The Improvement of The Total Flow-Time Problem under The Non-Idle Machine Constraint of Two Machine Open-Shop

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Abstract-**The total flow-time problem of two machine open-shop is NP-hard in strong sense. For the above problem without constrains, an explicit solution is constructed in this paper from the optimal schedule under the non-idle machine constraint, its total flow-time is reductive. The explicit solution under non-idle machine constraints must not be the explicit solution without constraints is proved under the consideration.**

Keywords-Total flow-time, Open-shop, Efficient algorithm, Scheduling

I. INTRODUCTION

For n job $J_j(j=1,2,\cdots,n)$ will be working in m machine $M_i(i=1,2,\cdots,m)$, and all job are worked once on any machine, also working sequence is arbitrarily. It is called open-shop. Achugbue and Chin [2]have proved that $O2\|\sum c_j$ is NP-hard in strong sense. For the problem of the total flow-time problem of two machine open-shop that working time rely on the machine, is denoted by

$$O2\left|P_{ij}=P_i\right|\sum C_j \quad (1)$$

For problem(1.1), M.Dror [3]give the optimal solution under $2P_2 \le P_1$,and give a efficient algorithm to following problem

$$O2\left|P_{ij}=P_i, P_2<P_1<2P_2\right|\sum C_j \quad (2)$$

Siming Xiang and Guochun Tang [4]proved that M.Dror algorithm is wrong conclusion. They proved that the problem can transform a assignment problem, then

$$O2\left|P_{ij}=P_i, P_2<P_1<2P_2, Non-Idle\right|\sum C_j \quad (3)$$

is polynomial solvable, Hungary algorithm i.e. Wenci Yu and Gang Ying[5]give the explicit solution for the problem. In this paper, an explicit solution of problem (2)is constructed and its total flow-time is more reductive than problem (3). It show that the proved that explicit solution of problem (2)is not explicit solution of problem (3).

Following are definitions and marks.

Definition1 For schedule S,if completion time of job j in M_1 is smaller than in M_2,then we called job j is forward job, otherwise backward job.

Definition2 The cycling sub-schedule of problem(1.3),corresponding job subset is $\{r,r+1,\cdots,r+l-1\}$,and r is called the origination job, l is called the length,processing sequencing as follows.

In M_1 : processing sequencing of jobs is $\{r,r+1,\cdots,r+l-1\}$,

In M_2 : processing sequencing of jobs is $\{r+1,\cdots,r+l-1,r\}$.

Definition3 The intersecting sub-schedule of problem(1.3), corresponding job subset is $\{r,r+1,r+2,r+3,r+4\}$,and r is called the origination job, its length is 5,processing sequencing as follows.

In M_1 : processing sequencing of jobs is $\{r,r+1,r+2,r+3,r+4\}$.

In M_2 : processing sequencing of jobs is $\{r+1,r+3,r,r+4,r+2\}$.

Definition4 The parallel sub-schedule of problem(1.3) $Ir(l)$,corresponding job subset is $\{r,r+1,\cdots,r+l-1\}$,and r is called the origination job, l is called the length, processing sequencing as follows.

In M_1 : processing sequencing of jobs is $\{r,r+1,\cdots,r+l-1\}$,

In M_2 : processing sequencing of jobs is $\{r,r+1,\cdots,r+l-1\}$.

The definitions and marks apply to problem(1.1)and problem 1.2).

II. MAIN RESULTS

For problem(1.3),namely $O2\left|P_{ij}=P_i, P_2<P_1<2P_2, Non-Idle\right|\sum C_j$,suppose m=$\left\lceil \frac{P_2}{P_1-P_2} \right\rceil$,then $\frac{m+1}{m}P_2 \le P_1 \le \frac{m}{m-1}P_2$.

Theorem 1 [5] If the number of jobs n satisfy $3 \le n \le m$,the optimal schedule S_* of problem (1.3)is listed below.

(1)If $n \equiv 0 \pmod 3$,then $S_* = T(3)T(3)\cdots T(3)$;

(2)If $n \equiv 1(\bmod 3)$,then $S_* = T(4)T(3)\cdots T(3)$;

(3)If $n \equiv 2(\bmod 3)$, then $S_* = T(3)T(3)\cdots T(3)Q(5)$.

