A Non-cooperative Game Model for Workload Allocation and Its Implementations in Plastic Mould Shop

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Abstract. It is proposed that a non-cooperative game model for workload allocation of the plastic mould shop based on the game theories. In this model, the manufacturing tasks were regarded as the players, and the candidate companies correspond to the manufacturing tasks were regarded as the strategies set. Then the multi-objective payoff is defined as the weighted composite of workloads of manufacturing equipments and finishing cost. The genetic algorithm is applied, and the payoff was converted as the fitness function based on the non-cooperative game theories. Therefore, obtaining the optimal results is determined by the Nash equili-brium point of this non-cooperation game. A numerical case study demonstrates the validity of the proposed model and method.

Introduction

Plastic mould manufacturing is typically resource-oriented order type single-piece production, customer order driver in shop by a project, and the manufacturing processes of each mould project are highly random and experience dependent. So, the planning and controlling of the production for plastic mould shop is very complex. In the one hand, it is need consider that the scheduling influence of different machine tools such as NC milling, wire-electrode cutting or electronic spark machining. In the other hand, the mould order should assure the workload equilibrium of the mould shop, and it make that every part of one mould can arrive at assembly shop on time. Thus, making a reasonable workload planning scheme for plastic mould shop is an important way that it can balance solve the conflict between the delivery time and the production cost.

Stevenson M.et al. (Stevenson, M. & Linda, C. H. 2006.) research the shop workload controlling technology and tactics of order entering layer, and set up the scheduling model involves the delivery time and balanced workload. Potts et al. (Potts, C.N. & Whitehead, J.D. 2001.) study built the function model which contain the factors such as the workload equilibrium, circulating time and integrated cost from the job scheduling problem. The method of LUMS (Lancaster University Management School) is the achievement from Lancaster University, and thinks that the suitable description of shop workload controlling technology is the workload accumulation of different stages (Stevenson, M. 2006) and Liu et al.(Liu et al. 2010) modeled the problem of workload controlling for mould shop as job scheduling for non identical parallel machine, and applied the hybrid genetic algorithm to solve the optimizing problem. The above research pay only attention to the workload equilibrium optimization of shop operation layer, simplify the complication of order entering layer and task circulating layer, and there are very large deviation between the result from the optimizing model and the actual production. In order to control the delivery time and production cost savings, it is necessary to planning the workload of shop.

The research for game theories are focused on the tactics antagonism, competing or strategy selection, and the inherent nature of conflict resolve or reconcile contradictions has largely the similarity with multi-objective optimization problem in engineering field. Xie et al. (Xie et al. 2006) made the multi-objective design problem to game strategy selection problem, and solve the optimization problem. It can be find that decision maker not need build special integrated evaluation function, but guide the competing and cooperation for each game player according to the rule of reason for game theory. The conception of equilibrium gives a new idea for solving the multi-objective optimization problem in engineering fields.

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In this paper, each machine tool in mould shop is described as decision-maker to illustrate the competing relationship in the production process. The payoff function is set up based on the workload equilibrium allocation of each machine tool, and a non-cooperative model is constructed. A genetic algorithm is applied to solve the model, then the optimization configuration for manufacturing resource is achieved.

Non-cooperative model of workload allocation in plastic mould shop

For the process of production machine tool and task optimization configuration, it can be supposed as follows.

the received order can be split into *n* manufacturing task.

There are *m* production machine tools in plastic mould shop, each machine tool can only and just undertake one task.

It involve several main elemnets such as workload and workload ratio in produciton process, and it can be described as follows.

(3)workload(WL), it indicates the total manufacutring time when the manufacturing task is assigned to the production machine tool.

(4)workload ratio(*p*), it is the ratio of workload and the maximum manufacutring time of the production machine tool(two-shift system for machine tool in this paper). p_{ij} is the workload ratio of production machine tool *j* for manufacturing task *i*.

