Dynamics and Vibration Analysis of Delta Robot

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Abstract. This paper deals with the dynamics and vibration analysis of an industrial Delta robot when performing high-speed pick-and-place operations. The transitions between the vertical and the horizontal segments of the pick-and-place trajectories are smoothed using either Lamé or cam curves. Using a simplified dynamics model of Delta robot, energy-based cost functions are defined and utilized to optimize Lamé and CAM curve's geometric parameters. A complete multi-body dynamic model of Delta robot is established with SolidWorks, and then transferred to ADAMS for simulation purpose. These two types of trajectories are compared in terms of residual vibrations and tracking accuracy. The results obtained could lead to practical merits in engineering applications.

1. Introduction

Delta parallel robot is a highly-used commercial parallel robot which has 3-DOF translational motion with high-speed capability. It was invented by Clavel early in 1985[1].This parallel robot has many advantages, such as large loading ability, high precision, simple inverse kinematic solutions , minor error, small deadweight load ratio, good dynamic performance, control ease and so on[2][3].There were several ways proposedfor inverse problem solution. The ways of getting direct solution are either Numerical Solution or Analytical Method, which are complex, but using the ADAMS software to get the forward solution is much easier. It will not generate any errors if the model of the robot is correct [4, 5, 6, 7].

This paper aims to build up a Delta parallel Robot by mechanism of constrain Equations. In Sec. A, further introduction and the application of Lamé curves [8] in high speed PPO trajectories are presented. In Sec. B, complete Mechanism of Delta parallel Robot by using SolidWorks and ADAMS software are presented in Sec. C, procedure of get the solution by using ADAMS software and Lamé curve and CAM curve trajectoriesand Direct solutions are followed by Sec. D, the simulation of Lamé and CAM curves related with acceleration and movement of position are discussed in Sec. E, the trajectories generated are tested on a physical prototype of Delta robot and the experiment results verify the effectiveness of the optimized trajectory and vibration analysis are define in Sec. F. Finally, the conclusion and the future research are discussed in Sec. G.

2. Modeling of Delta Parallel Robot

2.1 The Mechanism Constraint Equation.

Delta parallel robot is made up of one static platform, one moving platform, three driving rods and three driven mobile parallelograms branch chains (Fig.1 is its structure diagram).The three edges of the static platform have the same kinematic chain join to the moving platform. Each kinematic chain is a closed-loop parallelogram branch chain which is joined with two driven rods by four spherical hinge, this close-loop and one driving rodith rotation joint constitute a series mechanism, the driving rod is fixed to the static platform, the driving rod swing repeat by the drive of determined by these three kinematics chains. The moving platform has 3-DOF translational motion, which means that it can move only along the direction of x, y, z -axis in the space coordinate and can't rotate around the x, y, z- axis.

As shown in Figure 1, *O* is the center of the static platform, and *O'* is the center of the moving platform,A_i $B_i$are the driving rods with the length of l_1, B_i $C_i$are the driven rods with the length of l_2. To calculate expediently, setting up coordinate system on both staticplatform and moving platform named *O* - *XYZ* and*O'*- *X' Y' Z'*,q_1, q_2, $q_3$are the field angle betweenthe *Z* -axis and A_i B_i.

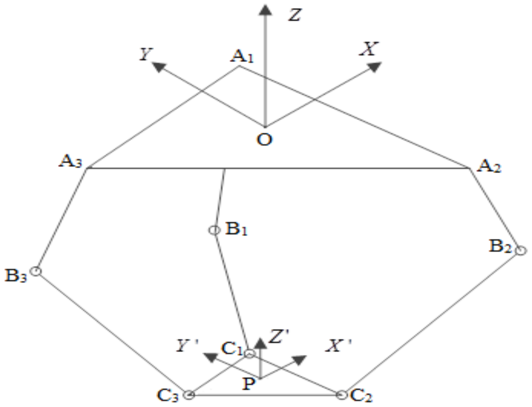

Figure 1. Kinematics Diagram of Delta Parallel Robot.

