The Evolutionary Relationships Between Tree Type of Network Models For The Internet of Things

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Abstract. One of efficient and powerful methods for stimulating real networks and tracking motion regulation of networks is to build up some network models. Investigating topological structure of networks and building up network models play an important role for understanding more microscopic and structural features of etworks. A class of growing tree type of network models has been constructed in this article. We focus on building up such models by constructive operations of graph theory, and try to determine topulogical properties of the models.

Introduction

Complex networks have become a hot academic research. One of the main reasons behind the popularity of complex network is their flexibility and generality for signifying real systems in nature and society and the Internet of Things. Many scholars have done a large amount of empirical research to optical communicationsand networking, mobile and wireless networks security in real-life networks. There are two network models: One is the scale-free distribution of degree, which implies that many real-life networks display a power law degree distribution, and another is the small-world behavior that characterizes those networks having an average clustering coefficient much larger than that of random networks, and at the same time the average shortest path distance is relatively small compared with the size of the network.

In 1998, Watts and Strogatz proposed the pioneering small-world network model [1]. Barabási and Albert presented the scale-free network model in 1999,, which has a degree distribution of power law form [2,3,4]. Inspired by small-world and scale-free two typical network models, Over the past decade, the academic community made a number of stochastic models to build scale-free or small-world networks and explain their topology in real systems. However, the formation mechanism of various real networks are distinct, there exists diversity, richness and complexity. Existing models is not far from contain the formation mechanism and methods of the real system. Currently, many real systems are not involved in studying complex networks. Apply the views of complex networks in these systems research, capture its formation mechanism, set the appropriate evolution model, and discuss the properties of the model on these basis, is of great theoretical and practical value. Nevertheless, in general, the network are connected, the spanning tree is one powerful tool for researching connectivity graph theory, which has been successfully applied to the study of practical problems, accumulated a large number of theoretical results [5,6,7]. One of the most typical applications of spanning tree, Radia Perlman utilized the structural relationship between the spanning tree and network and invent the spanning tree protocol which has been widely applied in network bridge and exchange board [8]. Looking spanning tree of scale-free network model for practical application provide guidance. Many authors consider the spanning trees of wireless sensor networks help us to establish a fully connected vertices with minimal relay vertices and base stations to ensure that, in order to improve network survivability [9-13]. The literature [14] gives some conclusions on spanning tree. There is no doubt that studying complex network will bring new problems and new issues to mathematical body, then resulting in a new research content, the accumulation and sublimation of research achievement will lead to new branches of mathematics in [15].

In general, people presented the construction process of network model, in order to optimize the utilization of the resources, by finding the spanning tree of network model to improve operation speed of the network model. In this paper, We focus on building up several tree network models by constructive operations of graph theory, and try to determine some properties of these tree network models, and then analyze the differences among these tree network models. Besides, we analysis the vertex, edge and cumulative distribution relationship among these tree network models, and then find out the leaf set and the maximum diameter of them. By reviewing some advances in the area we wish to convey the potential for understanding complex systems though the evolutionary relationship of the tree networks behind them.

Main Conclusion

For an initial network T_0 is a tree which contains q vertices and l edges, the notation $n_v(0)$ and $n_e(0)$ represents the number of vertices and edges of T_0. $V(0)$ and $E(0)$ denote the vertex set and edge set of T_0, respectively. Clearly, $n_v(0) = |V(0)|$, $n_e(0)=|E(0)|$. The notation $k(u, i)$ represents the number of edges connected with the vertex u in a tree network model at time step in time i.

***N*(*t*)-tree network model.** For each vertex $u \in V_N(0)$ of tree $T_0=N(0)$, add m vertices, and join these m vertices with u, the resulting tree network is denoted as $N(1)$, the notation X_{N_1} the set of new vertices adding to $N(0)$, Y_{N_1} the set of new edges adding to $N(0)$. Then we have

$Y_{N_1}=\{wu: u \in V_N(0), w \in X_{N_1}\}$, and $V_N(1)=V_N(0) \cup X_{N_1}$, $E_N(1)=E_N(0) \cup Y_{N_1}$, $Y_{N_1} = X_{N_1} = mn_v(0)$.

