Different configurations of 25-order structure of Steiner triple systems of counting

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Keywords: Triple System；Complete trimap；Count

Abstract. A method of constructing and technology view for sub graph decomposition of K_v is proposed.,and it clarifies the application completely trimap and complete bipartite graph in combination Design；Order to clarify the structure of the basic ideas of Steiner triple systems 25 Order introduced the whole process of construction Steiner triple systems and computing.The conclusion is: the text of the constructors and counting methods Steiner triple systems are effective and replicable.

Introduction

Block design theory is an important branch of the combinatorial mathematics and plays an important role in experiment design, competition arrangement, digital communication areas. In 1850 a British mathematician Thomas P.Kirkman posed a famous 15 schoolgirls problem and solved this problem in the same year [1-5]. In 1971 D.R. Ray-Chaudhari and R.M. Wilson published a paper with topic Solution of Kirkman's schoolgirls problem to show how to construct Kirkman triple systems of order $6n+3$ [6-10]. In 1961 a Chinese mathematician Lu Jiaxi posed the decomposable condition of BIBD design [11-12]. It has been an open problem for over hundred years to determine whether there exists a Kirkman triple system of order $6n+3$ for each $n=0,1,2,3$. In 2003 a method of constructing Kirkman triple systems of order $s\times t$ by using main matrix and subsidiary matrix is proposed by the author of this paper, and Kirkman triple systems of order $v=27,45,81,135$, are constructed [13-15], the open problem mentioned above can be solved hopefully[5].

1. Basic theory

Definition 1: Supposing Top Set $V(G)=\{c_1,c_2,\cdots,c_v\}$, Edge Set $E(G)=\{c_1c_2,c_1c_3,\cdots,c_1c_v,c_2c_3,\cdots,c_2c_v,\cdots,c_{v-1}c_v\}$ and G is complete to K_v, if we put $\left|E(G)\right|=v(v-1)/2$ sides into triangular matrix so that it can exist the relation with any edge c_ic_j , top c_i and top c_j, then this triangular matrix called side matrix, and referred to as K_v'.

Definition 2: Supposing Top Set is $V(G)=\{c_1,c_2,\cdots,c_v\}$ and $V(G)$ include $V_i=\{c_m,c_{m+1},\cdots,c_{m+t-1}\}$, $V_j=\{c_p,c_{p+1},\cdots,c_{p+t-1}\}$, $V_k=\{c_q,c_{q+1},\cdots,c_{q+t-1}\}$ three subsets, if each top of V_i neighbor top t of V_j and top t of V_k, then graph G called complete trimap and denoted $K_{t,t,t}^{(i,j,k)}$, which three sub-graph is called complete ipartite graph, respectively $K_{t,t}^{(i,j)}$, $K_{t,t}^{(i,k)}$, $K_{t,t}^{(j,k)}$, meanwhile , $K_{t,t,t}^{(i,j,k)}=K_{t,t}^{(i,j)}\bigcup K_{t,t}^{(i,k)}\bigcup K_{t,t}^{(j,k)}$

If $t\times t$ complete graph K_3 exist in the complete trimap $K_{t,t,t}^{(i,j,k)}$ has separated , then $t\times t$ triple system matrix $K_{t,t,t}^{(i,j,k)}$ can be delieved as

$$K_{t,t,t}^{(i,j,k)}=\left\{\begin{array}{cccc} \{m,p,q\} & \{m,p+1,q\} & \cdots & \{m,p+t-1,q+t-1\} \\ \{m+1,p+1,q\} & \{m+1,p+2,q\} & \cdots & \{m+1,p,q+t-1\} \\ \vdots & \vdots & \vdots & \vdots \\ \{m+t,p+t-1,q\} & \{m+t-1,p,q\} & \cdots & \{m+t-1,p+t-2,q+t-1\} \end{array}\right\}$$

Apparently, we put the fixed of minimum number from m,p,q , V_i,V_j,V_k into matrix $K_{t,t,t}^{(i,j,k)}$, then we can obtain $t\times t$ complete graph K_3, but $K_{t,t,t}^{(i,j,k)}$ has t kinds of structure rather than only one.

