solution to the model is obtained by the Bubnov-Galerkin method in the first, second and third approximations. After reducing to a system of ordinary differential equations the numerical stiffness switching method is applied since the problem appears to be stiff. Comparative analysis of the nonlinear model and its geometrically linear analogue is carried out, and the significance of application of the governing equations taking into account geometric nonlinearity is shown. Drill strings with various parameters of length, operating frequency, axial force, and air flow pressure are investigated.**

Keywords - drill string; nonlinear model; lateral vibrations; air flow; stiffness switching method.

I. INTRODUCTION

One of the major problems in the oil and gas industry is failures of drill strings. With the implementation of a Drill String Prevention Quality Improvement Project in 1992 the number of drill string failures has been reduced considerably. To achieve that a big number of mathematical models describing the behavior of drilling systems have been built up. However, at present one of seven drilling rigs still falls out because of the breaks of drill strings, which require large costs for their reconstruction [1].

During the process of drilling diverse types of vibrations such as axial, lateral and torsional ones can occur. Lateral vibrations are considered to be the most damaging of them, more often resulting in the drilling equipment breakdown.

Linear mathematical models describing the motion of drill strings under various types of vibrations were investigated in [2-4]. V.I. Gulyaev and O.I. Borshch [2] constructed vibration modes of drill string tubes with the longitudinal non-uniform preloading, calculated their frequencies and carried out numerical analysis. Dynamic model of a drill string including drill pipes and drill collars was formulated by Y.A. Khulief et al. [3]. For numerical solution the finite element method was applied. In [4] the effect of drilling mud flow speed, drill string weight, weight on bit and angular velocity on drill string vibrations and stability were analyzed. Dynamic nonlinear models of transverse and axial vibrations of rotating drill strings with axial loading and contact with the borehole wall were studied by [5, 6]. These works also took into account the presence of drilling mud outside the drill string. Models with geometric nonlinearity and initial curvature were investigated in [7, 8]. In [9] authors used the Floquet theory and partial discretization method for analysis of stability of a drill string, and studied its vibrations at finite strains.

In contrast to the previous research, this work aims at studying nonlinear lateral vibrations of a drill string taking into consideration the nonlinear influence of a supersonic air flow. The process of derivation of the nonlinear model is provided. The created model includes rotation of the drill string, impact of the axial load, and pressure of the air flow. Numerical analysis of the model is conducted, after that the obtained results are visualized.

II. DERIVATION OF THE NONLINEAR MODEL

When conducting shallow drilling operations the rotary way of drilling wells is mainly applied. In this case, modeling the motion of a drill string it is necessary to take into consideration its rotation as well.

The mathematical model of the drill string motion is constructed on the basis of the nonlinear theory of elasticity [10]. The drill string is presented as a one-dimensional rod of length l with symmetric cross-section.

Let us consider a global right-handed Cartesian coordinate system centered at point O with the x-, y- and z-axes. Let the z-axis be directed along the axis of the rod. Denote components of displacements over the x-, y- and z-axes by $U(x, y, z, t)$, $V(x, y, z, t)$, $W(x, y, z, t)$, respectively.

We apply the hypothesis of plane sections [11]. The rod is assumed to be isotropic, and the strain components (elongations and shears) to be infinitesimal. We also assume that the angles of rotation are negligible compared to unity, whereas the displacements are not restricted.

Generally, the displacements of any point of the rod can be written in the form [12, 13].

In this work the flat bending of the rod of length l is studied, that is the axis of the rod is supposed to be bent only in the Oyz-plane. The longitudinal displacement along the z-axis and torsion of the rod are neglected. Then the expressions for the displacements can be given in the following form:

$$\begin{cases} V(x,y,z,t) = v(z,t), \\ W(x,y,z,t) = -\dfrac{\partial v(z,t)}{\partial z} y, \end{cases} \tag{1}$$

where $v(z,t)$ is the displacement of the flexural center of the cross-section along the y-axis owing to bending; $-\dfrac{\partial v(z,t)}{\partial z}$ is the turning angle of the cross-section around the y-axis under bending.

Taking into account rotation of the drill string, let us introduce a local coordinate system $Ox'y'z'$ rotating along with the drill string [14, 15]. The z'-axis coincides with the direction of the axis of the rod. Let the rotation of the rod be anticlockwise. Then the position of any point of the rod in the local coordinate system relative to the global one can be defined as follows:

$$\begin{cases} x' = (x+U)\cos\phi + (y+V)\sin\phi \\ y' = -(x+U)\sin\phi + (y+V)\cos\phi \\ z' = z + W. \end{cases} \tag{2}$$

Here $\phi = \omega t$ is the turning angle of the rod; ω is the angular speed of the rod.