Let OA_i= *R*, OC_i= *r*, the coordinate of center*O'* in coordinate system $O-XYZ$ is$[xyz]^T$, and thenthe position vector of $A_i$in$O-XYZ$are:

$a_{iO}=\begin{bmatrix} R\cos\alpha_i \\ R\sin\alpha_i \\ 0 \end{bmatrix}$, among them$\alpha_i=\frac{4i-3}{6}\pi,\ Here\ (i=1,2,3)$, in the same time the position vector of C_i in coordinate system $O-XYZ$ can also be gained:

$c_{iO'}=\begin{bmatrix} r\cos\alpha_i \\ r\sin\alpha_i \\ 0 \end{bmatrix}$, and the same$\alpha_i=\frac{4i-3}{6}\pi,\ Here\ (i=1,2,3)$, by geometric relation, the position vector of B_i in coordinate system $O-XYZ$can also be expressed as:

$$B_{iO'}=\begin{bmatrix} (R+l_1\sin\theta_i)\cos\alpha_i \\ (R+l_1\sin\theta_i)\sin\alpha_i \\ -l_1\cos\theta_i \end{bmatrix}$$

Let's put coordinate of vector $\overline{OO'}$ is $C_o=[xyz]^T$ in coordinate system$O-XYZ$, then $\overline{O'C_\iota}$ can be expressed as: $c_{iO}=\begin{bmatrix} r\cos\alpha_i+x \\ r\sin\alpha_i+y \\ z \end{bmatrix}$,

So, based on$|C_iB_i|=l_2$, can be derived as:

$$[(R+l_1\sin\theta_i-r)\cos\alpha_i-x]^2+[(R+l_1\sin\theta_i-r)\sin\alpha_i-y]^2+(-l_1\cos\theta_i-z)^2={l_2}^2 \qquad (1)$$

Let be $F_i=[(R+l_1\sin\theta_i-r)\cos\alpha_i-x]^2+[(R+l_1\sin\theta_i-r)\sin\alpha_i-y]^2+(-l_1\cos\theta_i-z)^2-{l_2}^2=0 \qquad (2)$

After calculating the T from the equation (1), the *Jacobian* matrix can be gain as: $T=A^{-1}B$.So,

$A=\begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} & \frac{\partial F_3}{\partial z} \end{bmatrix}$, and as well $B=-\begin{bmatrix} \frac{\partial F_1}{\partial \theta_1} & & \\ & \frac{\partial F_2}{\partial \theta_2} & \\ & & \frac{\partial F_3}{\partial \theta_3} \end{bmatrix}$, so,

$\frac{\partial F_i}{\partial x}=[(R+l_1\sin\theta_i-r)\cos\alpha_i-x]$,

$\frac{\partial F_i}{\partial y}=[(R+l_1\sin\theta_i-r)\sin\alpha_i-y]$,

$\frac{\partial F_i}{\partial z}=z+l_1\cos\theta_i$,

Let

$M=(R+l_1\sin\theta_i-r)\cos\alpha_i$,

$N = (R + l_1 \sin\theta_i - r)\sin\alpha_i$,
$and\ P = -l_1 \cos\theta_i$,
Then:

$$\frac{\partial F_i}{\partial \theta_i} = \frac{\partial F_i}{\partial M_i}\frac{\partial M_i}{\partial \theta_i} + \frac{\partial F_i}{\partial N_i}\frac{\partial N_i}{\partial \theta_i} + \frac{\partial F_i}{\partial P_i}\frac{\partial P_i}{\partial \theta_i}$$
$$= -xl_1\cos\theta_i\cos\alpha_i - yl_1\cos\theta_i\sin\alpha_i - zl_1\sin\theta_i + (R - r)l_1\cos\theta_i$$

Is the required mechanism constraint equation.

2.2 Lamé Curves.

A Lamé curve, also known as a super ellipse is defined in the Cartesian coordinate system as:

$$\left(\frac{u}{d}\right)^m + \left(\frac{v}{e}\right)^m = 1 \quad d > 0, e > 0, m = 1,2, \ldots (1)$$

Where d and e are called the *semi-diameters* of the curve. The shape of a Lamé curve varies as the order m. Take the first quadrant for example. As shown in Figure 2, when m is 1, the curve is a straight line. When m is 2, the curve becomes a quarter of an ellipse. As m goes to positive infinity, the curve gets very close to two orthogonal straight lines, which are actually a quarter of a rectangle.

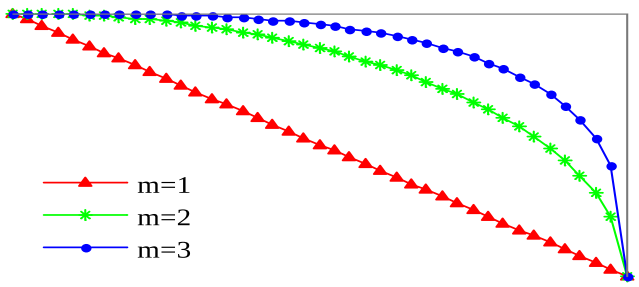

Figure 2. Lamé curve

In this paper, Lamé curves with order 3 are used as the transitions between the horizontal and the vertical segments of the PPO trajectory. The cubic Lamé curve is the smallest curve in its family that allows for G2 continuity when blended with lines. In additional, compared with higher order curves (m>3), the computation load of the cubic Lamé curve is relatively small.