The non-cooperative game theory is the process which all the players choose personal strategy to maximize his benefit. Standardized non-cooperative game problem contains three elements: player, strategy and payoff.

For the non-cooperative game problem in plastic mould shop, the manufacturing task is mapped as the player, and the production machine tool is mapped as the strategy set for the manufacturing task. The production workload ratio and the integrated cost are the payoff of the players, and it can obtain the Nash equilibrium solution. Thus, it can realize the manufacturing resource optimizing configuration, and balance the workload and reduce the running cost of production machine tool.

The game theory is introduced, and the non-cooperative model can be descrided by ternary group.

$$G = \{MT, E, U\}$$

The above variables can be described as follows.

The player $MT = MT_i\,(i = 1,2,\cdots,n)$ for *n* manufacturing tasks.

(2) E_i is the strategy set of player MT_i, and it is contracted the production machine for manufacturing task. The production machine set of all the players of manufacturing task indicates as $E = E_i\ (i = 1,2,\cdots,m)$. For the players of manufacturing task, the strategy set indicates as E_{MT_i}, $E_{MT_i} \le E_i$, $(i = 1,2,\cdots,m)$.

(3) U_i is the payoff of the players, and it shows minimizing the production workload ratio and the integrated cost.

Define the variables:

$$x_{ij} = \begin{cases} 1 & manufacturing\ i\ is\ contracted\ to\ machine\ tool\ \ j \\ 0 & otherwise \end{cases}$$

Then, the optimization problem can be described as following models.

$$U(x) = \lambda_1 \prod_{i=1}^{n} \sum_{j=1}^{m_i} p_{ij} \sum x_{ij} + \lambda_2 \sum_{i=1}^{n} \sum_{j=1}^{m_i} c_{ij} \sum x_{ij} \tag{1}$$

$$s.t.\quad \sum_{j=1}^{m_i} \sum_{i=1}^{n} x_{ij} = 1, \quad i = 1,2,\cdots,n \tag{2}$$

$$x_{ij} = 1 \text{ or } 0, \qquad \forall\ i, j \tag{3}$$

Equation (1) is the optimization function, and the function consists of two parts. $\lambda_1 \prod_{i=1}^{n} \sum_{j=1}^{m_i} p_{ij} \sum x_{ij}$ and $\lambda_2 \sum_{i=1}^{n} \sum_{j=1}^{m_i} c_{ij} \sum x_{ij}$ represent minimizing the production workload ratio, minimizing the processing cost respectively.

Constraint (2) indicates that only one production machine tool is selected for each manufacturing task.

Constraint (3) indicates that the production machine tool is selected or not selected.

where λ_1, λ_2 are the weights for production workload factor and cost factor, and $\lambda_1, \lambda_2 \in [0,1]$.

Players cooperative and compete in the process of the game, and at last there is an equilibrium status which this is the solution of game theories. Nash Equilibrium theory gives the definition of non-cooperative game and its equilibrium solution. It can be described as follows.

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \quad s_{-i} \subset S_{-i}$$

In generally, all the players were tabbed "- *i*" except the given player in game theories.

Non-cooperative game model solving using genetic algorithm

To obtain the effective solution for the non-cooperative game model of workload allocation in plastic mould shop, the genetic algorithm (GA) is introduced.

Encoding of chromosome

For our genetic algorithm, the natural number string is selected as the gene description because the format of integer encoding is easier to express the meaning of solution corresponding to the binary format. Each gene composes of n parameters, represents ith manufacturing task. Let $w = [w_1, w_2, \cdots, w_i, \cdots, w_n]$ $(i = 1,2,\cdots,n)$, where w_i is a integer. This stands for the criterion value w_i that is selected for ith criterion. Thus, a selection represents a chromosome or an individual in the genetic algorithm. The objective of optimization is to find out the chromosome, and make the value of the objective function minimum.

