

The comparison of bullwhip effect between AR(1) and ARMA(1,1) demand in supply chain

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Abstract. This paper constructs a supply chain consisting of two retailers and one supplier. Meantime retailers face two kinds of demand models there are AR (1) demand and ARMA (1, 1) demand. In order to choose the better demand model, we emphatically analysis the impact of the market competition parameters between two retailers and compare the bullwhip effect of two demand model. The result obtained that the value of coefficient in ARMA(1,1) demand is the main factor of choosing demand model in market competition.

1. Introduction

In supply chain, a well-known phenomenon is called bullwhip effect, which can be explained that the correct information and actual demand is amplified or distorted form the downstream to the upstream. The earliest studies can be traced back to [1]. Recently, many researches on the impact of bullwhip effect have focused on the demand model or forecasting method on supply chain . Generally, these major method that have been researching is: Exponential-smoothing [2,3], Moving-average [4], First-order autoregressive [5]. In addition, [6] comparison the bullwhip effect when using MA, ARIMA and EWMA method. In these papers, the supply chain consists of one retailers and one supplier. But in [7], a new supply chain working with two retailers and one supplier is established and investigate the effects of parameters on bullwhip effect .

According to above studies, we parallels the research of [6,7] on bullwhip effect. The main difference from previous studies is the comparison analysis of the bullwhip effect in supply chain between AR(1) demand and ARMA(1,1) demand. Our analyses show that in market competition the value of bullwhip effect is affected by the value of coefficient in ARMA(1,1) demand.

The paper is organized as follows: Section 2 establish supply chain two demand model. Section 3 measure the bullwhip effect with two demand model. Section 4 analysis the impact of the market competition parameters and compare the bullwhip effect. Finally in section 5 conclude the full text and proposal for the further studies.

2. Supply chain model

2.1 demand modeling

Firstly, we consider that the two retailers face AR(1) demand model:

$$D_{i,t} = \delta_{i,1} + \phi_i D_{i,t-1} + \varepsilon_{i,t} \quad (i = 1, 2) \quad (1)$$

Where, $D_{i,t}$ is the demand during period t ; $\delta_{i,1}$ is a basic demand constant for retailers, $\varepsilon_{i,t}$ is a normally distributed random error with mean 0 and variance σ_1^2 .

$$\text{And, } E(D_{i,t}) = E(D_{i,t-1}) = \frac{\delta_i}{1 - \phi_i}, \quad \text{Var}(D_{i,t}) = \text{Var}(D_{i,t-1}) = \frac{\sigma_i^2}{1 - \phi_i^2}.$$

In ARMA(1,1) demand, the formulate that retailers faced as :

$$D_{i,t} = \delta_{i,1} + \phi_i D_{i,t-1} + \varepsilon_{i,t} - \theta_1 \varepsilon_{i,t-1} \quad (i = 1, 2) \quad (2)$$

$$\text{And, } E(D_{i,t}) = E(D_{i,t-1}) = \frac{\delta_i}{1 - \phi_i}, \quad \text{Var}(D_{i,t}) = \text{Var}(D_{i,t-1}) = \frac{1 + \theta_1^2 - 2\phi_i \theta_1}{1 - \phi_i^2}.$$

2.2 Inventory policy

We assume that two retailers apply the order-up-to inventory policy in this supply chain, it can be given as :

$$q_{i,t} = S_{i,t} - S_{i,t-1} + D_{i,t-1}, \quad (3)$$

and $S_{i,t} = \hat{D}_{i,t} + z \hat{\sigma}_{i,t}$, as proved [5] that $\hat{\sigma}_{i,t}$ does not depend on t and has no influence on the bullwhip effect. So

$$q_{i,t} = S_{i,t} - S_{i,t-1} + D_{i,t-1} = \hat{D}_{i,t} - \hat{D}_{i,t-1} + D_{i,t-1}. \quad (4)$$

2.3 Forecasting method

Using the MA forecasting method, the forecast-updating demand for retailers is given as below:

$$\hat{D}_{i,t} = \frac{L}{k} \sum_{i=1}^k D_{i,t-i}, \quad (5)$$

Here k is the span (number of date points) for the MA forecast.