Since the PPO trajectory is geometrically symmetric, the following analysis only considers the right part of the path. As shown in Figure 3, curve *ABCDE* is half of the trajectory with the starting point *A* and the middle point *E*. In the trajectory fixed coordinate frame *U*-*V*, line segments $|CE| = a/2$ and $|AC|=b$, where a, b are the lengths of the horizontal segment and the vertical in the Adept cycle respectively. The square corner *BCD* is now smoothed by using a cubic Lamé curve in the first quadrant, with semi-diameters $|OB|=|CD|=d$ and $|OD|=|BC|=e$. As for other parameters, $|AB|=h$, $|DE|=c$, and it holds true that $d = a/2 - c$, $e = b - h$.

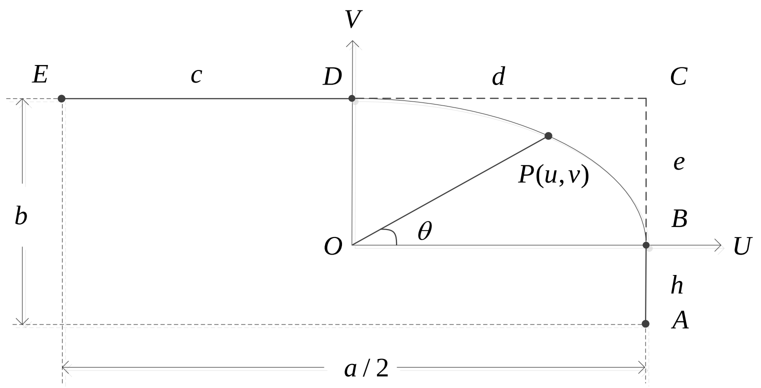

Figure 3. Half PPO Trajectory Smoothed Using Lamé curves

By limiting the cubic Lamé curve in the first quadrant, the absolute sign in Eq. (1) can be allotted with, thus making the curve analytical everywhere. Detailed discussion on the geometry of the cubic

Lamé curve can be found in Ref. [9]. Properties that are necessary for trajectory planning are listed as below:

The coordinates of a point *P* on the curve as shown in Fig.3, can be represented in analytical forms of θ, which is measured counterclockwise in coordinate frame *U-V*.

$$u(\theta) = \frac{d}{(1+\tan^3\theta)^{\frac{1}{3}}} \quad (2\text{-a})$$

$$v(\theta) = \frac{e\tan\theta}{(1+\tan^3\theta)^{\frac{1}{3}}} \quad (2\text{-b})$$

The length of the curve l_{BD} does not have a closed form, but can be written in an integration form of θ as:

$$l_{BD} = \int_0^{\frac{\pi}{4}} \frac{1+\tan^2\theta}{(1+\tan^3\theta)^{4/3}} \sqrt{d^2\tan^4\theta + e^2 d\theta} + \int_0^{\frac{\pi}{4}} \frac{1+\tan^2\theta}{(1+\tan^3\theta)^{4/3}} \sqrt{e^2\tan^4\theta + d^2 d\theta} \quad (3)$$

2.3 Simulation by SolidWorks and ADAMS software.

SolidWorks is the powerful software, especially in designing of parametric feature modeling of 3D design, feature modeling not only describes the information of the geometric shape but also highly expresses the function information of the products, and SolidWorks software have so powerful assembling function that the designed parts have beautiful shape and good visibility. Using the SolidWorks software to build the 3D model of Delta parallel robot can avoid repeating modeling work for assembling the part and a lot easier than the other software, According to the above described frame of Delta parallel robot, built the 3D model show as in Figure 4.