Theorem 2 [5] If the number of jobs n satisfy $3 \le n \le m$ $n > m$, the optimal schedule S_* of problem (1.3)is listed below.

(1)If $m \equiv 0(\bmod 3)$, then

$S_* = T(3)T(3)\cdots T(3)I(n-m)$;

(2)If $m \equiv 1(\bmod 3)$, then

$S_* = T(4)T(3)\cdots T(3)I(n-m)$;

(3)If $m \equiv 2(\bmod 3)$, then

$S_* = T(3)T(3)\cdots T(3)I(n-m-1)$.

Lemma 1 If the number of jobs n satisfy $3 \le n \le m$,the total flow-time of the optimal schedule S_* of problem (1.3)is listed below.

(1)If $n = 3k$, then $\sum C_j = \frac{3}{2}k(k+1)P_2 + k(3k+2)P_1$

(2)If $n = 3k+1$, then

$$\sum C_j = \frac{1}{2}k(3k+5)P_2 + (3k^2+4k+2)P_1$$

(3)If $n = 3k+2$, then

$$\sum C_j = (\frac{3}{2}k^2 + \frac{9}{2}k + 2)P_2 + (3k^2+5k+3)P_1 \cdot$$

Proof (1) Let $I = \{$ Forward jobs subscript of the optimal schedule S_* $\}$,

$\bar{I} = \{$ Backward jobs subscript of the optimal schedule $S_*\}$,

Then $I = \{3\mu+1 \mid \mu = 0,1,\cdots,k-1\}$,

$\bar{I} = \{3\mu+2, 3\mu+3 \mid \mu = 0,1,\cdots,k-1\}$,

and $\sum_{j\in I} C_j = \sum_{\mu=0}^{k-1} 3(\mu+1)P_2 = \frac{3}{2}k(k+1)P_2$,

$$\sum_{j\in \bar{I}} C_j = \sum_{\mu=0}^{k-1}[(3\mu+2)P_1 + (3\mu+2)P_1] = k(3k+2)P_1,$$

Therefore

$$\sum C_j = \sum_{j\in I} C_j + \sum_{j\in \bar{I}} C_j = \frac{3}{2}k(k+1)P_2 + k(3k+2)P_1 \cdot$$

The proof of(2)and(3)is slightly.

We improved the optimal schedule in theorem 1 allowing in the machine idle period of time below.

First, we consider $n = 3k$ .After completion of the processing job $3l$,processing job $3l+1$ in M_2 in first thing, after a period of time $t = (3l+1)P_2 - 3lP_1$, processing job $3l+1, 3l+2, \cdots, 3k$ in M_1 continuously, processing job $3l+2$, $3l+3$, $\cdots, 3k$ in M_2 continuously. It is that turn schedule S_* to $S_1 = T_1(3)T_4(3)\cdots T_{3l-2}(3)I_{3l+1}(3k-3l)$,among S_1 there are idle time $t = (3l+1)P_2 - 3lP_1$ in M_1 after completion of the processing job $3l$.

Lemma 2 The total flow-time of the schedule

$$S_1 = T_1(3)T_4(3)\cdots T_{3l-2}(3)I_{3l+1}(3k-3l)$$

is $\sum C_j = \frac{3}{2}l(l+1)P_2 + l(3l+2)P_1$

$+(k-l)[\frac{1}{2}(9l+9k+3)P_1 + 9lP_2 - 9lP_1 + 3P_2]$.

Proof $\sum_{j\le 3l} C_j = \frac{3}{2}l(l+1)P_2 + l(3l+2)P_1$,

$$\sum_{j>3l} C_j = \sum_{i=1}^{3k-3l}[(3l+i)P_1 + 3l(P_2-P_1) + P_2]$$

$$= (k-l)[\frac{1}{2}(9l+9k+3)P_1 + 9lP_2 - 9lP_1 + 3P_2]$$

And $\sum C_j = \sum_{j\le 3l} C_j + \sum_{j>3l} C_j$.