Let the top end of the drill string be affected by the longitudinal force $N(z,t)$ which is equal to the supporting reaction of the lower end of the drill string on the well bottom. The applied compressing force is considered to be positive.

To obtain the equations of drill string vibrations, we use the Hamilton principle which allows to characterize the drill string motion as a whole for an arbitrary interval of time from t_1 to t_2 [16]. According to this principle

$$\delta J = \int_{t_1}^{t_2} \delta(T - U_0 + \Pi)dt = 0, \tag{3}$$

where U_0 is the potential energy of rod strain, T is the kinetic energy, Π is the potential of external forces, allowing for the effect of the compressing load.

Potential energy U_0 of the rod is given by the Clapeyron formula:

$$U_0 = \iiint_{V_0} \Phi \, dV_0 = \frac{1}{2}\iiint_{V_0} \sigma_{ij}\varepsilon_{ij} dV_0 , \tag{4}$$

where Φ denotes the potential of elastic strain or the elastic potential, $i, j = 1, 2, 3$.

The components of stress tensor σ_{ij} and strain tensor ε_{ij} are connected with the potential as follows: $\sigma_{ij} = \dfrac{\partial \Phi}{\partial \varepsilon_{ij}}, \varepsilon_{ij} = \dfrac{\partial \Phi}{\partial \sigma_{ij}}.$

Kinetic energy T of the rod is defined as:

$$T = \frac{1}{2}\iiint_{V_0} \rho\left(\dot{\bar{R}} \cdot \dot{\bar{R}}\right) dV_0 = \frac{1}{2}\int_0^l \int_F \rho\left(\dot{\bar{R}} \cdot \dot{\bar{R}}\right) dF dz, \tag{5}$$

where $\bar{R}$ is the radius vector determining the position of any point of the rod, ρ is density.

In our case the radius vector $\bar{R}$ is

$$\begin{aligned} \bar{R} &= \left((x+U)\cos(\omega t) + (y+V)\sin(\omega t)\right)\bar{i} \\ &+ \left(-(x+U)\sin(\omega t) + (y+V)\cos(\omega t)\right)\bar{j} + (z+W)\bar{k}, \end{aligned} \tag{6}$$

where $\bar{i}, \bar{j}, \bar{k}$ are unit vectors.

As in [5, 6, 17], potential of external forces, Π, is given by:

$$\Pi = -\frac{1}{2}\int_0^l N(z,t)\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 + \left(\frac{\partial W}{\partial z}\right)^2\right] dz. \tag{7}$$

Substituting (1) into the formulae for the potential energy of the rod, its kinetic energy and the potential of external forces, we integrate them over the cross-section area F. Introducing geometric characteristics $I_x = \int_{F(z)} y^2 dF$, $S_x = \int_{F(z)} y dF = 0$, since the z-axis coincides with the neutral axis of the rod, the following expressions for the energies and potential of external forces are obtained:

$$U_0 = \frac{G(1-\nu)}{1-2\nu}\int_0^l v_{zz}{}^2 I_x dz + \frac{GF}{2(1-2\nu)}\int_0^l v_z{}^4 F dz, \tag{8}$$

where $G = \dfrac{E}{2(1+\nu)}$ is the shear modulus, E is Young's modulus, ν is Poisson's ratio;

$$T = \frac{1}{2}\rho\int_0^l \left[v_t{}^2 F + v_{zt}{}^2 I_x + \omega^2\left(I_x + I_y\right) + \omega^2 v^2 F\right] dz; \tag{9}$$

$$\Pi = \frac{1}{2}\int_0^l N(z,t)\left[\left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial^2 v}{\partial z^2}\right)^2 y^2\right] dz. \tag{10}$$

Substituting (8)-(10) in (3) and calculating the variation δJ of the action integral, we obtain the nonlinear dynamic model of motion of a rotating drill string compressed from both ends by the variable longitudinal force $N(z,t)$:

$$\begin{aligned} &\rho F \frac{\partial^2 v}{\partial t^2} + EI_x \frac{\partial^4 v}{\partial z^4} - \rho I_x \frac{\partial^4 v}{\partial z^2 \partial t^2} + \frac{\partial}{\partial z}\left(N(z,t)\frac{\partial v}{\partial z}\right) \\ &- \frac{EF}{1-\nu}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)^3 - \rho F \omega^2 v = 0. \end{aligned} \tag{11}$$

At that, we neglected the derivatives of order higher than two which are responsible for the contribution of the longitudinal force as in [18].