Designing the fitness function

For our non-cooperative game model, the workload allocation problem can be translated into solving the Nash equilibrium solution with the constraints.

$$U_i(s_0^*, s_1^*, \cdots, s_i^*, \cdots, s_{n-i}^*) \geq U_i(s_0, s_1, \cdots, s_i, \cdots, s_{n-i})$$

$$\forall s_i \in S_i \tag{4}$$

where s_i^* is the optimizing strategy of E_i.

Then, the fitness function can be formulated as equation (5) and (6).

$$F^k(U_0^k, U_1^k, \cdots, U_i^k, \cdots, U_{n-1}^k) = [\sum_{i=0}^{n-1} (U_i^k - \hat{U}_i)^2]^{1/2} \tag{5}$$

$$\hat{U}(x) = \lambda_1 \prod_{i=1}^{n} \sum_{j=1}^{m_i} p_{ij} \sum x_{ij}(t) + \lambda_2 \sum_{i=1}^{n} \sum_{j=1}^{m_i} c_{ij} \sum x_{ij}(t) \tag{6}$$

Solving the equation (5), the optimal Nash equilibrium solution is acquired.

Case study

We consider the real-world problem of plastic inject mould enterprise that bid for a project for seven manufacturing tasks for interior decoration. For the manufacturing task, there are three criteria that are the production machine tool MA, processing cost PC and workload ratio *p*.

Table 1. The parameters for manufacturing tasks

MT	MA	PC	p
MT1	A1	12	0.35
	A2	13	0.46
	A3	14	0.84
	A4	10.5	0.31
	A5	11	0.56
MT2	B1	23	0.78
	B2	22.5	0.45
	B3	24	0.37
MT3	C1	10	0.82
	C2	9.5	0.45
	C3	12	0.36
	C4	11	0.39
MT4	D1	78	0.5
	D2	81	0.7
MT5	E1	32	0.5
	E2	33	0.45
	E3	31	0.73
MT6	F1	8	0.56
	F2	7.5	0.47
	F3	9	0.68
	F4	8.5	0.91
MT7	G1	6.5	0.68
	G2	7.0	0.64
	G3	8.1	0.71
	G4	6.4	0.73

For the seven manufacturing task, there are several candidate production machine tools responding to the different tasks respectively (see Table 1). The final parameter settings were determined: population size=20; crossover rate=0.7, mutation rate=0.25, and number of generation=2000.The weight parameters were determined: $\lambda_1 = 0.6$, $\lambda_2 = 0.4$.

The GA is applied to the optimization problem and the optimal value of the objective function is obtained. The optimal Nash equilibrium solution is w^*=[A4, B2, C2, D1, E3, F2, G1]. At the same time, the simulation curve is shown in Fig. 1.

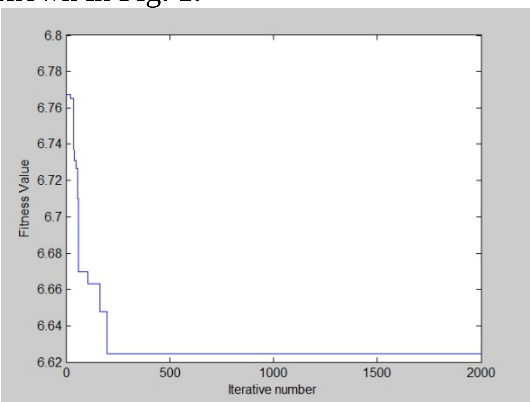

Figure1. Simulation curve

Conclusions

We have proposed a non cooperative model for workload allocation in plastic mould shop. In order to select prior the production machine tool which have minimum workload and processing cost, the game theory is introduced to construct model, and the GA is applied to obtain the optimal

Nash equilibrium solution. The GA based on integer encoding is applied to solve the optimization problem. The results of implementation indicate that the proposed methodology is helpful tool for a manufacturing task to select the right production machine tool.

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