Theorem 1:: Supposing Top Set is $V(G)=\{C_1,C_2,\cdots,C_v\}$ and $|V(G)|=v=s\times t-s+1$, s,t already exists Steiner triple's order,then it must be exist $s\times t-s+1$ order of steiner triple and this steiner triple's structure is equivalent to $v(v-1)/6$ kinds decompose from a complete graph K_v.

The specific steps of constructing the Steiner triple system with the order of $s\times t-s+1$ are that:

(1)The edge matrix $K_v^{'}$ can be divided as t edge matrices $K_t^{(1)},K_t^{(2)},\cdots,K_t^{(s)}$ of complete graphs $K_v^{'}$, and $s(s-1)/2$ edge matrices of $K_{t-1,t-1}^{(i,j)}$, $K_{t-1,t-1}^{(i,k)}$,……, $K_{t-1,t-1}^{(j,k)}$ with complete bipartite graph., then we get the edge matrix K_v^{1A}.

(2)Let s complete graphs $K_t^{(1)},K_t^{(2)},\cdots,K_t^{(s)}$ of $K_v^{'}$ with the order of t be devided into $t(t-1)/2$ complete graphs K_3 ,and $s(s-1)/2$ complete bipartite graphs $K_{t-1,t-1}^{(i,j)}$ $K_{t-1,t-1}^{(i,k)}$ $K_{t-1,t-1}^{(j,k)}$ can be merged as $s(s-1)/2$ triple systems matrix $K_{t-1,t-1,t-1}^{(i,j,k)}$, which be constituted with $(t-1)(t-1)$ complete graphs K_3.

2.method of construction Steiner triple systems of order 25

If edge matrix $K_{25}^{'}$ of complete graphs K_{25} can be divided into 3 complete graphs $K_9^{(1)},K_9^{(2)},K_9^{(3)}$ with order 9 of edge matrix $K_9^{1(1)},K_9^{1(2)},K_9^{1(3)}$,and edge matrix $K_{8,8}^{(1,2)},K_{8,8}^{(1,3)},K_{8,8}^{(2,3)}$ of 3 complete bipartite graphs $K_{8,8}^{(1,2)},K_{8,8}^{(1,3)},K_{8,8}^{(2,3)}$.

$$K_9^{1(1)}=\begin{array}{cccccccc|c} C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & \\ \hline 1_2 & 1_3 & 1_4 & 1_5 & 1_6 & 1_7 & 1_8 & 1_9 & C_1 \\ & 2_3 & 2_4 & 2_5 & 2_6 & 2_7 & 2_8 & 2_9 & C_2 \\ & & 3_4 & 3_5 & 3_6 & 3_7 & 3_8 & 3_9 & C_3 \\ & & & 4_5 & 4_6 & 4_7 & 4_8 & 4_9 & C_4 \\ & & & & 5_6 & 5_7 & 5_8 & 5_9 & C_5 \\ & & & & & 6_7 & 6_8 & 6_9 & C_6 \\ & & & & & & 7_8 & 7_9 & C_7 \\ & & & & & & & 8_9 & C_8 \end{array},\quad K_{8,8}^{1(1,2)}=\begin{array}{cccccccc|c} C_{10} & C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & C_{17} & \\ \hline 1_{10} & 1_{11} & 1_{12} & 1_{13} & 1_{14} & 1_{15} & 1_{16} & 1_{17} & C_1 \\ 2_{10} & 2_{11} & 2_{12} & 2_{13} & 2_{14} & 2_{15} & 2_{16} & 2_{17} & C_2 \\ 3_{10} & 3_{11} & 3_{12} & 3_{13} & 3_{14} & 3_{15} & 3_{16} & 3_{17} & C_3 \\ 4_{10} & 4_{11} & 4_{12} & 4_{13} & 4_{14} & 4_{15} & 4_{16} & 4_{17} & C_4 \\ 5_{10} & 5_{11} & 5_{12} & 5_{13} & 5_{14} & 5_{15} & 5_{16} & 5_{17} & C_5 \\ 6_{10} & 6_{11} & 6_{12} & 6_{13} & 6_{14} & 6_{15} & 6_{16} & 6_{17} & C_6 \\ 7_{10} & 7_{11} & 7_{12} & 7_{13} & 7_{14} & 7_{15} & 7_{16} & 7_{17} & C_7 \\ 8_{10} & 8_{11} & 8_{12} & 8_{13} & 8_{14} & 8_{15} & 8_{16} & 8_{17} & C_8 \end{array}$$