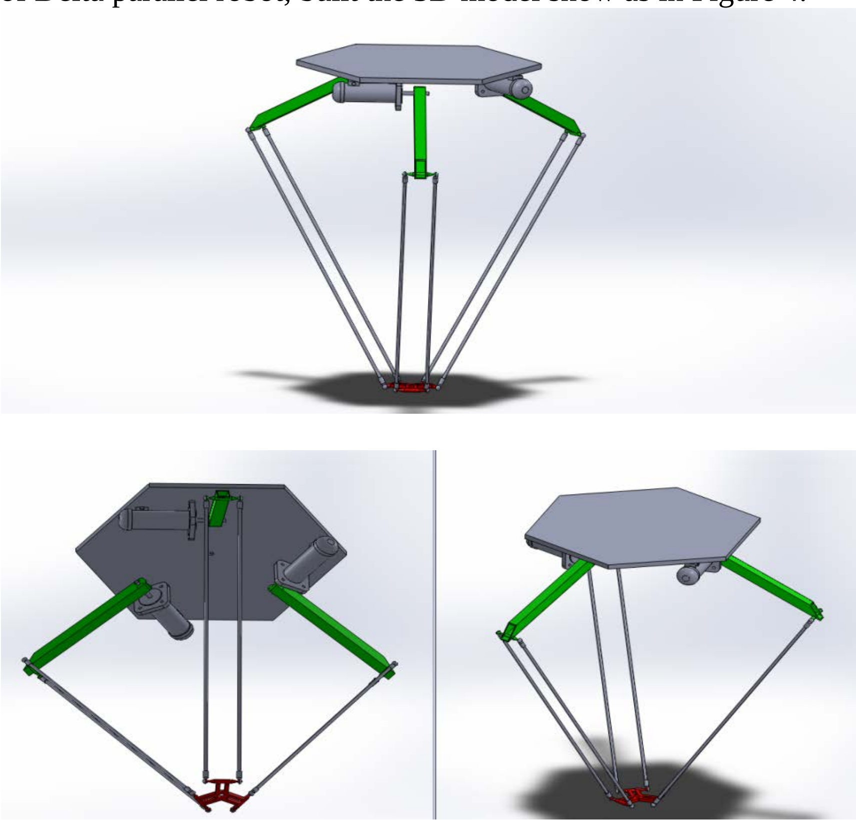

Figure 4. Delta parallel robot mechanism

The grey platform is static platform, the red platform is moving, the green rods are three driving rods, grey rods are six driven rods, and from their both ends they are connected with driving rods and with moving platform by using ball joints. The detailed parameters are shown in the following Table 1.

Here, r_1 is circumradius of static platform,

And r_2 is circumradius of moving platform.

MECHANISM/solid module is the interface between ADAMS software and SolidWorks software, the two use seamless interface and user can do according to the model define mechanical system and perform the simulation of kinematics and dynamics without exiting the application environment, and the model can also be transferred to ADAMS/View for the following comprehensive kinematics analysis. The Delta model in ADAMS can be shown as Figure 5.

Table 1.
PARAMETERS OF DELTA PARALLEL ROBOT

Structure Parameters	Length/mm
Driving Rod	351
Driven Rod	831.7
r1	200
r2	57.45

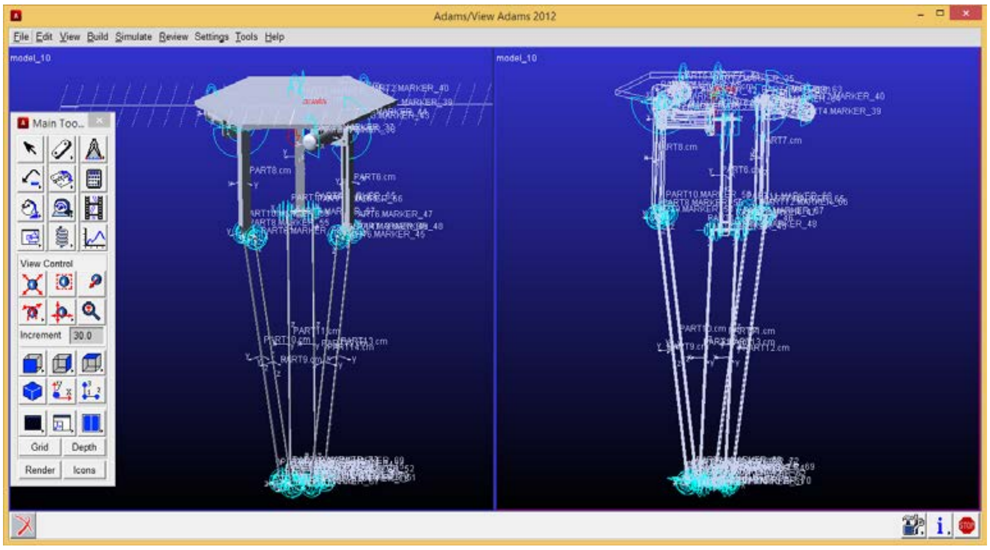

Figure 5. Delta parallel robot mechanism model and its Render form

2.4 Solution of Delta Parallel Robot:

The position kinematics problem of parallel robot including two facts: the inverse solution and the forward solution [10], generally speaking, the inverse solution is much easier to get [11], that we can say as, the position parameters of moving platform to seek the position parameters of the input joints, on the contrary, the forward solution is known the position parameters of input joints to seek the position parameters of moving platform which is hard [12], proposed a geometric solution of direct solution of Delta parallel robot, but using the ADAMS software to solve the forward solution is quite easy. The following figure 6 is the flow chart of direct solution.