Theorem 3 Let $v = \frac{k}{5} + \frac{3P_2 - P_1}{15(P_1 - P_2)}$,when l satisfy $v < l < k$,the total flow-time of the schedule $S_1 = T_1(3)T_4(3)\cdots T_{3l-2}(3)I_{3l+1}(3k-3l)$ is lesser than the schedule $S_* = T(3)T(3)\cdots T(3)$.

Proof When $v = \frac{k}{5} + \frac{3P_2 - P_1}{15(P_1 - P_2)} < l < k$,

$$\frac{1}{2}(9l+9k+3)P_1 + 9lP_2 - 9lP_1 + 3P_2 <$$

$$\frac{3}{2}(k+l+1)P_2 + (3k+3l+2)P_1,$$

Therefore the total flow-time of the schedule

$$S_1 = T_1(3)T_4(3)\cdots T_{3l-2}(3)I_{3l+1}(3k-3l)$$

is lesser than the schedule $S_* = T(3)T(3)\cdots T(3)$ from lemma1 and lemma 2.

Second, we consider $n = 3k+1$.After completion of the processing job $3l+4$,there is a period of idle time $t = (3l+5)P_2 - (3l+4)P_1$ in M_1 and processing job $3l+5$, $3l+6$, $\cdots, 3k+1$ in M_1 continuously and processing job $3l+5$, $3l+6$, $\cdots, 3k+1$ in M_2 continuously. It is that turn schedule S_* to $S_2 = T_1(4)T_5(3)\cdots T_{3l+2}(3)I_{3l+5}[3(k-l-1)]$,among S_2 there are idle time $t = (3l+5)P_2 - (3l+4)P_1$ in M_1 after completion of the processing job $3l+4$.

Lemma 3 The total flow-time of the schedule

$S_2 = T_1(4)T_5(3)\cdots T_{3l+2}(3)I_{3l+5}[3(k-l-1)]$

is $\sum C_j = (\frac{9}{2}k^2 - 9kl - \frac{15}{2}k + \frac{15}{2}l^2 + \frac{35}{2}l + 12)P_1 + (9kl - \frac{15}{2}l^2 - \frac{37}{2}l + 15k - 11)P_2$.

Proof

$$\sum_{j \le 3l+4} C_j = \frac{1}{2}[3(l+1)+5](l+1)P_2 + [3(l+1)^2 + 4(l+1)+2]P_2$$;

$$\sum_{j>3l+4} C_j = \sum_{u=3l+5}^{3k+1} \{uP_1 + [(3l+5)P_2 - (3l+4)P_1]\}$$ Then

$$= 3(k-l-1)[\frac{3}{2}(k-l)P_1 - P_1 + 3lP_2 + 5P_2]$$

$$\sum C_j = \sum_{j \le 3l+4} C_j + \sum_{j > 3l+4} C_j .$$

Theorem 4 Let $v = \frac{k}{5} + \frac{2P_2}{15(P_1 - P_2)} - \frac{4}{3}$, when l satisfy $v < l < k-1$, the total flow-time of the schedule $S_2 = T_1(4)T_5(3)\cdots T_{3l+2}(3)I_{3l+5}[3(k-l-1)]$ is lesser than the schedule $S_* = T_1(4)T_5(3)\cdots T_{3k-1}(3)$.

Proof From lemma1 and lemma2, the difference between the total flow-time of the schedule S_* and S_2 is

$$\sum_{S_*} C_j - \sum_{S_2} C_j = [(P_2 - P_1)(\frac{3}{2}k - \frac{15}{2}l - 10) - P_2](k-l-1)$$.

As a result, when $v = \frac{k}{5} + \frac{2P_2}{15(P_1 - P_2)} - \frac{4}{3} < l < k-1$, $\sum_{S_*} C_j - \sum_{S_2} C_j > 0$, that is $\sum_{S_*} C_j > \sum_{S_2} C_j$.

Finally, we consider $n = 3k$. After completion of the processing job $3l$, processing job $3l+1$ in M_2 in first thing, after a period of idle time $t = (3l+1)P_2 - 3lP_1$ in M_1, processing job $3l+1$, $3l+2$, $\cdots, 3k+2$ in M_1 continuously, processing job $3l+2, 3l+3, \cdots, 3k+2$ in M_2 continuously. It is that turn schedule $S_* = T(3)T(3)\cdots T(3)Q(5)$ to $S_3 = T_1(3)T_4(3)\cdots T_{3l-2}(3)I_{3l+1}[(3k+2)-3l]$, among S_3 there is idle time $t = (3l+1)P_2 - 3lP_1$ in M_1 after completion of the processing job $3l$.