$$K_9^{1(2)}=\begin{array}{cccccccc|c} C_{10} & C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & C_{17} & \\ \hline 9_{10} & 9_{11} & 9_{12} & 9_{13} & 9_{14} & 9_{15} & 9_{16} & 9_{17} & C_9 \\ & 10_{11} & 10_{12} & 10_{13} & 10_{14} & 10_{15} & 10_{16} & 10_{17} & C_{10} \\ & & 11_{12} & 11_{13} & 11_{14} & 11_{15} & 11_{16} & 11_{17} & C_{11} \\ & & & 12_{13} & 12_{14} & 12_{15} & 12_{16} & 12_{17} & C_{12} \\ & & & & 13_{14} & 13_{15} & 13_{16} & 13_{17} & C_{13} \\ & & & & & 14_{15} & 14_{16} & 14_{17} & C_{14} \\ & & & & & & 15_{16} & 15_{17} & C_{15} \\ & & & & & & & 16_{17} & C_{16} \end{array},\quad K_{8,8}^{1(1,3)}=\begin{array}{cccccccc|c} C_{18} & C_{19} & C_{20} & C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & \\ \hline 1_{18} & 1_{19} & 1_{20} & 1_{21} & 1_{22} & 1_{23} & 1_{24} & 1_{25} & C_1 \\ 2_{18} & 2_{19} & 2_{20} & 2_{21} & 2_{22} & 2_{23} & 2_{24} & 2_{25} & C_2 \\ 3_{18} & 3_{19} & 3_{20} & 3_{21} & 3_{22} & 3_{23} & 3_{24} & 3_{25} & C_3 \\ 4_{18} & 4_{19} & 4_{20} & 4_{21} & 4_{22} & 4_{23} & 4_{24} & 4_{25} & C_4 \\ 5_{18} & 5_{19} & 5_{20} & 5_{21} & 5_{22} & 5_{23} & 5_{24} & 5_{25} & C_5 \\ 6_{18} & 6_{19} & 6_{20} & 6_{21} & 6_{22} & 6_{23} & 6_{24} & 6_{25} & C_6 \\ 7_{18} & 7_{19} & 7_{20} & 7_{21} & 7_{22} & 7_{23} & 7_{24} & 7_{25} & C_7 \\ 8_{18} & 8_{19} & 8_{20} & 8_{21} & 8_{22} & 8_{23} & 8_{24} & 8_{25} & C_8 \end{array}$$