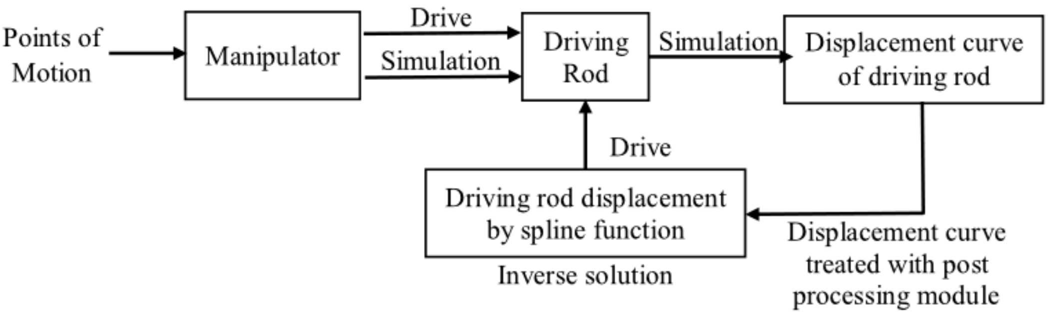

Figure 6. Flow chart of Direct Solution

2.5 Lamé and CAM curve simulation

After getting the forward solution of *pick and place* by using Lamé curve and CAM curve method from MATLAB software, the Lamé curve shown in the following Figure 7. , then save the measured curves Data and then it will be send to the post processing module, change the three curves data by using Lamé and CAM curves into spline curves data, in this way the discrete data of the driving joint and moveable platform of the Delta parallel robot during simulation by ADAMS software can be gain, the treated spline curves can be used as driving function, for the known condition to seek the forward solution of Delta parallel robot and its each joints.

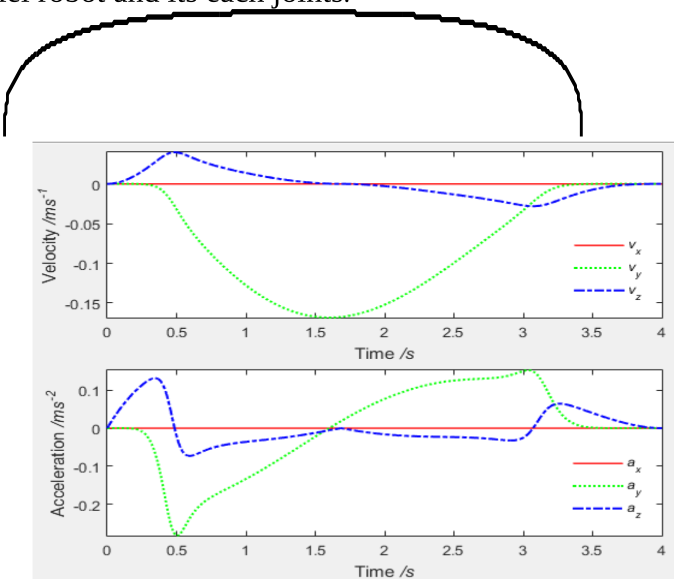

Figure 7. Curve Data by using Lamé curve method

According to the provided spline function by ADAMS, take the discrete date point as known conditions generate to driving function, then according to the driving function then moving function which about the angle and time can be added to the three moving actuators. The functions are:

Motion 1: CUBSPL (time, 0, SPLINE_1, 0)

Motion 2: CUBSPL (time, 0, SPLINE_2, 0)

Motion 3: CUBSPL (time, 0, SPLINE_3, 0)

After adding the moving function (Lamé curve and CAM curve of pick and place function) to the actuators of Delta Robot and Moveable platform of delta robot respectively, and properties of simulation is set to 4.0 seconds with 1000 steps size, the trajectory of the manipulator by simulating, as the trajectory track is visible in Figure 8. And the displacement curves of the manipulator-CM shown in Figure 9.

Put the result of trajectory of the manipulator to post processing module generated to spline function in ADAMS software, the dates of spline functions are the forward solution of the manipulator.