Similarly, for each vertex $v \in V_N(1)$ of the tree $N(1)$, we add m vertices, and join these vertices with v, thereby we obtain the tree network $N(2)$. The rest can be done in the same manner, according to the above-described structure, add m vertices for each vertex $x \in V_N(t-1)$ of tree $N(t-1)$, thus, the resulting tree network is denoted as $N(t)$. We called $N(t)$-tree network model. Since $n_v(1) = n_v(0) + mn_v(0)=(m+1)q$, $n_e(1)= n_e(0)+ mn_v(0)=mq+l$, $Y_{N_t} = X_{N_t}$

It is not difficult to calculate some basic parameters of $N(t)$-tree network model. when $t \geq 1$, it is easy to obtain the number of vertices and edges of $N(t)$-tree network model.

$$n_v(t) = n_v(t-1) + mn_v(t-1)=(m+1)^t n_v(0)= (m+1)^t q,\ n_e(t)= n_e(t-1)+ mn_v(t-1)= (m+1)^t q-1, \quad (1)$$

In addition, the number of newly added vertices of $N(i)$-tree network model

$$\left|X_{N_i}\right| = n_v(i)-n_v(i-1)=mq(m+1)^{i-1} \qquad i=1,2,\ldots,t. \quad (2)$$

In $N(t)$, the maximum degree is $\Delta(N(t))=\Delta(T_0)+tm$, the minimum degree is $\delta(N(t))=1$. The average degree $\langle k\rangle$ of $N(t)$-tree network model $\langle k\rangle=2n_e(t)/n_v(t)\to 2$ as $t\to\infty$., which implies that the tree network is sparse network with as few links as possible. According to the construction of the $N(t)$-tree network model, it is not difficult to find that the network was discussed in [16] when the tree network T_0 is a vertex and no edges.

***M*(*t*)-tree network model.** For each vertex $u \in V_M(0)$ of initial tree $T_0=M(0)$, mp newly added vertices, where p represents the probability, and join these mp vertices with u, and then the resulting tree network $M(1)$, the notation X_{M_1} the set of new vertices adding to $M(0)$ to form $M(1)$, Y_{M_1} the set of new edges adding to $M(0)$ to produce $M(1)$, there are $Y_{M_1}=\{wu: u \in V_M(0), w \in X_{M_1}\}$, and $V_M(1)=V_M(0) \cup X_{M_1}$, $E_M(1)=E_M(0) \cup Y_{M_1}$, $Y_{M_1} = X_{M_1} = mpm_v(0)$.

Analogously, for each vertex $v \in V_M(1)$ of tree network $M(1)$, we add mp vertices, and join these mp vertices with v, therefore the resulting tree network $M(2)$. By the parity of reasoning, on the basis of the above-mentioned structure, add mp vertices for each vertex $x \in V_M(t-1)$ of tree network $M(t-1)$. Thus, we can obtain tree network $M(t)$ from tree network $M(t\text{-}1)$. We named $M(t)$-tree network model. Since $m_v(1)= m_v(0)+mpm_v(0)=(mp+1)q$, $m_e(1)= m_e(0)+ mpm_v(0)=mpq+l$, $Y_{M_t} = X_{M_t}$

It is easily to compute some basic parameters $M(t)$-tree network model. For $t \geq 1$, it is easy to obtain the number of vertices and edges of $M(t)$-tree network model

$$m_v(t) = m_v(t-1) + mm_v(t-1)=(m+1)^t m_v(0)= (mp+1)^t q,\ n_e(t)= n_e(t-1)+ mn_v(t-1)= (mp+1)^t q-1, \quad (3)$$

Besides, the number of newly added vertices of $M(i)$-tree network model

$$\left|X_{M_i}\right|=m_v(i)-m_v(i-1)=mpq(mp+1)^{i-1} \qquad i=1,2,\ldots,t. \tag{4}$$

In $M(t)$, $\Delta(M(t))=\Delta(T_0)+tmp$ is the maximum degree, $\delta(M(t))=1$ is the minimum degree. The average degree $\langle k\rangle$ of $M(t)$-tree network model $\langle k\rangle=2\, m_e(t)\,/m_v(t) \rightarrow 2$ as $t\rightarrow\infty$. We can see that the proposed tree network model is a sparse tree with as few links as possible.

