Adaptive Impulsive Noise Control Algorithms based on The Fractional Lower-order Statistics

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Abstract. Noise control of signals is a key challenge problem in signal enhancement, signal recognition, communication, radar, and sonar applications. The most widely used method is adaptive linear filtering method, which can adaptive change filter parameters with the stochastic property of the stationary Gaussian noise. The representative algorithms of this include least mean square (LMS) adaptive filter and recursive least squares (RLS) adaptive filter. The conventional adaptive filtering algorithms suffer from severe degradation in impulsive noise environments. Although the fractional lower-order statistics (FLOS) based adaptive methods are robust to the impulsive noise, the complexity of these algorithms is a main problem in real world implementation. In this paper, two adaptive algorithms for impulsive noise reduction are proposed. The proposed methods have the capability of steady-state in the presence of impulsive noise. Simulation results illustrate an improvement in terms of convergence and steady-state performance.

Introduction

We live in a natural environment where noise is inevitable and ubiquitous, the signals are generally contaminated by acoustic background noise. As a result, noise reduction algorithms and systems for signal enhancement have received considerable interest in the past, primarily because the reduced signal intelligibility under noisy conditions is one of the major complaints in signal processing applications [1]. Therefore, noise reduction has been in great demand for an increasing number of audio applications, communications systems, and so on.

The noise control process, which is often referred to as rather noise reduction or signal enhancement, can be achieved in many different ways, such as beamforming, adaptive filtering, temporal filtering, spatial-temporal filtering, etc [2], [3]. Two technique dominate the adaptive filter arena, namely, the least mean square (LMS) algorithm, and the recursive least squares (RLS) algorithm. It is well known that under the independence assumptions, the LMS algorithm convergences in the mean, and RLS algorithm convergences in the mean squares sense to the Wiener optimal filter [4], [5]. The LMS is known to track remarkably well the variations of a slowly time-varying model [3]. In many applications, LMS adaptive filtering algorithms are widely used, partly because they require less calculation and are simple to implement. It can also be delineated in the frequency domain, resulting in various derivative techniques [6]. However, it is slow-converging as initialization or the impulse response estimation phase. In general, the RLS algorithm converges relatively fast but is computationally extremely complex compared to the LMS approach. On the other hand, it has been shown that the RLS exhibits unstable behavior leading to divergence. To overcome the problem, some algorithms have been developed. The divergent behavior of the RLS has been attributed to round-off errors. The effects of round-off errors have been analyzed in [7]. One solution to this problem is to use different techniques during initialization and the steady state, use the LMS during the steady state, and the RLS during the initialization.

Most noise reduction algorithms work well for stationary or slowly varying noise, but less so for heavily non-stationary noise. Without specific knowledge about the noise, it can be difficult to the signal from the noise for these methods. The conventional LMS and RLS algorithms assume that the environmental noise is Gaussian noise. However, the assumption of Gaussian noise is often unrealistic. Studies have shown that an important class of noise as underwater acoustic noise, atmospheric noise, multiuser interference, and radar clutters in real world applications are non-Gaussian processes [8], [9]. It has been shown that a class of α–stable distribution is more appropriate for modeling impulsive noise than Gaussian noise [10]. In order to robust against the impulsive noise, some fractional lower-order statistics (FLOS) based adaptive algorithms were developed in [11]-[14]. Although the FLOS based methods are robust to the impulsive noise, the computation of these methods is too complex to implement.

In this paper, we analyze the performance of the conventional adaptive algorithms in the presence of stable distribution impulsive noise. Then we propose two adaptive noise reduction method based on the fractional lower-order statistics. Simulation results show the robustness and effectiveness of the proposed algorithms.

Problem Formulation

The noise control problem considered in this paper is to recover a signal of *x(k)* from the received signal *y(k)* which is corrupted by the noise,

$$y(k) = x(k) + n(k) \tag{1}$$

where *n(k)* is the additive symmetric α-stable (*SαS*) noise. An univariate symmetric α-stable (*SαS*) probability density function (PDF) is best defined via the inverse Fourier transform integral [13], which is completely characterized by the three parameters, characteristic exponent α ($-\infty < \alpha < \infty$), dispersion γ ($\gamma > 0$), and location a ($-\infty < a < \infty$). The characteristic exponent α relates directly to the heaviness of the tails of the stable distribution. The smaller the characteristic exponent is, the heavier the tails of the distribution. The value where α=1 corresponds to a Cauchy case, the value α=2 corresponds to a Gaussian case.