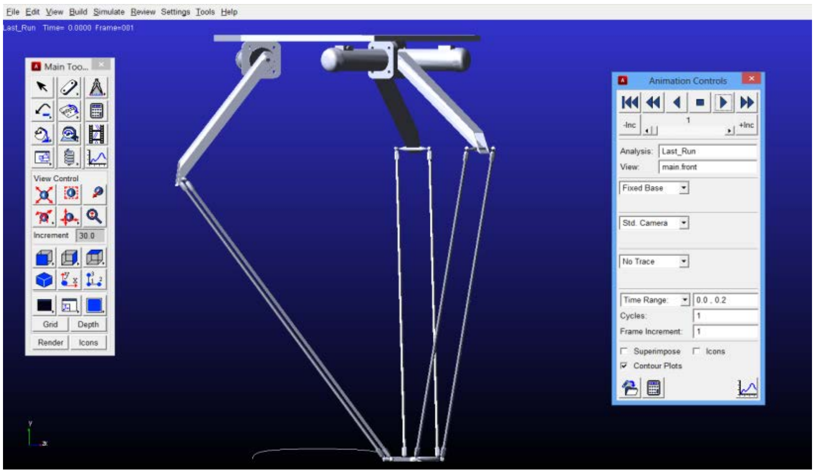

Figure 8. The trajectory of the pick and place manipulator

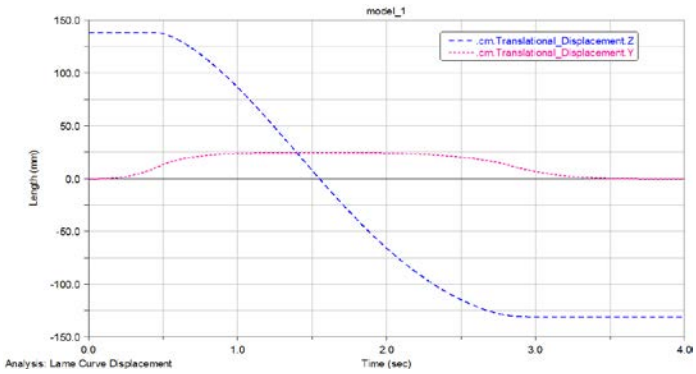

Figure 9. The displacement curves of Y, Z of moveable platform by Lamé method

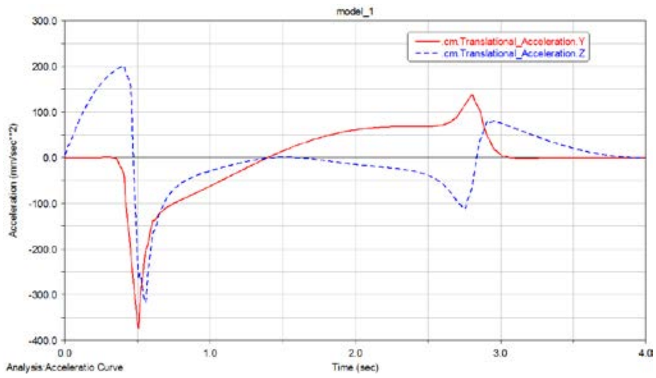

Figure 10. The acceleration curves of Y, Z of moveable platform by Lamé method

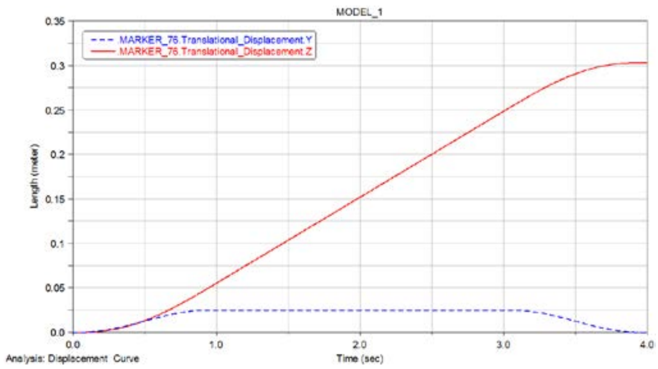

Figure 11. The displacement curves of Y, Z of moveable platform by CAM method

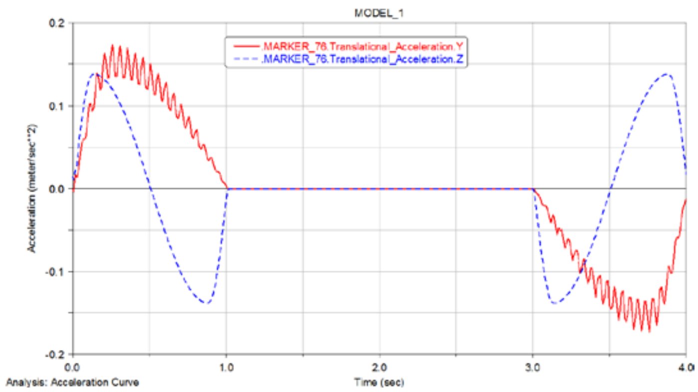

Figure 12. The acceleration curves of Y, Z of moveable platform by CAM method

2.6 Experiment.

The optimized trajectory not only renders the Delta robot reduced energy cost, but also reduces residual vibrations. The latter can be verified by the experiment results in this section, and also can compare the both trajectories.