***R*(*t*)-tree network model.** For initial tree $T_0 = R(0)$, we add $mk(u,0)$ vertices for each vertex $u\in V_R(0)$, and connect each of them with u, we obtain the tree network $R(1)$. Let X_{R_1} be the set of vertices newly added into $R(0)$, and let Y_{R_1} be the set of edges newly added into $R(0)$. Thus we have

$$Y_{R_1}=\{wu\colon u\in V_R(0),\, w\in X_{R_1}\}, \text{ and } V_R(1) = V_R(0)\cup X_{R_1},\, E_R(1)= E_R(0)\cup Y_{R_1},\, Y_{R_1} = X_{R_1} =2mr_e(0).$$

Similarly, add $mk(v,1)$ vertices for each vertex $v\in V_R(1)$ of the tree network $R(1)$, and connect each of them with v, the resulting tree network $R(2)$. And so on, in the light of the above iteration process, we add $mk(u,t\text{-}1)$ vertices for each vertex $x\in V_R(t\text{-}1)$, thus we obtain tree network $R(t)$ from tree network $R(t\text{-}1)$. It denotes as $R(t)$-tree network model. Obviously,

$$r_v(1)=r_v(0)+mr_e(0)=2ml+q,\ r_e(1) =r_e(0)+2mr_e(0)=(2m+1)l,\, Y_{R_t} = X_{R_t}.$$

From the above steps, we can easily find some basic parameters of $R(t)$-tree network model. when $t\geq1$, it is easy to generate the number of vertices and edges of $R(t)$-tree network model.

$$r_v(t) = r_v(t\text{-}1) +2mr_e(t\text{-}1)=(2m+1)^t\, r_e(0)= (2m+1)^t l+1\ \ r_e(t)= r_e(t-1)+2mr_e(t-1)= (2m+1)^t l, \tag{5}$$

Furthermore, the number of newly added vertices of $R(i)$-tree network model

$$\left|X_{R_i}\right| = r_v(i)-r_v(i-1)=2ml(2m+1)^{i-1},\ i=1,2,\ldots, t. \tag{6}$$

In $R(t)$, the maximum degree is $\Delta(R(t))=(m+1)^{t}\Delta(T_0)$, the minimum degree is $\delta(R(t))=1$. The average degree $\langle k\rangle$ of $R(t)$-tree network model $\langle k\rangle=2\, r_e(t)\,/r_v(t) \rightarrow 2$ as $t\rightarrow\infty$, which implies that is a sparse tree network. According to the construction of the $R(t)$-tree network model, the network in coincide with the phenomenon that "the richer get richer and the poorer get poorer".

***Q*(*t*)-tree network model.** For initial tree $T_0 = Q(0)$, add $mh(b)k(u,0)$ vertices for each vertex $u\in V_Q(0)$, where $h(b)$ is environment factor and $h(b)>0$, and connect these vertices with u, produces the tree network $Q(1)$ at time step $t=1$. Let X_{Q_1} be the set of vertices newly added into $Q(0)$, and let Y_{Q_1} be the set of edges newly added into $Q(0)$. Therefore, $Y_{Q_1}=\{wu\colon u\in V_Q(0), w\in X_{Q_1}\}$, and

$$V_Q(1)= V_Q(0)\cup X_{Q_1},\, E_Q(1)= E_Q(0)\cup Y_{Q_1},\, Y_{Q_1} = X_{Q_1} =2mh(b)q_e(0).$$

Similarly, add $mh(b)k(v,1)$ vertices for each vertex $v\in V_Q(1)$ of the tree network $Q(1)$, and connect each of them with v, the resulting tree network $Q(2)$. And so on, in the light of the evolution process of the model, we add $m\ h(b)k(u,t-1)$ vertices for each vertex $x\in V_Q(t-1)$, thus we obtain tree network $Q(t)$ from tree network $Q(t-1)$. It denotes as $Q(t)$-tree network model. Since $q_v(1)=q_v(0)+2mh(b)$, $q_e(0)=2mlh(b)+q$, $q_e(1)=q_e(0)+2\,h(b)r_e(0)=2mh(b)\mathrm{l}+l$, $Y_{Q_t} = X_{Q_t}$.