The important difference between Gaussian and non-Gaussian *SαS* distribution is that the moments of *SαS* distribution is finite only for *p*th-order ($p<\alpha$) moments. Since only the *p*th-order ($p<\alpha$) moments are finite for the stable distribution variables, the fractional lower-order statistics (FLOS) have become one of the significant signal processing techniques in impulsive noise environments. For two *SαS* random variables ζ and η, the *p*th-order fractional correlation is defined as

$$\langle \varsigma, \eta \rangle_p = E(\varsigma \eta^{\langle p-1 \rangle}) \tag{2}$$

where $z^{\langle p \rangle} = |z|^{p-1} z$, and the $\operatorname{sign}(\cdot)$ function is defined by

$$sign(z) = \begin{cases} 1, z > 0 \\ 0, z = 0 \\ -1, z < 0 \end{cases} \tag{3}$$

Impulsive Noise Control Algorithms based on the Fractional Lower-order Statistics

The least-mean-square (LMS) algorithm is widely used in adaptive signal processing for its robustness and simplicity. It is known for its simplicity and its good steady-state performance in stationary context [12]. An adaptive filtering algorithm adjusts the filter tap weight $\mathbf{w}(k)$ at each time instant according to the measured value of $e(k)$. The standard LMS algorithm updates as [10]

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu e(k)\mathbf{x}(k) \tag{4}$$

where μ is defined as the step-size parameter which affects the convergence property of the filter weights, and $e(k) = d(k) - y(k)$ is the estimation error between the desired signal $d(k)$ and the received signal $y(k)$.

In theory the cost function of LMS algorithm is

$$\eta(k) = E(e^2(k)) \tag{5}$$

The filter parameters convergent to the optimal condition,

$$\min_{w(k)} \eta(k) = e^T(k)e(k) \tag{6}$$

The RLS algorithm is one of the most important adaptive filter algorithms due to its fast convergence rate in non-stationary environment, insensitivity to the eigenvalue spread of the input correlation matrix. The modular structure of it provides fast implementations. Therefore, it is desirable to have RLS-type algorithms that are roust in impulsive noise. The cost function of RLS algorithm is

$$\varsigma(k) = \sum_{i=1}^{k} \lambda^{k-i} e^2(i) \tag{7}$$

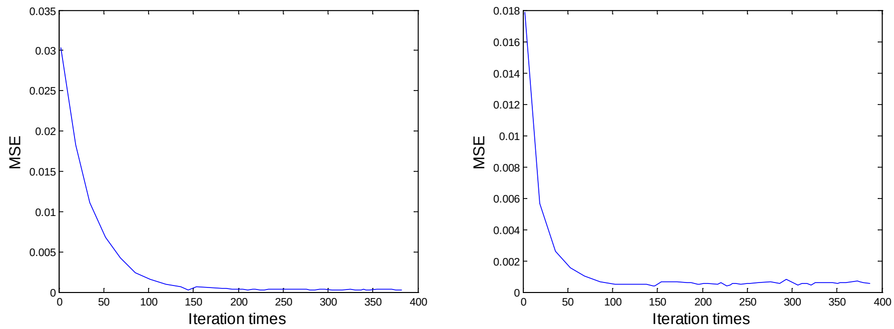
where $\lambda(k) = d(k+1) - x^T(k+1)W(k)$, $W(k)$ is the weight vector of the RLS. The filter parameters convergent to the optimal condition,

$$\min_{W(k)} \varsigma(k) = e^T(k)\mathbf{\Lambda}(k)e(k) \tag{8}$$

where $\mathbf{\Lambda}(k) = Diag(\lambda^{k-1}, \cdots, \lambda, 1)$, and $e(k) = d(k) - x^T(k)W(k-1)$.

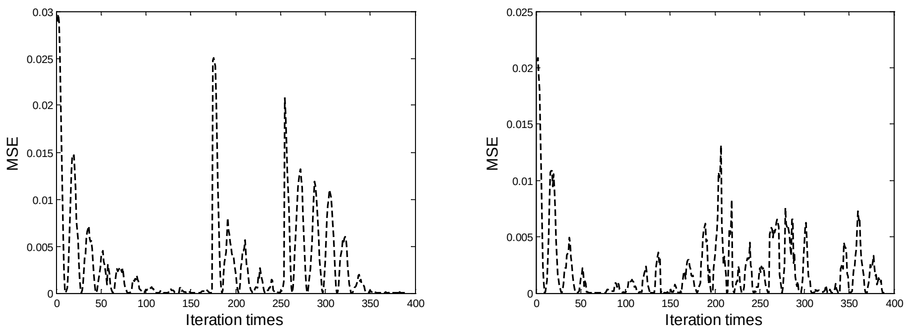
It has been well established the performance of the conventional adaptive algorithm (iterms of the speed and convergence performance) change with a change in the characteristics [3]. It follows from equation (5) and (7) that the LMS and RLS algorithms will convergent to the steady state, when parameters convergent to the optimal conditions (6) and (8). However, when the noise in the received signal contain impulsive noise components, the variance of $e(k)$ will become unbound, $E\{e^T(k)e(k)\} \to \infty$. Especially, the mean of $e(k)$ will also infinite ($E\{e(k)\} \to \infty$), when the characteristic exponent is $0 < \alpha < 1$. In this case, the LMS and RLS algorithms cannot convergent to the steady state, thus, they are useless in the presence of impulsive noise.