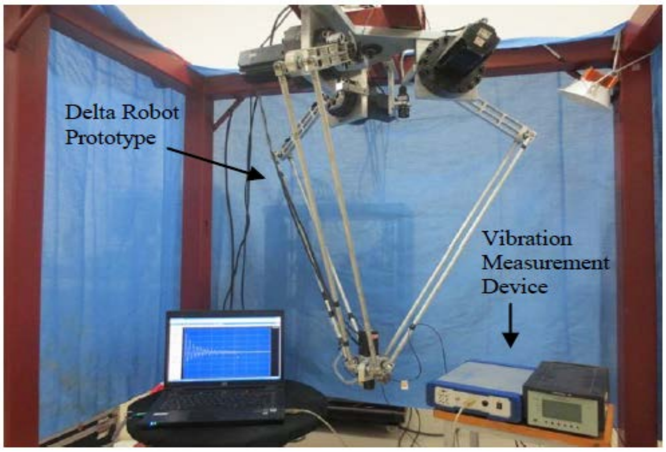

Figure 13. Delta Robot Prototype and Vibration Measurement Device

As shown in Figure 13, a prototype of Delta robot is fabricated to test the PPO trajectories. All the links of this prototype are made of carbon fiber and no composite material is used. During high speed operations, the elasticity of the links and the clearance at the joints lead to unwanted vibrations. As a manner to measure and record the residual vibrations at the terminal position, accelerometers are attached on the end-effector such that vibration signals can be transmitted to the measurement device.

For the PPO trajectories, we used each method with three different time cycles to get the appropriate result of comparison between Lamé and CAM curves. But in each manipulation performed with the same height and the distance as in vertical distance is 25mm and in horizontal is about 305mm. The measured dates of Lamé and CAM trajectories with 4 second of time period from the measuring device are given below.