From above description, we can count the order and size of the $Q(t)$-tree network model. when $t\geq1$, it is easy to figure up the number of vertices and edges of $Q(t)$-tree network model

$$\begin{gathered} q_v(t) = q_v(t-1)+2mh(b)q_e(t-1)=(2m\,h(b)+1)^t \\ q_e(0)= (2mh(b)+1)^t l+1\ \ q_e(t)= q_e(t\text{-}1)+2mh(b)q_e(t-1)= (2m\,h(b)+1)^t l, \end{gathered} \tag{7}$$

Moreover, the number of newly added vertices of $Q(i)$-tree network model

$$\left|X_{Q_i}\right|=q_v(i)-q_v(i-1)=2mh(b)l(2mh(b)+1)^{i-1},\ i=1,2,\ldots, t. \tag{8}$$

In $Q(t)$, the maximum degree is $\Delta(Q(t))=(m\,h(b)+1)^t\Delta(T_0)$, the minimum degree is $\delta(Q(t))=1$.The average degree $\langle k\rangle$ of $Q(t)$-tree network model $\langle k\rangle=2q_e(t)/q_v(t)\rightarrow2$ as $t\rightarrow\infty$. the resulting network is a sparse tree whose vertices have many fewer connection than is possible.

The distribution of the tree network model. Let $d_1, d_2,\cdots, d_a$ be the different degree of vertex of the initial tree network T_0, without loss of generality $d_1 < d_2<\cdots < d_a$. let $n_{d_j}(0)$ denotes the number of degree d_j in T_0, where $j = 1,2, \cdots, a$. According to the structure of tree network model, we can see that the model is connected and acyclic.

The distribution of *N*(*t*)-tree network model. By the above facts and $\left|X_{N_i}\right|=mq(m+1)^{i-1}$, we are not difficult to obtain the degree spectrum of $N(t)$-tree network model at time step $t\geq 1$ as follows.

d	1	$m+1$	$2m+1$	$\cdots$	$(t-1)m+1$	$tm+d_1$	$tm+d_2$	$\cdots$	$tm+d_a$
$n_d(t)$	$\left\|X_{N_i}\right\|$	$\left\|X_{N_{i-1}}\right\|$	$\left\|X_{N_{t-2}}\right\|$	$\cdots$	$\left\|X_{N_1}\right\|$	$n_{d_1}(0)$	$n_{d_2}(0)$	$\cdots$	$n_{d_a}(0)$

Clearly, it shows that the degree spectrum of $N(t)$-tree network model is discrete. By the method used in [17]. We compute $P_{cum}^{N}(k)$ in the following. The cumulative distribution is given by

$$P_{cum}^{N}(k)=\frac{1}{n_v(t)}[n_v(0)+\sum_{j=0}^{\tau}\left|X_j\right|]=\frac{(m+1)^{\tau}q}{(m+1)^{t}q}\propto(m+1)^{\tau-t}$$

Substituting $\tau=t-(k-1)/m$ in the above expression, then we have $(m+1)^{\tau-t}\propto(m+1)^{-(k-1)/m}$. Hence $P_{cum}^{N}(k)\propto(m+1)^{-(k-1)/m}$. Obviously, when the size of the network is large, the cumulative degree distribution $P_{cum}^{N}(k)$ is a power of degree k.