$$
K_9^{1(3)}=\begin{array}{c} \begin{array}{cccccccc} C_{18} & C_{19} & C_{20} & C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \end{array} \\ \left(\begin{array}{cccccccc} 9_{18} & 9_{19} & 9_{20} & 9_{21} & 9_{22} & 9_{23} & 9_{24} & 9_{25} \\ & 18_{19} & 18_{20} & 18_{21} & 18_{22} & 18_{23} & 18_{24} & 18_{25} \\ & & 19_{20} & 19_{21} & 19_{22} & 19_{23} & 19_{24} & 19_{25} \\ & & & 20_{21} & 20_{22} & 20_{23} & 20_{24} & 20_{25} \\ & & & & 21_{22} & 21_{23} & 21_{24} & 21_{25} \\ & & & & & 22_{23} & 22_{24} & 22_{25} \\ & & & & & & 23_{24} & 23_{25} \\ & & & & & & & 24_{25} \end{array}\right)\begin{array}{c} C_9 \\ C_{18} \\ C_{19} \\ C_{20} \\ C_{21} \\ C_{22} \\ C_{23} \\ C_{24} \end{array} \end{array},\quad
K_{8,8}^{1(2,3)}=\begin{array}{c} \begin{array}{cccccccc} C_{18} & C_{19} & C_{20} & C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \end{array} \\ \left(\begin{array}{cccccccc} 10_{18} & 10_{19} & 10_{20} & 10_{21} & 10_{22} & 10_{23} & 10_{24} & 10_{25} \\ 11_{18} & 11_{19} & 11_{20} & 11_{21} & 11_{22} & 11_{23} & 11_{24} & 11_{25} \\ 12_{18} & 12_{19} & 12_{20} & 12_{21} & 12_{22} & 12_{23} & 12_{24} & 12_{25} \\ 13_{18} & 13_{19} & 13_{20} & 13_{21} & 13_{22} & 13_{23} & 13_{24} & 13_{25} \\ 14_{18} & 14_{19} & 14_{20} & 14_{21} & 14_{22} & 14_{23} & 14_{24} & 14_{25} \\ 15_{18} & 15_{19} & 15_{20} & 15_{21} & 15_{22} & 15_{23} & 15_{24} & 15_{25} \\ 16_{18} & 16_{19} & 16_{20} & 16_{21} & 16_{22} & 16_{23} & 16_{24} & 16_{25} \\ 17_{18} & 17_{19} & 17_{20} & 17_{21} & 17_{22} & 17_{23} & 17_{24} & 17_{25} \end{array}\right)\begin{array}{c} C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \\ C_{16} \\ C_{17} \end{array} \end{array}
$$

Then edge matrix $K_9^{1(1)}, K_9^{1(2)}, K_9^{1(3)}$ of 3 complete graphs can be divided into $9(9-1)/6$ complete graphs K_3,according to Steiner triple systems $ST_1(9), ST_2(9), \cdots ST_9(9)$ of order 9,and 3 complete bipartite graphs $K_{8,8}^{(1,2)}, K_{8,8}^{(1,3)}, K_{8,8}^{(2,3)}$ can be merged as Steiner triple systems matrix $K_{8,8,8}^{(1,2,3)}$ of 8×8 complete graphs K_3.Finally, $9(9-1)/2$ complete bipartite graphs K_3 and matrix $K_{8,8,8}^{(1,2,3)}$ constructed Steiner triple systems $ST_1(25)$ of order 25.Supposing the division of $9(9-1)/6$ complete graphs K_3 of $K_9^{(1)}, K_9^{(2)}, K_9^{(3)}$ with $ST_2(9), ST_3(9), \cdots, ST_7(9)$ respectively,and division of proposal 8×8 complete graphs K_3 of complete trimp graphs $K_{8,8,8}^{(1,2,3)}$ should be transformed accordingly,so we get some different configurations triple systems of Steiner triple systems $ST_2(25), ST_3(25), \cdots, ST_7(25)$ of order 25.