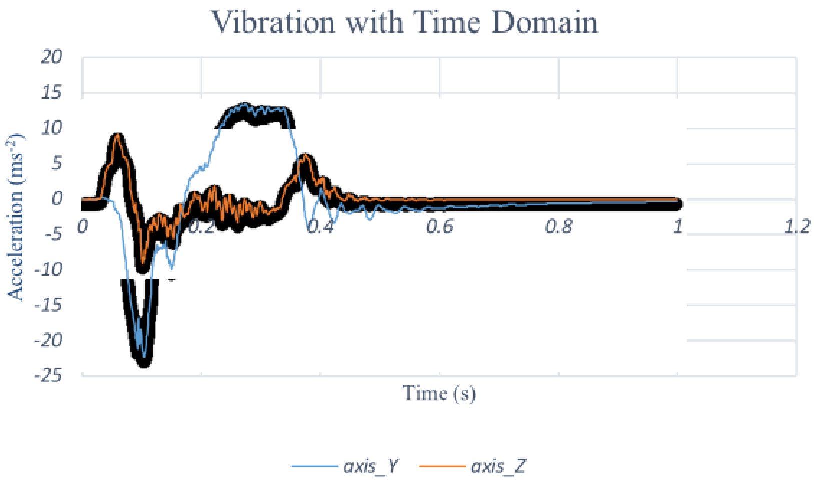

Figure 14. Lamé trajectory's residual Vibration with time domain

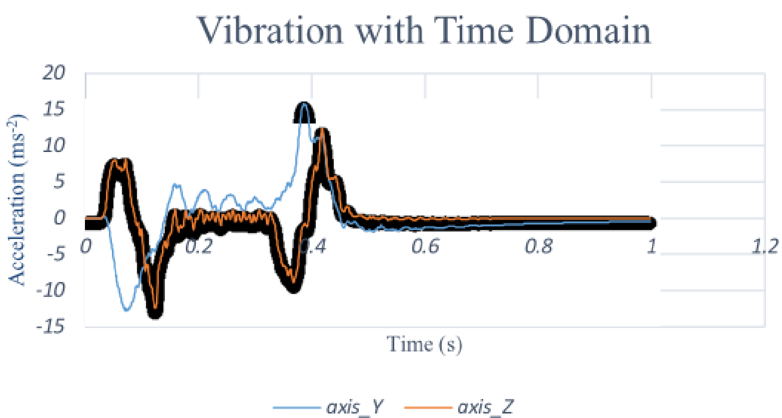

Figure 15. CAM trajectory's residual Vibration with time domain

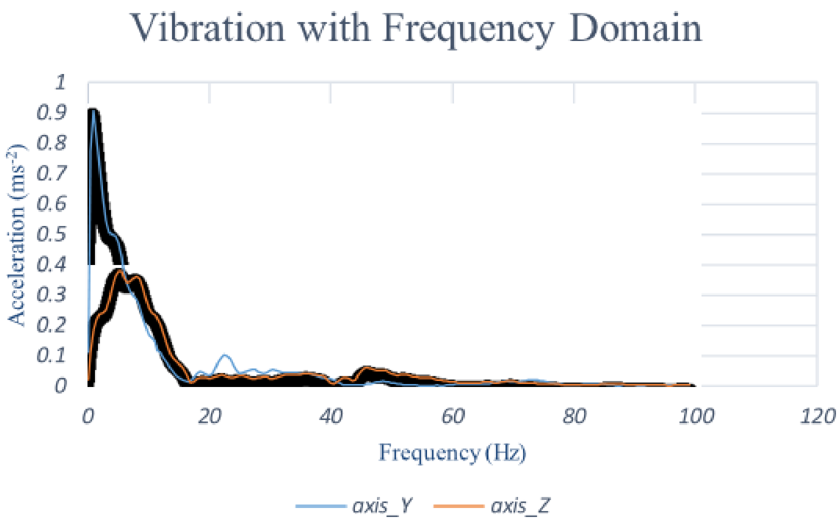

Figure 16. Lamé trajectory's residual Vibration with frequency domain

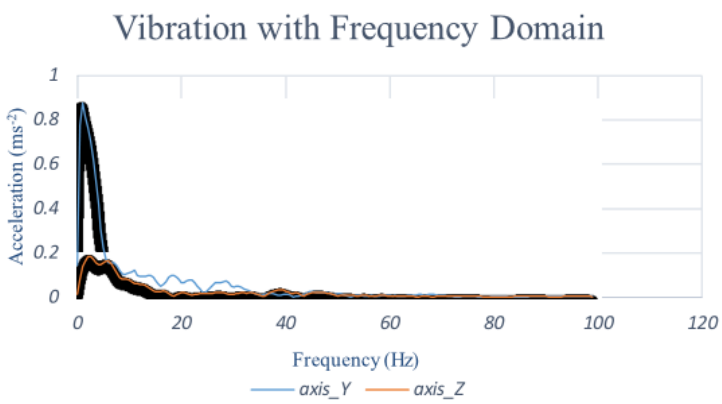

Figure 17. CAM trajectory's residual Vibration with frequency domain

From the above figures,under the vibration is given with the time priority, Lamé curve compared with the CAM curve according to y axis, at 0.1 s Lamé curve acceleration amplitude is smaller than CAM curve acceleration amplitude, And within time duration from 0.2s to 0.35s, the Lamé curve have bigger acceleration amplitude than the CAM curve. According to z axis, the comparison between Lamé curve with the CAM, during 0.2s to 0.35s, the curve of the Lamé behave unstable, in other hand the CAM curve is relatively flat, the CAM curve within time duration of 0.35s to 0.45s behave alternatingly as first in positive side and then in negative direction, But the Lamé curve at that time is in positive direction. The same result we already got by simulation in Adams.

Under the vibration analysis according to the frequency domain, the Lamé and the CAM curves have not big difference related to the y axis, But according to the z axis curve near the 10Hz, Lamé curve acceleration is about 0.3-2ms-2, in other side the CAM curve acceleration is about 0.15-2ms-2. Which shows that the CAM curve is more precise and have less vibration than the Lamé curve during Pick-and-Place operation.

3. Conclusions.

This paper presents the optimization of a PPO trajectory as an approach to improve industrial Delta robots' high speed performances. The trajectory is smoothed by using Lamé and CAM curves. The optimized trajectory is tested on a prototype. Experiment results verify that the optimized trajectory can improve the motion quality of high speed PPOs in terms of energy efficiency, residual vibration and terminal state accuracy. The analysis in this paper shows that the comparison between Lamé and CAM curves. Although the both curves are effective tool for the planning of standard PPO trajectories, in some aspects CAM curve is more precise and have less residual frequency than the Lamé curve.

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