The distribution of *M*(*t*)-tree network model. By the above analysis and $X_{M_i}=mpq(mp+1)^{i-1}$. For the purpose of simplicity we define a function $f(x)=(t-x)mp+1$ which denote the degree enter in the tree network at time step t. We can obtain the degree spectrum of the $M(t)$-tree network model.

d	1	$f(1)$	$f(2)$	$\cdots$	$f(-1)$	$tmp+d_1$	$tmp+d_2$	$\cdots$	$tmp+d_a$
$n_d(t)$	$\left\|X_{M_i}\right\|$	$\left\|X_{M_{i-1}}\right\|$	$\left\|X_{M_{t-2}}\right\|$	$\cdots$	$\left\|X_{M_1}\right\|$	$n_{d_1}(0)$	$n_{d_2}(0)$	$\cdots$	$n_{d_a}(0)$

Due to the discreteness of this degree spectrum, it is convenient to obtain the cumulative distribution $P_{cum}^{M}(k)$, according to the statistical techniques used in literature[17], it is the cumulative distribution.

$$P_{cum}^{M}(k)=\frac{1}{m_v(t)}[m_v(0)+\sum_{j=0}^{\tau}\left|X_j\right|]=\frac{q(mp+1)^{\tau}}{q(mp+1)^{t}}\propto(mp+1)^{\tau-t}$$

Using $\tau=t-(k-1)/mp$, then we have $(mp+1)^{\tau-t}\propto(mp+1)^{-(k-1)/mp}$, $P_{cum}^{M}(k)\propto(mp+1)^{-(k-1)/mp}$. It indicates that the $M(t)$-tree network model obeys the exponential law form.

The distribution of *R*(*t*)-tree network model. By the above analysis and $\left|X_{R_i}\right|=2ml(2m+1)^{i-1}$, we can easily to gain the degree spectrum of the$R(t)$-tree network model

d	1	$m+1$	$(m+1)^2$	$\cdots$	$(m+1)^{t-1}$	$(m+1)^t d_1$	$(m+1)^t d_2$	$\cdots$	$(m+1)^t d_a$
$n_d(t)$	$\left\|X_{R_i}\right\|$	$\left\|X_{R_{i-1}}\right\|$	$\left\|X_{R_{t-2}}\right\|$	$\cdots$	$\left\|X_{R_1}\right\|$	$n_{d_1}(0)$	$n_{d_2}(0)$	$\cdots$	$n_{d_a}(0)$

Note that the degree spectrum of $R(t)$-tree network model is discrete, according to the statistical techniques used in [17], it is the cumulative distribution

$$P_{cum}^{R}(k)=\frac{1}{r_v(t)}[r_v(0)+\sum_{j=0}^{\tau}\left|X_j\right|=\frac{(2m+1)^{\tau}l+1}{(2m+1)^{t}l+1}\propto(2m+1)^{\tau-t}$$

Plugging $\tau=t-\ln k/\ln(m+1)$, thus we obtain $(2m+1)^{\tau-t}\propto(2m+1)^{-\ln k/\ln(m+1)}$,therefore $P_{cum}^{R}(k)\propto k^{-\ln(2m+1)/\ln(m+1)}$. It indicates that the $R(t)$-tree network model obeys the power law, and is a scale-free tree.

The distribution of *Q*(*t*)-tree network model. By the above analysis and $\left|X_{Q_i}\right|=2mlh(b)(2m\,h(b)+1)^{i-1}$. we can easily to gain the degree spectrum of $Q(t)$-tree network model

d	1	$mh(b)+1$	$(mh(b)+1)^2$	$\cdots$	$(mh(b)+1)^{t-1}$	$(mh(b)+1)^t d_1$	$(mh(b)+1)^t d_2$	$\cdots$	$(mh(b+1)^t d_a$
$n_d(t)$	$\left\|X_{Q_i}\right\|$	$\left\|X_{Q_{i-1}}\right\|$	$\left\|X_{Q_{t-2}}\right\|$	$\cdots$	$\left\|X_{Q_1}\right\|$	$n_{d_1}(0)$	$n_{d_2}(0)$	$\cdots$	$n_{d_a}(0)$

Note that the degree spectrum of $R(t)$-tree network model is discrete, according to the statistical techniques used in [17], it is the cumulative distribution

$$P_{cum}^{Q}(k)=\frac{1}{q_v(t)}\sum_{j=0}^{\tau}q_v(j)=\frac{(2mh(b)+1)^{\tau}l}{(2mh(b)+1)^{t}l+1}\propto[2mh(b)+1]^{\tau-t}$$

Substituting $\tau=t-\ln k/\ln(mh(b)+1)$, thus we have $(2mh(b)+1)^{\tau-t}\propto[2mh(b)+1]^{-\ln k/\ln(mh(b)+1)}$, so $P_{cum}^{Q}(k)\propto k^{-\ln(2mh(b)+1)/\ln(mh(b)+1)}$. It indicates that the $Q(t)$-tree network model exhibits the power law, which has the same degree exponent as the scale-free network model.