Boundary conditions for the rod with pinned ends are presented in the form:

$$\begin{aligned} &v(z,t) = 0 \qquad (z = 0, z = l) \\ &EI_x \frac{\partial^2 v(z,t)}{\partial z^2} = 0 \qquad (z = 0, z = l). \end{aligned} \tag{12}$$

In order to take into account the impact of a supersonic air flow on the drill string, we use the formula based on the hypothesis of plane sections [19]:

$$P = P_0\left(1 - \frac{\kappa - 1}{2}\cdot\frac{U_n}{C_0}\right)^{\frac{2\kappa}{\kappa-1}}, \quad (13)$$

where U_n is the normal component of the air flow speed on the string surface; C_0 is the speed of sound for the unperturbed flow; P_0 is the pressure of the unperturbed flow; κ is the polytropic exponent.

We consider the air moving in the direction opposite to the drill string motion.

The normal component of the air flow speed U_n can be written in the form:

$$U_n = V_g \frac{\partial v}{\partial z}, \quad (14)$$

where V_g denotes the speed of the unperturbed flow.

Expanding (13) in a power series and retaining the first three terms in this expansion, we obtain the final nonlinear model of lateral vibrations of a drill string in a supersonic air flow:

$$\begin{gathered}\rho F\frac{\partial^2 v}{\partial t^2} + EI_x\frac{\partial^4 v}{\partial z^4} - \rho I_x\frac{\partial^4 v}{\partial z^2\partial t^2} + \frac{\partial}{\partial z}\left(N(z,t)\frac{\partial v}{\partial z}\right) \\ -\frac{EF}{1-\nu}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)^3 - \rho F\omega^2 v + \Delta P = 0,\end{gathered} \quad (15)$$

where

$$\begin{gathered}\Delta P = P - P_0 = P_0\kappa\left(-M\frac{\partial v}{\partial z} + \frac{\kappa+1}{4}M^2\left(\frac{\partial v}{\partial z}\right)^2\right. \\ \left.-\frac{\kappa+1}{12}M^3\left(\frac{\partial v}{\partial z}\right)^3\right),\end{gathered} \quad (16)$$

$M = \dfrac{V_g}{C_0}$ is the Mach number.

III. Model Calculation

To obtain the solution of the model (15) the Bubnov-Galerkin method is applied. The compressing load is supposed to be constant and distributed along the rod length. The displacement $v(z,t)$ is given by the following expansion:

$$v(z,t) = \sum_{i=1}^{n} f_i(t)\sin\left(\frac{i\pi z}{l}\right), \quad (17)$$

where $n \le 3$, that is one, two and three modal approximations of the solution are considered.

Substituting (17) in (15) at $n = 3$ and utilizing the Bubnov-Galerkin technique, we obtain a system of three second-order ordinary differential equations:

$$\begin{aligned}&a_1 f_1'' + b_1 f_1 + f_1\left(c_{11}f_1^2 + c_{12}f_2^2 + c_{13}f_3^2 + c_{14}f_1f_3\right) \\ &+ c_{15}f_2^2f_3 + d_{11}f_2 + d_{12}f_1f_3 + d_{13}f_1^2 + d_{14}f_2^2 + d_{15}f_3^2 \\ &+ f_2\left(d_{16}f_1^2 + d_{17}f_2^2 + d_{18}f_3^2 + d_{19}f_1f_3\right) = 0, \\ &a_2 f_2'' + b_2 f_2 + f_2\left(c_{21}f_2^2 + c_{22}f_1^2 + c_{23}f_3^2 + c_{24}f_1f_3\right) \\ &+ d_{21}f_1 + d_{22}f_3 + d_{23}f_1f_2 + d_{24}f_2f_3 + f_1\left(d_{25}f_1^2 + d_{26}f_2^2\right) \\ &+ f_3\left(d_{27}f_1^2 + d_{28}f_2^2 + d_{28}f_3^2 + d_{2,10}f_1f_3\right) = 0, \\ &a_3 f_3'' + b_3 f_3 + f_3\left(c_{31}f_1^2 + c_{32}f_2^2 + c_{33}f_3^2\right) + f_1\left(c_{34}f_1^2\right. \\ &\left.+ c_{35}f_2^2\right) + d_{31}f_2 + d_{32}f_1f_3 + d_{33}f_1^2 + d_{34}f_2^2 + d_{35}f_3^2 \\ &+ f_2\left(d_{36}f_1^2 + d_{37}f_2^2 + d_{38}f_3^2 + d_{39}f_1f_3\right) = 0,\end{aligned} \quad (18)$$

where a_i are the constants responsible for the contribution of inertial terms; the constants b_i include the influence of the longitudinal compressing load, centrifugal force and rigidity of the rod; c_{ij} contain geometric nonlinearity; d_{ik} bring the influence of the supersonic air flow, $i = 1,2,3;\ j = \overline{1,4(5)};\ k = \overline{1,9(10)}$.