The performance of the LMS and RLS algorithms in Gaussian noise and impulsive noise are shown in Figure 1 and Figure 2, respectively. The signal is a single frequency sine signal and the signal to noise ratio is 3 dB, the characteristic exponent of the impulsive noise is $\alpha = 1.8$. It can be seen from Figure 1 and Figure 2 that the LMS and RLS algorithms can reduce the influence of the Gaussian noise, however, they cannot circumvent the impulsive noise.



(a) The LMS algorithm (b) The RLS algorithm

Fig.1. The convergence of LMS and RLS algorithms in Gaussian noise.



(a) The LMS algorithm (b) The RLS algorithm

Fig.2. The convergence of LMS and RLS algorithms in impulsive noise.

The work [9] introduces the fractional lower-order statistics is an effective technique to robust against the stable distribution noise. Although several robust adaptive algorithms have been proposed, they are difficult to implementation with real world applications. Therefore, it is necessary to develop new robust algorithms that are not only robust to the impulsive noise, but also easy to apply in real world. Here, we first define two new cost functions for the LMS and RLS algorithms. The new cost function of LMS algorithm is defined as

$$\eta'(k) = E[(e^{\langle p \rangle}(k))^T e^{\langle p \rangle}(k)],\ p < \alpha \tag{9}$$

The weight vectors updates as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mu(e(k))^{\langle p \rangle}\mathbf{x}(k)}{(\mathbf{x}^{\langle p \rangle}(k))^T \mathbf{x}^{\langle p \rangle}(k)} \tag{10}$$

where μ is the step-size parameter which affects the convergence behavior of the filter weights. The new cost function of RLS algorithm is defined as

$$\varsigma'(k) = \sum_{i=1}^{k} \lambda^{k-i} (e^{\langle p \rangle}(i))^T e^{\langle p \rangle}(i)),\ p < \alpha \tag{11}$$

The weight vector is given by

$$W(k) = W(k-1) + g(k)[d(k) - x^T(k)W(k-1)] \tag{12}$$

where $g(k) = C(k-1)(x(k))^{\langle p \rangle} / (\lambda + \mu(k))$, and $C(k)$ and $\mu(k)$ are recursively computed from,

$$C(k) = \lambda^{-1}[C(k-1) - g(k)(x^{\langle p \rangle}(k))^T C(k-1)] \tag{13}$$

$$\mu(k) = (x^{\langle p \rangle}(k))^T C(k-1) x^{\langle p \rangle}(k) \tag{14}$$

where $C(0) = \delta^{-1} I$ ($\delta > 0$).

Although $n(k)$ contains impulsive components, the $e(k)$ becomes to a second-order moment process by using nonlinear operation. Thus, the performance of the adaptive algorithms with impulsive noise will be much better than the conventional methods. Furthermore, the complexity of the proposed FLOS based LMS and RLS algorithms is similar to the conventional algorithms, except for the nonlinear operation $(e(k))^{\langle p \rangle}$.

Simulation Results

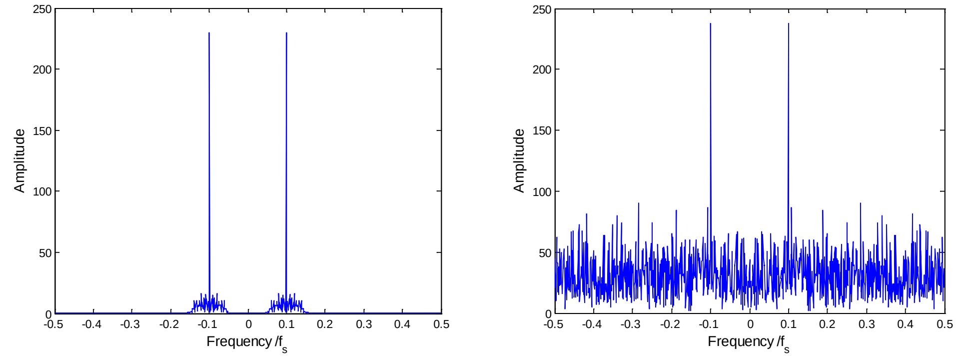
In this section, we apply the proposed algorithms to noise control in communications signals processing applications. The signal of interest (SOI) is an AM signal with carrier frequency of $f_c = 0.1 f_s$, bandwidth of $B = 0.04 f_s$. We use generalized signal-to-noise ratio (GSNR) as the ratio of the signal power over the impulsive noise dispersion,

$$\text{GSNR} = 10\lg[\frac{1}{N\gamma}\sum_{1}^{N} |s(k)|^2] \tag{15}$$

The step-size parameter of the FLOS based LMS is $\mu = 0.01$, the λ of the FLOS based RLS is 0.99.

