The Analysis of Convection Experiment

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Abstract. The paper mainly discusses the temperature distribution in eggs from the heating process. For eggs' heating process, the paper would explore the temperature change rules from the analysis, the lumped parameter method, numerical method of three aspects. Based on each methods, diagrams would be drawn, the features of temperature is going to analyze and the errors would be calculated.

1. Problem Analysis

Heating an egg and putting it into the boiling water at 100℃, then the analysis objective is the variation of temperature in the center of the egg. It is assume that egg density is 1000kg/m^3, the specific heat of egg is 3310J/(kg*k) and during the heating process, the surface heat transfer coefficients for h equals 1200W/(m^2*k). The paper uses lumped parameter method, numerical analysis method and conducts two-dimensional mathematical simulation.

2. Heating Process

The arranged recording data of experiment is shown in Appendix 1. The real heating curve can be obtained by data fitting using Matlab.

Table 1The arranged recording data of experiment

Time /s	Temp. /℃	Time /s	Temp./℃	Time /s	Temp. /℃	Time /s	Temp./℃	Time /s	Temp. /℃	Time /s	Temp. /℃
0	21	35	32	78	43	137	54	198	65	324	76
2	22	45	33	81	44	142	55	217	66	335	77
6	23	47	34	86	45	147	56	224	67	347	78
10	24	51	35	92	46	153	57	229	68	361	79
14	25	53	36	97	47	157	58	234	69	375	80
16	26	55	37	104	48	160	59	238	70	388	81
19	27	59	38	108	49	163	60	255	71	401	82
21	28	66	39	112	50	170	61	274	72		
23	29	72	40	123	51	173	62	290	73		
27	30	76	41	129	52	189	63	303	74		
30	31	77	42	133	53	192	64	315	75		

Analysis calculation is performed first, then the lumped parameter method is used. The process of heating eggs belongs to unsteady heat conduction. The eggs can be seen as high thermal conductive objects and it also can be considered as a whole isothermal egg. The egg temperature of lumped-parameter equation can be written. Calculating internal egg temperature changes over time, the temperature distribution mapping can finally obtained.

The use of numerical method is to discrete the region first and divide egg into 11 regions and then the forward and implicit difference equations can be displayed respectively and using iterative method to solve forward difference equation, it can obtain temperature change results and finally

the comparison of temperature distribution is made.

3. Model Assumption

It is assumed that egg is a sphere with 5cm diameter. In the heating process, the convective heat transfer coefficient remains unchanged. In the lumped parameter method, egg is assumed to be a sphere with uniform temperature.

4. Symbol Illustration

λ: Interior thermal conductivity of egg; ε: egg surface heat conduction coefficient; r: the sphere radius of egg; ρ: egg density; c: egg specific heat; h: egg surface heat transfer coefficient; τ: time; u: egg temperature; M: node location

5. Model Building and Solution

5.1 Heating process analysis

Making fitting of data, the real temperature variation over time can be obtained.

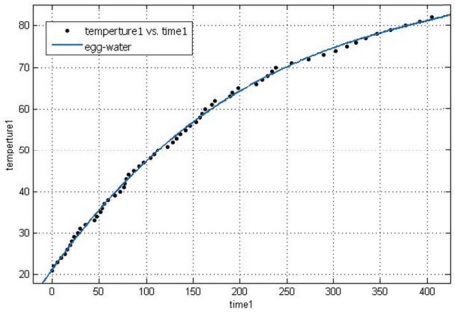

Fig. 1 real temperature variation

Using the analysis method, it is calculated that when the temperature in the center of egg comes to 82℃, it needs 635s.

Hurstwood: $Bi = \frac{hR}{\lambda}, Bi = \frac{1200 * 0.025}{0.7} = 42.86$

Coefficient of volume expansion: $a = \frac{\lambda}{\rho c}; a = \frac{0.7}{3310 * 1000} = 2.415 * 10^{-7}$

It is assumed that when the center temperature of eggs reaches82 degrees, it comes into the formal stage (Fig. 2).

Transcendental equations characteristic root:

$1 - u_n \cot u_n = Bi \quad n = 1.2.....$

$u_1 = 3.032$; $\frac{\theta_m}{\theta_0} = \frac{82 - 100}{20 - 100} = 0.025$; $2\frac{\sin u_1 - u_1 \cos u_1}{u_1 - \cos u_1 \sin u_1} = -2.0$

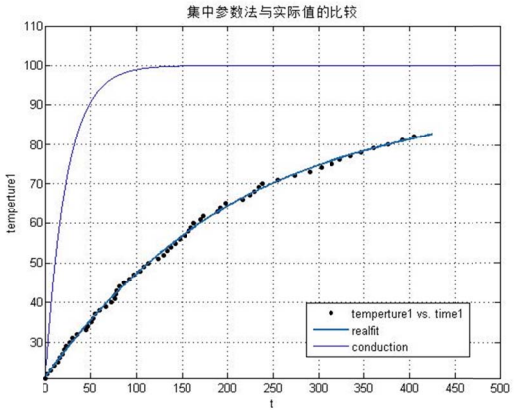

Fig. 2 formal stage

In the center of egg:

$$\eta = \frac{r}{l} = 0; \lim_{x \to 0} \frac{\sin x}{x} = 1; \frac{-2.0}{3.092 * \eta} \sin(3.092 * \eta) \exp(-3.092^2 \bullet Fo) = -2.0 * \exp(-3.0322 \bullet Fo) = 0.222$$

the root is:

$$F_0 = 0.24; F_0 = \frac{at}{R^2} = \frac{1.51 * 10^{-7}}{0.022} \tau \Rightarrow \tau = 635;$$

5.2 heating process lumped parameter method

In the heating process, it is seen that internal thermal conductivity of the eggs is much larger than the convective heat transfer coefficient of the egg surface, so it can be concluded that the temperatures inside the egg are basically consistent.