IV. Comparative Analysis of Mathematical Models

To be convinced of the need for application of the nonlinear model, it is compared to another model not involving geometric nonlinearity, which we call geometrically linear model, keeping in mind that it retains nonlinearity from the air flow.

Let us consider single-mode approximation of the solution:

$$v(z,t) = f(t)\sin\left(\frac{\pi z}{l}\right). \quad (19)$$

Numerical solution is carried out in the Wolfram Mathematica computational package by the stiffness switching method. This method uses an explicit Runge-Kutta method and a linearly implicit Euler method, and allows to switch between them when the system appears to be stiff. Due to that we can reduce computation time and keep high numerical precision.

Computations were conducted at the following values of the system parameters: $D = 0.2m$ (outer diameter of the drill string), $d = 0.12m$ (inner diameter), $N = 2.2\times10^3 H$, $E_{st} = 2.1\times10^5 MPa$, $\rho_{st} = 7800\,kg/m^3$, $P_0 = 1.013\times10^3 Pa$, $\nu_{st} = 0.28$, $M = 2.5$, $\kappa = 1.4$.

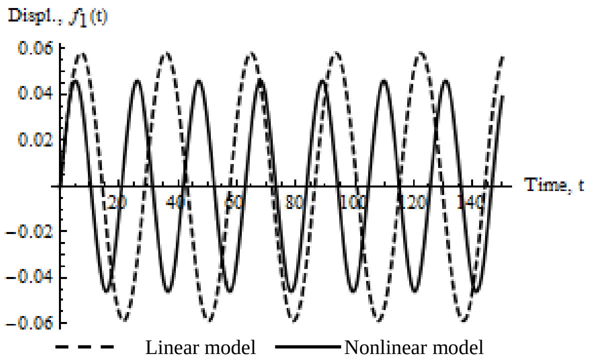

Figure 1. Drill string vibrations at $l = 100m, \omega = 10rpm$.

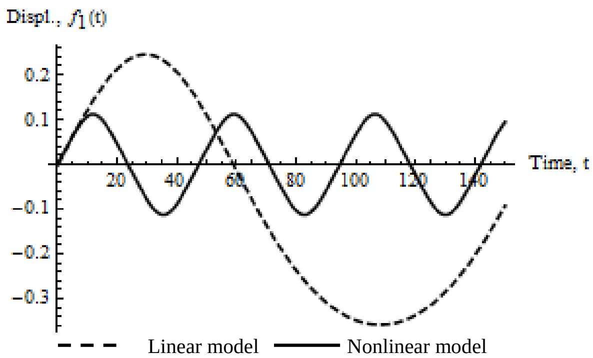

Figure 2. Drill string vibrations at $l = 200m, \omega = 0$.

Fig. 1-2 show that the nonlinear model gives smaller amplitude vibrations of the rod than the linear model. With increase in length of the rod or its rotation frequency linear vibrations rise unboundedly, whereas using the nonlinear model we can observe a stable oscillatory process. A similar phenomenon is observed when the pressure of the unperturbed air flow, P_0, is higher than $10^4 Pa$. These results demonstrate the need for application of nonlinear models when examining lateral vibrations of a drill string moving in a supersonic air flow.

V. Analysis of the Effect of the System Parameters

For a more detailed analysis of drill string vibrations let us consider the nonlinear model (17) with the buckling $v(z,t)$ in the second and third approximations.

Initial values of the length and rotation frequency are $l = 100m, \omega = 10rpm$.

Fig. 3-4 were constructed for the double-mode approximation of the solution, Fig. 5-6 for the triple-mode approximation.