Lemma 4 The total flow-time of the schedule

$S_3 = T_1(3)T_4(3)\cdots T_{3l-2}(3)I_{3l+1}[(3k+2)-3l]$

is $\sum C_j = (\frac{15}{2}l^2 + \frac{9}{2}k^2 - 9kl + \frac{15}{2}k - \frac{11}{2}l + 3)P_1 + (9kl - \frac{15}{2}l^2 + \frac{9}{2}l + 3k + 2)P_2$.

Proof: $\sum_{j \le 3l} C_j = \frac{3}{2}l(l+1)P_2 + l(3l+2)P_1$,

$$\sum_{j>3l} C_j = \sum_{u=3l+1}^{3k+2} [uP_1 + (3l+1)P_2 - 3lP_1]$$

$$= (3k - 3l + 2)[\frac{3}{2}(k-l+1)P_1 + (3l+1)P_2],$$

then $\sum C_j = \sum_{j \le 3l} C_j + \sum_{j>3l} C_j$.

Theorem 5 Let $l < k$, when l satisfy

$$(\frac{3}{2}k^2 - 9kl + \frac{15}{2}l^2)(P_2 - P_1) + (\frac{11}{2}l - \frac{5}{2}k)P_1 + (\frac{3}{2}k - \frac{9}{2}l)P_2 > 0$$.

The total flow-time of the schedule $S_3 = T_1(3)T_4(3)\cdots T_{3l-2}(3)I_{3l+1}[(3k+2)-3l]$ is lesser than the schedule $S_* = T(3)T(3)\cdots T(3)Q(5)$.

Proof:From lemma 1 and lemma 4, the difference between the total flow-time of the schedule S_* and S_2 is

$$\sum_{S_*} C_j - \sum_{S_3} C_j = [(\frac{3}{2}k^2 - 9kl + \frac{15}{2}l^2)(P_2 - P_1) + (\frac{11}{2}l - \frac{5}{2}k)P_1 + (\frac{3}{2}k - \frac{9}{2}l)P_2$$.

There is the conclusion of theorem 5.

III. EXAMPLES

Example 1

$n = 18, P_1 = 22, P_2 = 21, k = \frac{n}{3} = 6, v = \frac{6}{5} + \frac{41}{15} = \frac{59}{15}$.

The total flow-time of the schedule $S_* = T_1(3)T_4(3)T_7(3)T_{10}(3)T_{13}(3)T_{16}(3)$ is $\sum C_j = 3963$ .Let $l = 4$, the total flow-time of the schedule $S_1' = T_1(3)T_4(3)T_7(3)T_{10}(3)I_{13}(6)$ is $\sum C_j = 3962$, among S_1' there is idle time 9 units in M_1 after completion of the processing job 12. Let $l = 5$, the total flow-time of the schedule $S_1'' = T_1(3)T_4(3)T_7(3)T_{10}(3)T_{13}(3)I_{16}(3)$ is $\sum C_j = 3955$, among S_1'' there is idle time 6 units in M_1 after completion of the processing job 15.

Example 2

$n = 19, P_1 = 22, P_2 = 21, k = \frac{n-1}{3} = 6, v = \frac{8}{3}$..

The total flow-time of the schedule $S_* = T_1(4)T_5(3)T_8(3)T_{11}(3)T_{14}(3)T_{17}(3)$ is

$\sum C_j = 4397$ .Let $l = 3$,the total flow-time of the schedule

$S_2' = T_1(4)T_5(3)T_8(3)T_{11}(3)I_{14}(6)$ is $\sum C_j = 4392$, among S_2' there is idle time 8 units in M_1 after completion of the processing job 13. Let $l = 4$, the total flow-time of the schedule

$S_2'' = T_1(4)T_5(3)T_8(3)T_{11}(3)T_{14}(3)I_{17}(3)$ is $\sum C_j = 4387$, among S_2'' there is idle time 5 units in M_1 after completion of the processing job 15.

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