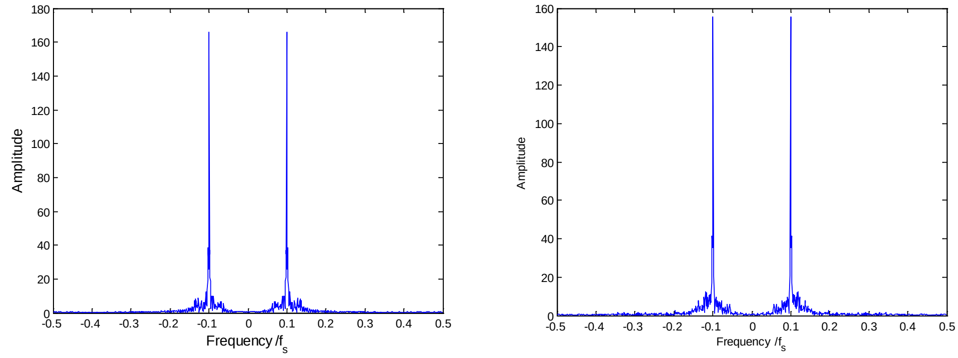
The power spectra of the AM signal and the noisy signal which includes AM signal and impulsive noise are shown in Figure 3. Figure 4 shows the power spectra of the signals processed by the proposed FLOS based LMS and RLS algorithms in impulsive noises. Figure 5 shows the convergence process of the proposed LMS and RLS algorithms. The characteristic exponent of the impulsive noise is $\alpha = 1.8$, the GSNR is 3dB.

It is easy to see from Figure 3 that the AM signal is completely corrupted by the impulsive noise. However, it can be seen from Figure 4 that the proposed LMS and RLS can reduce the impulsive noise. We note also that the FLOS based LMS reveals a similar suppression capability compared to the FLOS based RLS method. Although both the proposed LMS and RLS algorithms can convergent to the steady state, but from Figure 5 we can see that the FLOS based RLS convergence faster than the FLOS based LMS.



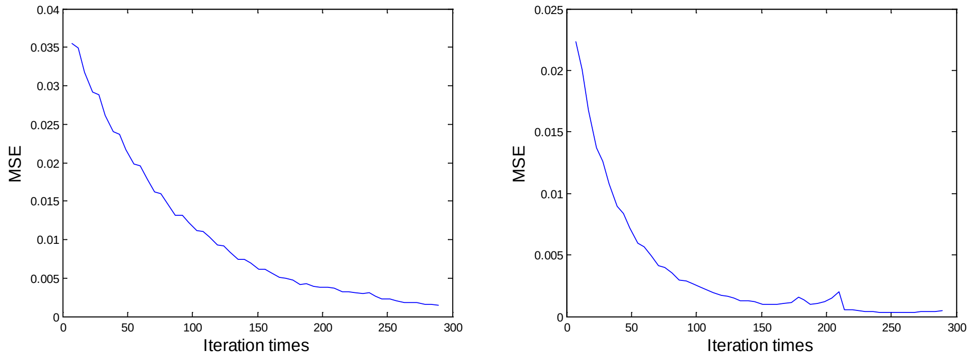
(a) The AM signal (b) AM signal with impulsive noise

Fig.3. The power density of the signals



(a) Proposed LMS algorithm (b) proposed RLS algorithm

Fig.4. The power density of the signals after noise reduction



(a) Proposed LMS algorithm (b) proposed RLS algorithm

Fig.5. The convergence of proposed FLOS based LMS and RLS algorithms in impulsive noise

Conclusion

In this paper, we analyze the adaptive impulsive noise reduction methods. It is shown that the conventional LMS and RLS algorithms are not robust to impulsive noise, two new FLOS based adaptive algorithms are proposed. The proposed algorithms are not only robust to the alpha stable impulsive noise, but also easier to implement than the conventional FLOS based methods. Simulation results illustrate the robustness and effectiveness of the proposed methods against the impulsive noise.

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