$$
ST_1(25)=\left\{\begin{array}{l}
(1,2,3),(1,4,7),(1,5,6),(1,8,9),(1,10,18),(1,11,19),(1,12,20),\\
(1,13,21),(1,14,22),(1,15,23),(1,16,24),(2,4,5),(2,6,8),(2,7,9),\\
(2,10,19),(2,11,20),(2,12,21),(2,13,22),(2,14,23),(2,15,24),(2,16,25),(2,17,18),\\
(3,4,8),(3,5,9),(3,6,7),(3,10,20),(3,11,21),(3,12,22),(3,13,23),(3,14,24),\\
(3,15,25),(3,16,18),(3,17,19),(4,6,9),(4,10,21),(4,11,22),(4,12,23),(4,13,24),\\
(4,14,25),(4,15,18),(4,16,19),(4,17,20),(5,7,8),(5,10,22),(5,11,23),(5,12,24),\\
(5,13,25),(5,14,18),(5,15,19),(5,16,20),(5,17,21),(6,10,23),(6,11,24),(6,12,25),\\
(6,13,18),(6,14,19),(6,15,20),(6,16,21),(6,17,22),(7,10,24),(7,11,25),(7,12,18),\\
(7,13,19),(7,14,20),(7,15,21),(7,16,22),(7,17,23),(8,10,25),(8,11,18),(8,12,19),\\
(8,13,20),(8,14,21),(8,15,22),(8,16,23),(8,17,24),(9,10,11),(9,12,15),(9,13,14),\\
(9,16,17),(9,18,19),(9,20,23),(9,21,22),(9,24,25),(10,12,13),(10,14,16),(10,15,17),\\
(11,12,16),(11,13,17),(11,14,15),(12,14,17),(13,15,16),(18,20,21),(18,22,24),(18,23,25),\\
(19,20,24),(19,21,25),(19,22,23),(20,22,25),(21,23,24)
\end{array}\right\}
$$

$$\ldots,$$

$$
ST_7(25)=\left\{\begin{array}{l}
(1,2,9),(1,3,8),(1,4,6),(1,5,7),(1,10,24),(1,11,25),(1,12,18),\\
(1,13,19),(1,14,20),(1,15,21),(1,16,22),(1,17,23),(2,3,5),(2,4,8),\\
(2,6,7),(2,10,25),(2,11,18),(2,12,19),(2,13,20),(2,14,21),(2,15,22),\\
(2,16,23),(2,17,24),(3,4,7),(3,6,9),(3,10,18),(3,11,19),(3,12,20),\\
(3,13,21),(3,14,22),(3,15,23),(3,16,24),(3,17,25),(4,5,9),(4,10,19),\\
(4,11,20),(4,12,21),(4,13,22),(4,14,23),(4,15,24),(4,16,25),\\
(4,17,18),(5,6,8),(5,10,20),(5,11,21),(5,12,22),(5,13,23),\\
(5,14,24),(5,15,25),(5,16,18),(5,17,19),(6,10,21),(6,11,22),(6,12,23),\\
(6,13,24),(6,14,25),(6,15,18),(6,16,19),(6,17,20),(7,8,9),(7,10,22),\\
(7,11,23),(7,12,24),(7,13,25),(7,14,18),(7,15,19),(7,16,20),(7,17,21),\\
(8,10,23),(8,11,24),(8,12,25),(8,13,18),(8,14,19),(8,15,20),(8,16,21),\\
(8,17,22),(9,10,17),(9,11,16),(9,12,14),(9,13,15),(9,18,25),(9,19,24),\\
(9,20,22),(9,21,23),(10,11,13),(10,12,16),(10,14,15),(11,12,15),(11,14,17),\\
(12,13,17),(13,14,16),(14,16,17),(18,19,21),(18,20,24),(18,22,23),(19,20,23),\\
(19,22,25),(20,21,25),(21,22,24),(23,24,25)
\end{array}\right\}
$$

3.Counting of Steiner triple system with 25 order

The number of Steiner triple systems with 25 order depended on division proposal number $N^{(1)}$ of $9(9-1)/6$ complete graphs K_3 of complete graphs $K_9^{(i)}$ with order 9,coming from matrix $K_{25}^{'}$ of complete graphs K_{25} with order 25, and division proposal number $N^{(2)}$ of 8×8 complete graphs K_3,and $N^{(1)}$ decideed number of Steiner triple systems with order 9.Construction result and enumeration of 105 Steiner triple systems with order 9 be proved, and $N^{(1)}=105$, $N^{(2)}=8$. Obviously.According to Multiplication principle,number of 25 Order Steiner triple systems are that: $N=105\times 8$.

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