The two retailers order quantity of period t under the MA forecasting method is:

$$q_t = q_{1,t} + q_{2,t} = \left(1 + \frac{L_1}{k}\right) D_{1,t-1} - \frac{L_1}{k} D_{1,t-k-1} + \left(1 + \frac{L_2}{k}\right) D_{2,t-1} - \frac{L_2}{k} D_{2,t-k-1}. \quad (6)$$

3. Measure of bullwhip effect

Total demand of two retailers face is:

$$D_t = D_{1,t} + D_{2,t}. \quad (7)$$

Thus , the variance of D_t is written as:

$$Var(D_t) = Var(D_{1,t} + D_{2,t}) = Var(D_{1,t}) + Var(D_{2,t}) + 2Cov(D_{1,t}, D_{2,t}). \quad (8)$$

We assume the correlation coefficient is ψ ($-1 \leq \psi \leq 0$) that presents the degree of market competition. The greater the absolute value of ψ , the fiercer of the market competition.

Then, we know that

$$Cov(D_{1,t}, D_{2,t}) = \psi \sqrt{Var(D_{1,t})} \sqrt{Var(D_{2,t})}.$$

$$\text{And } Var(D_t) = Var(D_{1,t} + D_{2,t}) = Var(D_{1,t}) + Var(D_{2,t}) + 2\psi \sqrt{Var(D_{1,t})} \sqrt{Var(D_{2,t})}.$$

For AR(1) demand model :

$$\begin{aligned} Var(q_t) &= \left(1 + \frac{L_1}{k}\right)^2 + \left(\frac{L_1}{k}\right)^2 - 2\frac{L_1}{k} \left(1 + \frac{L_1}{k}\right) \phi_1^k Var(D_{1,t}) \\ &\quad + \left(1 + \frac{L_2}{k}\right)^2 + \left(\frac{L_2}{k}\right)^2 - 2\frac{L_2}{k} \left(1 + \frac{L_2}{k}\right) \phi_2^k Var(D_{2,t}) \\ &\quad + 2\left(\left(1 + \frac{L_1}{k}\right)\left(1 + \frac{L_2}{k}\right) - \frac{L_2}{k} \left(1 + \frac{L_1}{k}\right) \phi_1^k - \frac{L_1}{k} \left(1 + \frac{L_2}{k}\right) \phi_2^k + \frac{L_1 L_2}{k^2}\right) \psi \sqrt{Var(D_{1,t})} \sqrt{Var(D_{2,t})}. \\ &= H_1 Var(D_{1,t}) + H_2 Var(D_{2,t}) + H_3 \psi \sqrt{Var(D_{1,t})} \sqrt{Var(D_{2,t})} \end{aligned} \quad (9)$$

For ARMA(1,1) demand model :

$$\begin{aligned} Var(q_t) &= Var(q_{1,t} + q_{2,t}) = \left(1 + \frac{L_1}{k}\right)^2 + \left(\frac{L_1}{k}\right)^2 - 2\left(1 + \frac{L_1}{k}\right) \frac{L_1}{k} \frac{(\phi_1 - \theta_1)(1 - \phi_1 \theta_1)}{1 + \theta_1^2 - 2\phi_1 \theta_1} \phi_1^{k-1} Var(D_{1,t}) \\ &\quad + \left(1 + \frac{L_2}{k}\right)^2 + \left(\frac{L_2}{k}\right)^2 - 2\left(1 + \frac{L_2}{k}\right) \frac{L_2}{k} \frac{(\phi_2 - \theta_2)(1 - \phi_2 \theta_2)}{1 + \theta_2^2 - 2\phi_2 \theta_2} \phi_2^{k-1} Var(D_{2,t}) \\ &\quad + 2\left(\left(1 + \frac{L_1}{k}\right)\left(1 + \frac{L_2}{k}\right) - \frac{L_2}{k} \left(1 + \frac{L_1}{k}\right) \phi_1^k - \frac{L_1}{k} \left(1 + \frac{L_2}{k}\right) \phi_2^k + \frac{L_1 L_2}{k^2}\right) \psi \sqrt{Var(D_{1,t})} \sqrt{Var(D_{2,t})} \\ &= H_1 Var(D_{1,t}) + H_2 Var(D_{2,t}) + H_3 \psi \sqrt{Var(D_{1,t})} \sqrt{Var(D_{2,t})} \end{aligned} \quad (10)$$