The evolutionary relationship between the tree type of network models

The vertex and edge relationship among tree type of network models. Since the number of vertices and edges of the $N(t)$-tree network model respectively, $n_v(t)=(m+1)^t q$, $n_e(t)=(m+1)^t q-1$. The number of vertices and edges of the $M(t)$-tree network model respectively, $m_v(t)=(mp+1)^t q$, $m_e(t)=(mp+1)^t q-1$. It implies that when $p=1$, the number of vertices and edges of the $N(t)$-tree network model equal to the number of vertices and edges of the $M(t)$-tree network model. Besides, the number of vertices and edges of the $R(t)$-tree network model respectively, $r_v(t)=(2m+1)^t l+1$, $m_e(t)=(2m+1)^t l$, the number of vertices and edges of the $Q(t)$-tree network model respectively, $q_v(t)=(2mh(b)+1)^t l+1$, $q_e(t)=(2mh(b)+1)^t l$. It means that when $h(b)=1$, the number of vertices and edges of the $R(t)$-tree network model equals to the number of vertices and edges of the $Q(t)$-tree network model.

The cumulative distribution relationship among tree network models. According to the above computation the $P_{cum}^{N}(k), P_{cum}^{M}(k)$ obeys the exponent law, $P_{cum}^{R}(k), P_{cum}^{Q}(k)$ exhibits the power law, belongs to scale-free network model. Because the $N(t)$-tree network model and the $R(t)$-tree network model have no relationship with the probability and the environmental factors, we called this regular network model. The $M(t)$-tree network model and the $Q(t)$-tree network model have an relation on probability and the environmental factors, we call them the *irregular network models*.

The leaf sets and diameters of four models. The leaf sets of four models are $X_{N_t}, X_{M_t}, X_{R_t}, X_{Q_t}$, respectively. According to the increasing ways we know that the diameters of four models are identical. Hence, we only give a sharp proof of the $N(t)$-tree network model. Since we can obtain the network $N(t)$ from the network $N(t-1)$, the diameter of the initial tree T_0 is equal to $D(T_0)$. At each time step $t\geq1$, we can easily see that the diameter always lies between a pair of vertices that have just been created at this time step. We will call such newly-created vertices the *outer vertices*. At each time step $t\geq1$, we note that an outer vertices cannot be connected with two or more vertices that are created during the same time step $t'\geq t-1$. Indeed, we know that from step 1, no outer vertex is connected to a vertex of the initial network T_0. Thus, for any step $t\geq1$, any outer vertex is connected with the vertices that appeared as pairwise different steps. Now consider two outer vertices are created at time step $t\geq1$, say u_t and v_t. Then v_t is connected to the last layer vertices, which is must be created before or at time step $t-1$. We repeat this argument, if we make m jumps, from u_t we arrive in the initial T_0, in which we reach any v_t by using an edge of T_0 and making m jumps to v_t in a similar way. Thus the diameter $D(N(t))\leq D(T_0)+2t$.

Conclusion

We have constructed four tree type of network models, tand given the distribution of these models, verified the scale-free properties on them, found out the leaf set and the maximum diameter of the model, and analyzed the relationship among four models. The $M(t)$-tree network model and the $Q(t)$-tree network model can be established in the research of non-tree network model has been widely applied in many areas. These two models not only provide a reliable theoretical basis for studying of the actual network construction, but also provides the efficient tool which easy to understand and master for the actual network. As a further study, we can consider the network model from the tree network model to the general network model, tree network model with randomly add or delete some edges, whether have a decisive function for the initial network model. So one can better simulate actual network in real-life network models as diverse as the cell or WWW.

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