Boundary condition: $\tau = 0, \theta = t_0 - t_\infty = \theta_0$;

Average temperature changes over time: $$t = \exp\left(-\frac{hA\tau}{\rho c V} \right)(t_0 - t_\infty) + t_\infty$$

Through the calculated result, it can obtain that (Fig. 3):

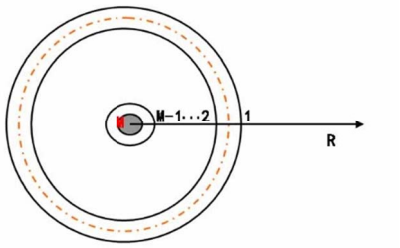

Fig. 3 the temperatures inside the egg

It can be concluded that the image obtained by lumped parameter method has great error comparing with the actual value, so it can't be selected as the mathematical model of heating the eggs.

5.3 Numerical method of heating process

First, the regional discrete inside the egg is made and the number of discrete regions are 11 which can be seen in Fig. 4.

The mathematical model is : $$\frac{\partial t}{\partial \tau} = \frac{a}{r^2} \left(\frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) \right)$$

Boundary condition: $$\tau = 0, t = t_0; r = 0, \frac{\partial t}{\partial r} = 0; r = r_3, -\lambda \frac{\partial t}{\partial r} = h(t - t_\infty); \nabla r = 0.15 / M$$

Forward difference format equations (one node):

$$\rho c\frac{4}{3}\pi(\frac{\nabla r}{2})^3\frac{t_0^{i+1}-t_0^i}{\nabla r}=4\pi\lambda(\frac{\nabla r}{2})^2\frac{t_1^i-t_0^i}{\nabla r}$$

$$\Rightarrow Fo=\frac{a\nabla\tau}{\nabla r^2}\le\frac{1}{6}$$

Mnodes:

$$\rho c\frac{4}{3}\pi\{[(m+\frac{1}{2})\nabla r]^3-[(m-\frac{1}{2})\nabla r]^3\}\frac{t_m^{i+1}-t_m^i}{\nabla\tau}$$
$$=4\pi\lambda[(m-\frac{1}{2})\nabla r]^2\frac{t_{m-1}^{i+1}-t_m^i}{\nabla r}+4\pi\lambda[(m+\frac{1}{2})\nabla r]^2\frac{t_{m+1}^i-t_m^i}{\nabla r}$$

Convergence condition:

$$1-\frac{8m^2+2}{4m^2+\frac{1}{3}}F_0\ge 0$$

Eleven nodes:

$$\rho c\frac{4}{3}\pi\{(m\nabla r)^3-[(m-\frac{1}{2})\nabla r]^3\}\frac{t_m^{i+1}-t_m^i}{\nabla\tau}$$
$$=4\pi\lambda[(m-\frac{1}{2})\nabla r]^2\frac{t_{m-1}^{i+1}-t_m^i}{\nabla r}+h4\pi\lambda m^2\nabla r^2(t_\infty-t_m^i)$$

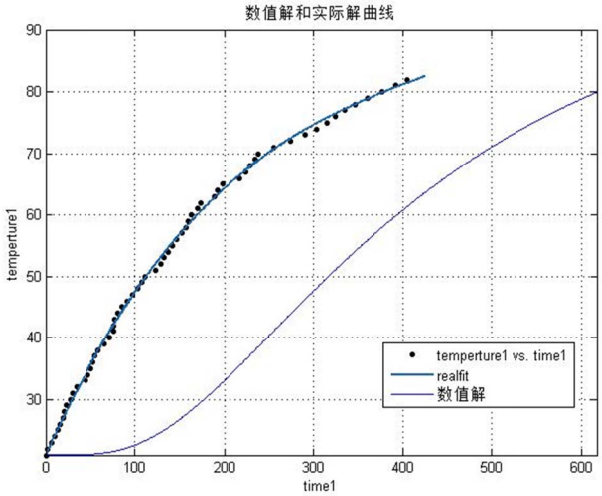

Fig. 4 the regional discrete inside the egg

Because the region is divided into 11 nodes, it gets the Fo=0.053 which meets the convergence criteria, therefore, the iteration result can be obtained by writing MATLAB program as shown in Table 2.

Table 2 rror between the images of numerical method and the real one

Node	1	2	3	4	5	6	7	8	9	10	11
Temperature℃	99.5276	97.3078	94.8187	92.1754	89.5078	86.9522	84.6427	82.7023	81.236	80.3232	80.0144

Plotting the curves:

From the results, there is still great error between the images of numerical method and the real one. And at the beginning, there is a temperature change delay in the numerical image. Through the analysis of heat conduction model, it concludes that because it is assumed that the coefficient of convective heat transfer of boiling water is a constant, while the actual convective heat transfer coefficient may be a change in value, and the egg is assumed as a pure thermal conductivity model but actually it exists two states: convection heat transfer of egg white and conduction after solidification of eggs.

6. Conclusion

Substances in eggs is complex, thermal conductivity, surface emissivity, inaccuracy can cause errors. Analysis error comes from the whole egg assumption and thermocouple movement within the egg.

References

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