Because the bullwhip effect can be defined the variance of upstream members' orders ratio the variance of actual downstream demand during the lead time. Hence, the bullwhip under the MA forecasting method with AR(1) or ARMA(1,1) demand model for two retailers can be determine as:

$$\begin{aligned}
 BWE &= \frac{Var(q_t)}{Var(D_t)} = \frac{H_1 Var(D_{1,t}) + H_2 Var(D_{2,t}) + H_3 \psi \sqrt{Var(D_{1,t}) Var(D_{2,t})}}{Var(D_{1,t}) + Var(D_{2,t}) + 2\psi \sqrt{Var(D_{1,t}) Var(D_{2,t})}} \\
 &= \frac{H_1 + H_2 \gamma^2 + H_3 \psi \gamma}{1 + \gamma^2 + 2\psi \gamma},
 \end{aligned} \tag{11}$$

Here, $\gamma = \sqrt{\frac{Var(D_{2,t})}{Var(D_{1,t})}}$, which means the consistency of demand volatility between two retailers.

4. The comparison of bullwhip effect

Fig.1 compares the bullwhip effect under two demand models. It simulates the bullwhip effect with the change of market competition as the function of the series parameter ψ both in AR(1) and ARMA(1,1). The two graphs indicate the value of bullwhip effect is different while the other same parameters ($\delta, \varepsilon, \gamma$) existing in supply chain. When the parameter θ is positive, the bullwhip effect in the ARMA(1,1) demand is smaller than the AR(1) demand. When the parameter θ is negative, the bullwhip effect in the AR(1) demand is smaller than the ARMA(1,1) demand. The managerial insight is that if there existed completion between the retailers in the market, the upstream can advice the downstream to choose the optimization demand model according the value of parameter θ , so that can minimize the bullwhip effect through the whole supply chain.

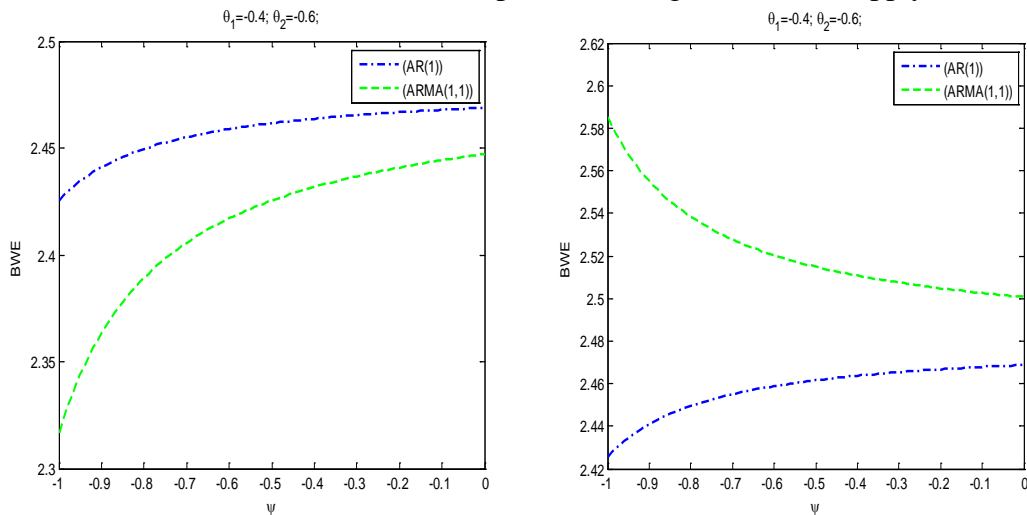


Fig.1 The comparison of the bullwhip effect in AR(1) and in ARMA(1,1)

5. Conclusion

The impact of demand model on bullwhip effect have been studies in previous literature. However the preference for using one or other of demand model in a supply chain that working with two retailers and one supplier is limited. In this paper, in order to can find out the appropriate demand model that will result into minimum the bullwhip whip of the supply chain, we analysis the impact of the market competition parameters and compared the bullwhip effect of two different demand model. It is proposed that the value of coefficient in ARMA(1,1) demand is the main influenced factor in the case of choosing AR(1) demand or ARMA(1,1) demand in market competition. Further study may can include the comparison between other demand model with different forecast-updating method as MA,AR(1) or EWMA, besides the future research may analyze the impact of the other parameters based on this paper.

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