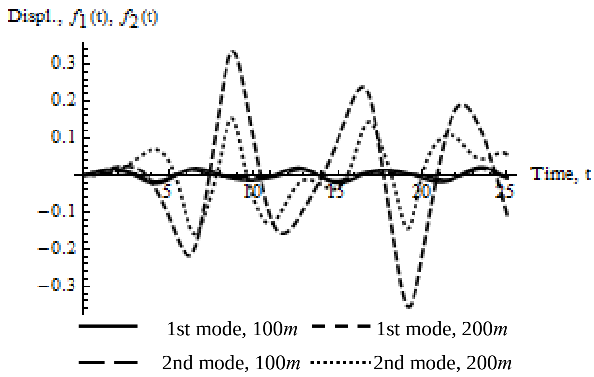

Figure 3. Vibrations of the drill strings of various lengths.

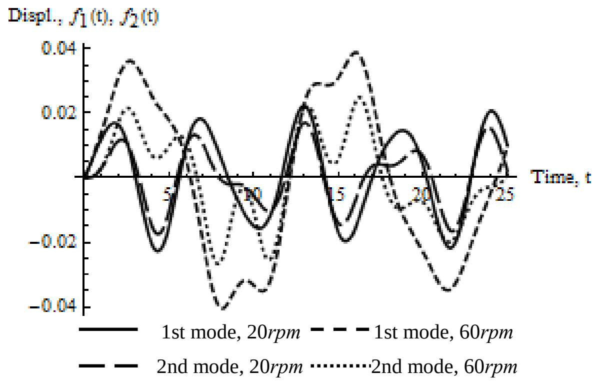

Figure 4. Vibrations of the drill string at different rotation frequencies.

As can be seen from Fig. 3, increase in length of the drill string leads to sharp rise in the amplitude of its lateral vibrations. Moreover, the higher the angular speed of rotation, the more intensive the drill string vibrations can be observed (Fig. 4). At the same time the second mode has a considerable influence on the vibrations and should be taken into account while modeling.

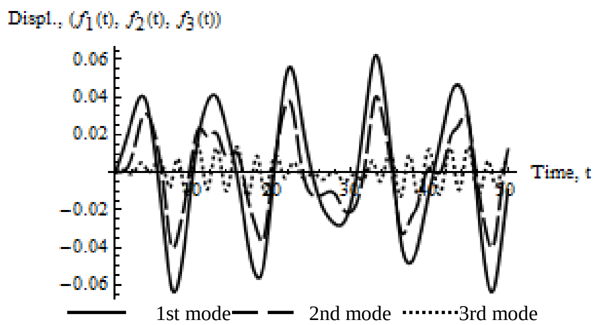

Figure 5. Vibrations of the drill string under the load $N = 3.5 \times 10^4 H$.

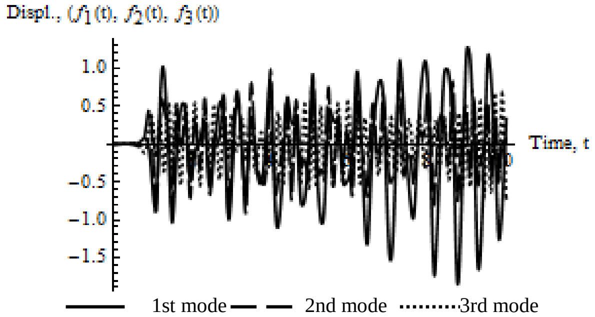

Figure 6. Effect of the air flow parameters: Mach number $M = 4$, pressure $P_0 = 1.013 \times 10^5 Pa$.

With increase in the value of the compressing load the contribution of the third mode to the oscillatory process grows (Fig. 5). Simultaneous rise in speed and pressure of the air flow results in the multiple increase in amplitude and frequency of the drill string vibrations, leading to instability of the system (Fig. 6).

VI. Conclusion

In this work the motion of a drill string in a supersonic air flow was modeled. Applying the Hamilton principle a nonlinear mathematical model of lateral vibrations of the drill string presented as a one-dimensional rod was constructed. The use of the Bubnov-Galerkin method allowed to reduce the model to a system of three second-order ordinary differential equations which were solved by means of the stiffness switching method. Comparison with the geometrically linear model showed that the nonlinear model gives smaller amplitude vibrations of the drill string and can be applied at larger values of the system parameters. The influence of various lengths of the drill string, frequency of rotation, compressing loading and the supersonic air flow was studied in double- and triple-mode approximation of the solution. It was found out that the second and third modes contribute significantly to the oscillatory process, and they should be taken into consideration when modeling. Also it was obtained that increase in the parameters leads to the rise of lateral vibrations of the drill string. The longer the string, the greater the influence the external loadings have. At the same time the speed and pressure of the air flow have a great impact on the amplitude of the drill string vibrations as well.

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